

Tutorial 10
IP Part 1

Question 1

Harrison Electric Company produces two expensive products that are popular with renovators of historic old homes: ornate lamps and old-fashioned ceiling fans. Both lamps and ceiling fans require a two-step production process involving wiring and assembly time. It takes about 2 hours to wire each lamp and 3 hours to wire a ceiling fan. Final assembly of each lamp and fan requires 6 and 5 hours, respectively. The production capability in this period is such that only 12 hours of wiring time and 30 hours of assembly time are available. Each lamp produced nets the firm \$600, and each fan nets \$700 in profit.

- (a) Formulate the LP relaxation problem.
- (b) Solve the problem graphically and interpret the results.
- (c) Create the spreadsheet to solve this model.

Solutions:

- (a) If we let L denote the number of lamps to make and F denote the number of ceiling fans to make, Harrison's product mix decision can be formulated using LP as follows:

$$\text{Maximize profit} = \$600L + \$700F$$

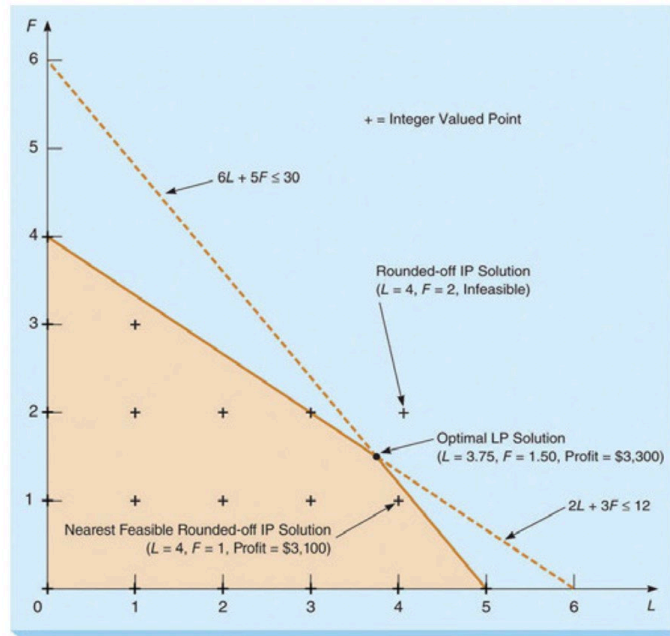
subject to the constraints:

$$2L + 3F \leq 12 \text{ (wiring hours)}$$

$$6L + 5F \leq 30 \text{ (assembly hours)}$$

$$L, F \geq 0$$

- (b) Because there are only two decision variables, let's employ the graphical approach to visualize the feasible region. The shaded region in Figure below shows the feasible region for the LP problem. The optimal corner point solution turns out to be $L = 3.75$ lamps and $F = 1.50$ ceiling fans, for a profit of \$3,300.



Since the company cannot produce and see a fraction of a product, this problem is actually an IP problem. First, if we use the traditional rounding rule (i.e., round down if the fraction is below 0.5 and round up otherwise), the resulting integer solution may not even be in the feasible region. For example, using this rule, we would round Harrison's LP solution to $L = 4$ lamps and $F = 2$ fans. As we can see from the figure, that solution is not feasible. It is, hence, not a practical answer. Second, even if we manage to round the LP solution in such a way that the resulting integer solution is feasible, there is no guarantee that it is the optimal IP solution.

You may also want to try listing the entire set of integer-valued solutions for this problem.

- (c) Set up this IP problem in Excel.

Question 2

Solutions:

X_1 = number of larger posters

X_2 = number of smaller posters

Maximize profit = $3X_1 + 2X_2$

subject to

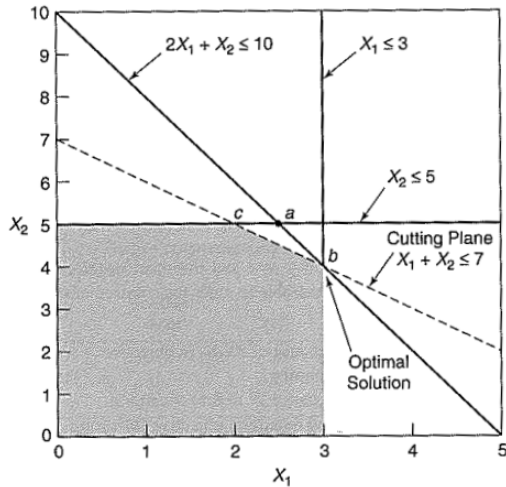
$$X_1 \leq 3$$

$$X_2 \leq 5$$

$$2X_1 + X_2 \leq 10$$

$$X_1, X_2 \geq 0$$

See graph below



Optimal LP solution at a is $(X_1 = 2.5, X_2 = 5, \text{profit} = \$17.5)$ while Integer solution at b is $(X_1 = 3, X_2 = 4, \text{profit} = \$17)$. Integer solution at c is $(X_1 = 2, X_2 = 5, \text{profit} = \$16)$. Hence the optimal integer solution is $X_1 = 3$ large posters and $X_2 = 4$ small posters (point b).

Question 3

Solutions:

(a) Maximize profit = $\$85X + \$1.5Y$

Subject to

$$30X + 0.5Y \leq 2,000$$

$$18X + 0.4Y \leq 800$$

$$2X + 0.1Y \leq 200$$

$$X, Y \geq 0 \text{ and } X \text{ integer}$$

Note that Y represents the bulk weight of hexall and is not required to be integer valued.

(b) The solution to Bagwell's problem is to produce 44 bags of xylene and 20 pounds of hexall, yielding a profit of \$3,770.