

2 Minimization using Quine-McCluskey (QM) Method

- Developed by W. V. Quine and E. J. McCluskey in 1956
- Functionally identical to Karnaugh map
- More efficient in computer algorithms
- Ease to handle large number of variables

For number of variables is less than 4, we use K-map; otherwise, QM method will be more efficient.

Procedure of QM-method

- Partitioning
- Combining
- Identifying Prime Implicants (PI)
- Generating PI chart
- Reducing chart
- Reporting result

Partitioning

Simplify $f(a,b,c,d) = \sum m(1,4,5,6,8,9,10,12,14)$

List all minterms of the function first

Minterms	<i>abcd</i>
m_1	0001
m_4	0100
m_5	0101
m_6	0110
m_8	1000
m_9	1001
m_{10}	1010
m_{12}	1100
m_{14}	1110



Partition them into groups

Minterms	<i>abcd</i>	
m_1	0001	} 1 1s
m_4	0100	
m_8	1000	
m_5	0101	} 2 1s
m_6	0110	
m_9	1001	
m_{10}	1010	
m_{12}	1100	} 3 1s
m_{14}	1110	

Combining

Compare the partitioned terms that follows Gary code property

Combine adjacent group implicants into $(n-1)$ -variable implicants

Minterms	<i>abcd</i>
m_1	0001 ✓
m_4	0100 ✓
m_8	1000 ✓
m_5	0101 ✓
m_6	0110 ✓
m_9	1001 ✓
m_{10}	1010 ✓
m_{12}	1100 ✓
m_{14}	1110 ✓

implicants

Mark the changed bit as “-” and give a ✓ to the combined implicants

Minterms	<i>abcd</i>
m_1, m_5	0-01
m_1, m_9	-001
m_4, m_5	010-
m_4, m_6	01-0
m_4, m_{12}	-100
m_8, m_9	100-
m_8, m_{10}	10-0
m_8, m_{12}	1-00
m_6, m_{14}	-110
m_{10}, m_{14}	1-10
m_{12}, m_{14}	11-0

implicants

0001
0101

Combining

Further compare the new partitioned terms that follows Gary code property

Further combine adjacent group implicants into $(n-2)$ -variable implicants

Minterms	<i>abcd</i>
m_1, m_5	0-01
m_1, m_9	-001
m_4, m_5	010-
m_4, m_6	01-0 ✓
m_4, m_{12}	-100 ✓
m_8, m_9	100-
m_8, m_{10}	10-0 ✓
m_8, m_{12}	1-00 ✓
m_6, m_{14}	-110 ✓
m_{10}, m_{14}	1-10 ✓
m_{12}, m_{14}	11-0 ✓

Again, mark the changed bit as “-” and give a ✓ to the combined implicants

Minterms	<i>abcd</i>
m_4, m_6, m_{12}, m_{14}	-1-0
$m_8, m_{10}, m_{12}, m_{14}$	1--0

No more combination

$$\begin{array}{ll}
 m_4, m_6: \underline{0}1-0 & m_4, m_{12}: -1\underline{0}0 \\
 m_{12}, m_{14}: \underline{1}1-0 & m_6, m_{14}: -1\underline{1}0
 \end{array}$$

Identifying Prime Implicants (PI)

Minterms	<i>abcd</i>	Minterms	<i>abcd</i>	Minterms	<i>abcd</i>
m_1	0001 ✓	m_1, m_5	0-01 PI_3	m_4, m_6, m_{12}, m_{14}	-1-0 PI_1
m_4	0100 ✓	m_1, m_9	-001 PI_4	$m_8, m_{10}, m_{12}, m_{14}$	1--0 PI_2
m_8	1000 ✓	m_4, m_5	010- PI_5	The remaining unmarked implicants are prime implicants. Label them as PI_i (label the rightmost column first)	
m_5	0101 ✓	m_4, m_6	01-0 ✓		
m_6	0110 ✓	m_4, m_{12}	-100 ✓		
m_9	1001 ✓	m_8, m_9	100- PI_6		
m_{10}	1010 ✓	m_8, m_{10}	10-0 ✓		
m_{12}	1100 ✓	m_8, m_{12}	1-00 ✓		
m_{14}	1110 ✓	m_6, m_{14}	-110 ✓		
		m_{10}, m_{14}	1-10 ✓		
		m_{12}, m_{14}	11-0 ✓		

