

EF2452: Mathematics for Economics and Finance

Lecture 8

Part I of II

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Main Topics

- Matrix algebra: addition, multiplication, transpose

What is a Matrix?

- A matrix is an array of numbers

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix}$$

- The above matrix A has m rows and n columns; it is an $m \times n$ matrix. Its size is $m \times n$.

Matrix: Multiplying by a number

- If k is a number, we define kA to be the matrix whose elements are k times those of A ; that is kA has (i,j) th entry ka_{ij} . For example,

$$3 \begin{pmatrix} 3 & 8 & 1 & 3 \\ 4 & 1 & 3 & 5 \\ 2 & 5 & 2 & 3 \end{pmatrix} = \begin{pmatrix} 9 & 24 & 3 & 9 \\ 12 & 3 & 9 & 15 \\ 6 & 15 & 6 & 9 \end{pmatrix}$$

Matrix Addition

- If A and B are two matrices of the same size, then we define $A + B$ to be the matrix whose elements are the sums of the corresponding elements in A and B .
- The (i,j) th entry of the matrix $A + B$ is $a_{ij} + b_{ij}$, where a_{ij} and b_{ij} are the (i,j) th entries of A and B , respectively.
- For example,

$$\begin{pmatrix} 3 & 8 & 1 & 3 \\ 4 & 1 & 3 & 5 \\ 2 & 5 & 2 & 3 \end{pmatrix} + \begin{pmatrix} 2 & 1 & 7 & 4 \\ 4 & 3 & 2 & 1 \\ 2 & 1 & -3 & 6 \end{pmatrix}$$
$$= \begin{pmatrix} 3+2 & 8+1 & 1+7 & 3+4 \\ 4+4 & 1+3 & 3+2 & 5+1 \\ 2+2 & 5+1 & 2-3 & 3+6 \end{pmatrix} = \begin{pmatrix} 5 & 9 & 8 & 7 \\ 8 & 4 & 5 & 6 \\ 4 & 6 & -1 & 9 \end{pmatrix}$$

Matrix Addition

$$A + (B + C) = (A + B) + C$$

$$k(A + B) = kA + kB$$

Matrix Multiplication

- Suppose A and B are matrices such that the number (say n) of columns of A is equal to the number of rows of B , then the product is defined. Otherwise, A and B cannot be multiplied.
- If you multiply a matrix of size $m \times n$ and $n \times p$, then the resulting matrix is of size $m \times p$.
- For example, consider the following matrices (with A being of size 2×3 and B being of size 3×4)

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix} \quad B = \begin{pmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \end{pmatrix}$$

- Then the matrix product is the following (with size 2×4)

$$A = \begin{pmatrix} c_{11} & c_{12} & c_{13} & c_{14} \\ c_{21} & c_{22} & c_{23} & c_{24} \end{pmatrix}$$

Matrix Multiplication

$$c_{11} = (a_{11} \times b_{11}) + (a_{12} \times b_{21}) + (a_{13} \times b_{31})$$

$$c_{12} = (a_{11} \times b_{12}) + (a_{12} \times b_{22}) + (a_{13} \times b_{32})$$

$$c_{13} = (a_{11} \times b_{13}) + (a_{12} \times b_{23}) + (a_{13} \times b_{33})$$

$$c_{14} = (a_{11} \times b_{14}) + (a_{12} \times b_{24}) + (a_{13} \times b_{34})$$

$$c_{21} = (a_{21} \times b_{11}) + (a_{22} \times b_{21}) + (a_{23} \times b_{31})$$

$$c_{22} = (a_{21} \times b_{12}) + (a_{22} \times b_{22}) + (a_{23} \times b_{32})$$

$$c_{23} = (a_{21} \times b_{13}) + (a_{22} \times b_{23}) + (a_{23} \times b_{33})$$

$$c_{24} = (a_{21} \times b_{14}) + (a_{22} \times b_{24}) + (a_{23} \times b_{34})$$

Matrix Multiplication

$$A(BC) = (AB)C$$

$$A(B + C) = AB + AC$$

$$AB \neq BA$$

For example, if

$$A = \begin{pmatrix} 5 & 1 \\ 2 & 3 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

then

$$AB = \begin{pmatrix} 8 & 14 \\ 11 & 16 \end{pmatrix} \quad BA = \begin{pmatrix} 9 & 7 \\ 23 & 15 \end{pmatrix}$$

Transpose of a Matrix

- The transpose of a matrix is an operator that switches the rows and columns so that the matrix flips over its diagonal.
- Suppose we have

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}$$

- Then, the transpose of A is given by

$$A^T = \begin{pmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{pmatrix}$$

- Note that

$$(A^T)^T = A$$

Identity Matrix

- A particularly useful matrix is the **identity matrix**, which has the number 1 in each of the positions on the ‘main diagonal’ and 0 elsewhere

$$I = \begin{pmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \end{pmatrix}$$

- Note that I is a square matrix; the number of rows equals the number of columns.

References

Following chapters of the textbook:

15 Matrix Algebra