

Tutorial 8
Linear Programming Part 2**Question 1**

A bank employs 12 full-time tellers. Part-time employee (working four hours per day) are also available. Tell requirements of each hour are as follow:

Time Period	# of Tellers Required
9am – 10am	10
10am – 11am	12
11am – 12noon	14
12noon – 1pm	16
1pm – 2pm	18
2pm – 3pm	17
3pm – 4pm	15
4pm – 5pm	10

According to the regulations and contract, full-timers:

- work from 9am to 5pm
- allowed 1 hour for lunch
- half of full-timers eat at 11am and the other half at noon
- full-timers thus provide 35 hours per week of productive labor time
- earn \$1000 per day in salary and benefits, on average

To secure the position of full-timers, part-time hours employed are limited to a maximum of 50% of a day's total requirement, and earn \$80 per hour (or \$320 per day) on average.

Determine the optimal employee scheduling that minimize the total daily labor cost.

Solutions:

Decision variables

F = no of full-time tellers

P1 = no of part-time tellers from 9am – 1pm

P2 = no of part-time tellers from 10am – 2pm

P3 = no of part-time tellers from 11am – 3pm

P4 = no of part-time tellers from 12noon – 4pm

P5 = no of part-time tellers from 1pm – 5pm

Objective

Minimize cost: $\$1000F + \$320(P1+P2+P3+P4+P5)$

CB2203 Data-Driven Business Modeling

Constraints

1st – 8th constraints: time period

9th – 10th constraints: FT limits & PT hour limits

11th constraint: non-negative

1. $F + P1 \geq 10$ (time period: 9-10)
2. $F + P1 + P2 \geq 12$ (time period: 10-11)
3. $0.5F + P1 + P2 + P3 \geq 14$ (time period: 11-12)
4. $0.5F + P1 + P2 + P3 + P4 \geq 16$ (time period: 12-1)
5. $F + P2 + P3 + P4 + P5 \geq 18$ (time period: 1-2)
6. $F + P3 + P4 + P5 \geq 17$ (time period: 2-3)
7. $F + P4 + P5 \geq 15$ (time period: 3-4)
8. $F + P5 \geq 10$ (time period: 4-5)
9. $F \leq 12$ (full time tellers limits)
10. $4(P1+P2+P3+P4+P5) \leq 56$ (part time working hours limits) ($56 = 112/2$)
11. $F, P1, P2, P3, P4, P5 \geq 0$

Question 2

The Weigelt Corporation has three branch plants with excess production capacity. Fortunately, the corporation has a new product ready to begin production, and all three plants have this capability, so some of the excess capacity can be used in this way. This product can be made in three sizes: large, medium, and small that yield a net unit profit of \$420, \$360, and \$300, respectively. Plants 1, 2, and 3 have the excess capacity to produce 750, 900, and 450 units per day of this product, respectively, regardless of the size or combination of sizes involved.

The amount of available in-process storage space also imposes a limitation on the production rates of the new product. Plants 1, 2, and 3 have 13,000, 12,000, and 5000 square feet, respectively, of in-process storage space available for a day's production of this product. Each unit of the large, medium, and small sizes produced per day requires 20, 15, and 12 square feet, respectively.

Sales forecasts indicate that if available, 900, 1,200, and 750 units of the large, medium, and small sizes, respectively, would be sold per day.

At each plant, some employees will need to be laid off unless most of the plant's excess production capacity can be used to produce the new product. To avoid layoffs if possible, management has decided that the plants should use the same percentage of their excess capacity to produce the new product.

Management wishes to know how much of each of the sizes should be produced by each of the plants to maximize profit. Formulate a linear program for this problem and solve it.

CB2203 Data-Driven Business Modeling

Solutions:

Decision Variables:

Let X_{ij} = number of units produced at plant i ($= 1, 2, 3$) of product j ($= L, M, S$).

Plant \ Product	L	M	S
1	X_{1L}	X_{1M}	X_{1S}
2	X_{2L}	X_{2M}	X_{2S}
3	X_{3L}	X_{3M}	X_{3S}

Objective:

$$\text{Max } Z = 420(X_{1L} + X_{2L} + X_{3L}) + 360(X_{1M} + X_{2M} + X_{3M}) + 300(X_{1S} + X_{2S} + X_{3S})$$