Generating PI chart

Mark an X in the corresponding columns if this PI covers that minterms
e.g. PI_1 involves m_4, m_6, m_{12}, m_{14}

Essential
Prime
implicants

PI	Numeric	1	4	5	6	8	9	10	12	14
PI_1	-1-0		x		x				x	x
PI_2	1--0					x		x	x	x
PI_3	0-01	x		x						
PI_4	-001	x					x			
PI_5	010-		x	x						
PI_6	100-					x	x			

These two columns have one X only !
Minterms 6 & 10 are covered by only one prime implicant
i.e. PI_1 and PI_2 are essential prime implicants

Reducing chart

Reduce the chart by removing those rows of essential prime implicants and columns that covered by them

PI	Numeric	1	4	5	6	8	9	10	12	14
PI_1	-1-0				⊗					
PI_2	1--0					x		⊗	x	x
PI_3	0-01	x		x						
PI_4	-001	x					x			
PI_5	010-		x	x						
PI_6	100-					x	x			

Reducing chart

The reduced PI chart may not have any essential prime implicants

PI	Numeric	1	5	9
PI_3	0-01	x	x	
PI_4	-001	x		x
PI_5	010-		x	
PI_6	100-			x

How to further reduce the chart ?

Reducing chart (covering rule)

Now go back to the reduced PI chart

PI	Numeric	1	5	9
PI_3	0-01	x	x	
PI_4	-001	x		x
PI_5	010-		x	
PI_6	100-			x



			✓	✓	✓

By choosing PI_3 , m_5 is covered (PI_5 is not required)
 By choosing PI_4 , m_9 is covered (PI_6 is not required)

PI_3 and PI_4 have been selected to cover Minterms (m_1, m_5, m_9)

So combining with previous result (all rows with ✓) the minimized function is:

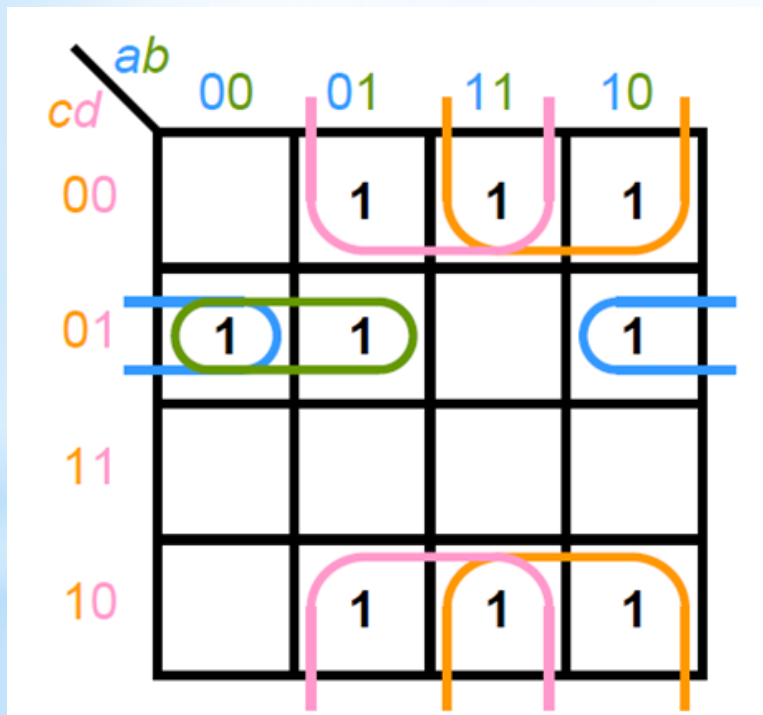
$$\begin{aligned}
 f(a, b, c, d) &= PI_1 + PI_2 + PI_3 + PI_4 \\
 &= -1-0 + 1--0 + 0-01 + -001 \\
 &= bd' + ad' + a'c'd + b'c'd
 \end{aligned}$$