Subject to:

$$X_{1L} + X_{1M} + X_{1S} \leq 750 \quad (\text{plant 1's capacity})$$

$$X_{2L} + X_{2M} + X_{2S} \leq 900 \quad (\text{plant 2's capacity})$$

$$X_{3L} + X_{3M} + X_{3S} \leq 450 \quad (\text{plant 3's capacity})$$

$$X_{1L} + X_{2L} + X_{3L} \leq 900 \quad (\text{product L sold per day})$$

$$X_{1M} + X_{2M} + X_{3M} \leq 1200 \quad (\text{product M sold per day})$$

$$X_{1S} + X_{2S} + X_{3S} \leq 750 \quad (\text{product S sold per day})$$

$$20X_{1L} + 15X_{1M} + 12X_{1S} \leq 13,000 \quad (\text{plant 1's space})$$

$$20X_{2L} + 15X_{2M} + 12X_{2S} \leq 12,000 \quad (\text{plant 2's space})$$

$$20X_{3L} + 15X_{3M} + 12X_{3S} \leq 5000 \quad (\text{plant 3's space})$$

$$6X_{1L} + 6X_{1M} + 6X_{1S} - 5X_{2L} - 5X_{2M} - 5X_{2S} = 0 \quad (\text{from } \frac{X_{1L} + X_{1M} + X_{1S}}{750} = \frac{X_{2L} + X_{2M} + X_{2S}}{900} \text{ (layoff policy)})$$

$$3X_{1L} + 3X_{1M} + 3X_{1S} - 5X_{3L} - 5X_{3M} - 5X_{3S} = 0 \quad (\text{from } \frac{X_{1L} + X_{1M} + X_{1S}}{750} = \frac{X_{3L} + X_{3M} + X_{3S}}{450} \text{ (layoff policy)})$$

$$(\text{N.B.: } \frac{X_{1L} + X_{1M} + X_{1S}}{750} = \frac{X_{2L} + X_{2M} + X_{2S}}{900} = \frac{X_{3L} + X_{3M} + X_{3S}}{450}, \text{ choose any two equalities})$$

with all $X_{ij} \geq 0$

Question 3

The Harrison and Keith Brokerage firm has just been instructed by one of its clients to invest \$250,000 for her money obtained recently through the sale of land holdings in Ohio. The client has a good deal of trust in the investment house, but she also has her own ideas about the distribution of the funds being invested. In particular, she requests that the firm selects whatever stocks and bonds they believe are well rated, but within the following guidelines:

- (i) Municipal bonds should constitute at least 20% of the investment.
- (ii) At least 40% of the funds should be placed in a combination of electronics firms, aerospace firms, and drug manufacturers.
- (iii) No more than 50% of the amount invested in municipal bonds should be placed in a high-risk, high-yield nursing home stock.

Subject to these restraints, the client's goal is to maximize projected return on investments. The analysts at Harrison and Keith, aware of these guidelines, prepare a list of high-quality stocks and bonds and their corresponding rates of return.

INVESTMENT	PROJECTED RATE OF RETURN (%)
Los Angeles municipal bonds	5.3
Thompson Electronics, Inc.	6.8
United Aerospace Corp.	4.9
Palmer Drugs	8.4
Happy Days Nursing Homes	11.8

- (a) Formulate this portfolio selection problem using LP.
- (b) Solve this problem.

Solutions:

- (a) Let L be dollars invested in Los Angeles municipal bonds
 T be dollars invested in Thompson Electronics
 U be dollars invested in United Aerospace
 P be dollars invested in Palmer Drugs
 H be dollars invested in Happy Days Nursing Homes

Maximize return, $Z = 0.053L + 0.068T + 0.049U + 0.084P + 0.118H$

CB2203 Data-Driven Business Modeling

Subject to

1. Municipal bonds
 $L \geq 0.2(L + T + U + P + H)$
 $\rightarrow 0.8L - 0.2T - 0.2U - 0.2P - 0.2H \geq 0$ [Rewrite the equation]
2. Combination of electronics, aerospace, and drugs
 $T + U + P \geq 0.4(L + T + U + P + H)$
 $\rightarrow -0.4L + 0.6T + 0.6U + 0.6P - 0.4H \geq 0$ [Rewrite the equation]
3. Nursing home as percent of bonds
 $H \leq 0.5L$
 $\rightarrow H - 0.5L \leq 0$ [Rewrite the equation]
4. Fund to be invested
 $L + T + U + P + H \leq 250,000$
5. Non-negativity
 $L, T, U, P, H \geq 0$

(b) Using Excel's Solver:

Harrison and Keith Brokerage

	L	T	U	P	H			
	Los Angeles	Thompson	United	Palmer	Happy Days			
\$ invested	\$50,000.00	\$0.00	\$0.00	\$175,000.00	\$25,000.00	[Optimal solution of each decision variable]		
Return	5.30%	6.80%	4.90%	8.40%	11.80%	\$20,300.00	[Objective]	
Constraints								
Total	1	1	1	1	1	250000.0	<=	250000
Nursing	-0.5	0	0	0	1	0.0	<=	0
Electr, Aero, Drug	-0.4	0.6	0.6	0.6	-0.4	75000.0	>=	0
Muni Bonds	0.8	-0.2	-0.2	-0.2	-0.2	0.0	>=	0
						LHS	Sign	RHS

- Total returns = \$20,300
- \$50,000 invested in Los Angeles municipal bonds ($L = 50,000$)
- No investment in Thompson Electronics and United Aerospace ($T, U = 0$)
- \$175,000 invested in Palmer Drugs ($P = 175,000$)
- \$25,000 invested in Happy Days Nursing Homes ($H = 25,000$) per house each day.