or

$$\begin{aligned}
 f(a, b, c, d) &= PI_1 + PI_2 + PI_3 + PI_6 \\
 f(a, b, c, d) &= PI_1 + PI_2 + PI_4 + PI_5
 \end{aligned}$$

Verify result by K-map

Simplify $f(a,b,c,d) = \Sigma m(1,4,5,6,8,9,10,12,14)$



$$f(a, b, c, d) = bd' + ad' + a'c'd + b'c'd$$

Don't Care Conditions

How to minimize incompletely specified functions using QM method?

- The first three steps are the same (list, partition and combine)
- But do **NOT** list the don't care minterms in the PI chart in step 4

The reason is

- We set don't care terms to minterms so as to find PIs
- But omit them as the don't care terms are not essential to be covered

Example: Don't Care

Simplify $f(a,b,c,d) = \Sigma m(4,8,9,10,12,15) + \Sigma d(2,6,13)$

The don't care terms are listed together

Minterms	<i>abcd</i>
m_2	0010
m_4	0100
m_6	0110
m_8	1000
m_9	1001
m_{10}	1010
m_{12}	1100
m_{13}	1101
m_{15}	1111



Partition the terms as usual

Minterms	<i>abcd</i>
m_2	0010
m_4	0100
m_8	1000
m_6	0110
m_9	1001
m_{10}	1010
m_{12}	1100
m_{13}	1101
m_{15}	1111

Example: Don't Care Conditions

Combine them to form prime implicants

Minterms	<i>abcd</i>	Minterms	<i>abcd</i>	Minterms	<i>abcd</i>
m_2	0010 ✓	m_2, m_6	0-10 PI_2	m_8, m_9, m_{12}, m_{13}	1-0- PI_1
m_4	0100 ✓	m_2, m_{10}	-010 PI_3		
m_8	1000 ✓	m_4, m_6	01-0 PI_4		
m_6	0110 ✓	m_4, m_{12}	-100 PI_5		
m_9	1001 ✓	m_8, m_9	100- ✓		
m_{10}	1010 ✓	m_8, m_{10}	10-0 PI_6		
m_{12}	1100 ✓	m_8, m_{12}	1-00 ✓		
m_{13}	1101 ✓	m_9, m_{13}	1-01 ✓		
m_{15}	1111 ✓	m_{12}, m_{13}	110- ✓		
		m_{13}, m_{15}	11-1 PI_7		

Example: Don't Care Conditions

Note that the don't care terms m_2 , m_6 , m_{13} are not listed in this chart !!

Find the essential prime implicants and reduce the chart as usual

PI	Numeric	4	8	9	10	12	15
PI_1	1-0-		x	(x)		x	
PI_2	0-10						
PI_3	-010				x		
PI_4	01-0	x					
PI_5	-100	x				x	
PI_6	10-0		x		x		
PI_7	11-1						(x)



PI	Numeric	4	10
PI_3	-010		x
PI_4	01-0	x	
PI_5	-100	x	
PI_6	10-0		x



$$\begin{aligned}
 f(a, b, c, d) &= PI_1 + PI_3 + PI_4 + PI_7 \\
 &= 1-0- + -010 + 01-0 + 11-1 \\
 &= ac' + b'cd' + a'bd' + abd
 \end{aligned}$$



PI	Numeric	4	10
PI_3	-010		x
PI_4	01-0	x	