Question 4

Comfortable Hands is a company that features a product line of winter gloves for the entire family: men, women, and children. They are trying to decide what mix of these three types of gloves to produce.

Comfortable Hands' manufacturing labor force is unionized. Each full-time employee works 40-hour per week. In addition, by union contract, the number of full-time employees can never drop below 20. Non-union, part-time workers also can be hired with the following union-imposed restrictions: (1) each part-time worker works 20 hours per week and (2) there must be at least two full-time employees for each part-time employee.

All three types of gloves are made out of the same 100 percent genuine cowhide leather. Comfortable Hands has a long-term contract with a supplier of the leather and receives a 5000-square-foot shipment of the material each week. The material requirements and labor requirements, along with the gross profit per glove sold (not considering labor costs), are given in the following table.

Glove	Material required (square feet)	Labor required (minutes)	Gross profit per pair (\$)
Men's	2	30	8
Women's	1.5	45	10
Children's	1	40	6

Each full-time employee earns \$13 per hour, while each part-time employee earns \$10 per hour. Management wishes to know what mix of each of the three types of gloves to produce per week, as well as how many full-time and part-time workers to employ. They would like to maximize their net profit (their gross profit from sales minus their labor costs). Formulate a linear program for this problem algebraically.

Solutions:

Let X_1, X_2, X_3 = number of men's, women's, children's gloves to be produced,
 Y_1, Y_2 = number of full-time, part-time workers to be employed.

$$\begin{aligned} \text{Max } Z &= 8X_1 + 10X_2 + 6X_3 - 13 \times 40Y_1 - 10 \times 20Y_2 \\ &= 8X_1 + 10X_2 + 6X_3 - 520Y_1 - 200Y_2 \end{aligned}$$

s.t.

$$2X_1 + 1.5X_2 + X_3 \leq 5000$$

$$30X_1 + 45X_2 + 40X_3 \leq 40 \times 60Y_1 + 20 \times 60Y_2$$

$$30X_1 + 45X_2 + 40X_3 - 2400Y_1 - 1200Y_2 \leq 0 \quad (\text{from } 30X_1 + 45X_2 + 40X_3 \leq 40 \times 60Y_1 + 20 \times 60Y_2)$$

$$Y_1 \geq 20$$

$$Y_1 - 2Y_2 \geq 0 \quad \text{from } \left(\frac{Y_1}{Y_2} \geq 2 \right)$$

$$X_i, Y_j \geq 0$$

Question 5

The Learning Center runs a day camp for 6-10 years old children during the summer. Its manager, Elizabeth Reed, is trying to reduce the center's operating costs to avoid having to raise the tuition fee. Elizabeth is currently planning what to feed the children for lunch. She would like to keep costs to a minimum, but also wants to make sure she is meeting the nutritional requirements of the children. She has already decided to go with peanut butter and jelly sandwiches, and some combination of apples, milk, and/or cranberry juice. The nutritional content of each food choice and its cost are given in the table that accompanies this problem.

Food item	Calories from fat	Total calories	Vitamin C (mg)	Fiber (g)	Cost (¢)
Bread (1 slice)	15	80	0	4	6
Peanut butter (1 tbsp)	80	100	0	0	5
Jelly (1 tbsp)	0	70	4	3	8
Apple	0	90	6	10	35
Milk (1 cup)	60	120	2	0	20
Cranberry juice (1 cup)	0	110	80	1	40

The nutritional requirements are as follows: Each child should receive between 300 and 500 calories, but no more than 30 percent of these calories should come from fat. Each child should receive at least 60 milligrams (mg) of vitamin C and at least 10 grams (g) of fiber.

To ensure tasty sandwiches, Elizabeth wants each child to have a minimum of 2 slices of bread, 1 tablespoon (tbsp) of peanut butter, and 1 tbsp of jelly, along with at least 1 cup of liquid (milk and/or cranberry juice). Elizabeth would like to select the food choices that would minimize cost while meeting all these requirements.

- Formulate a linear program for this problem algebraically.
- Solve it on a spreadsheet for this problem and interpret the result.

Solutions:

- Let B be slices of bread selected
 P be table spoons of the peanut butter selected
 J be table spoons of the jelly selected
 A be numbers of apple selected
 M be cups of milk selected
 C be cups of cranberry juice selected

Minimize Cost, $Z = 6B + 5P + 8J + 35A + 20M + 40C$

CB2203 Data-Driven Business Modeling

Subject to

- Total calories
 $80B + 100P + 70J + 90A + 120M + 110C \geq 300$
 $80B + 100P + 70J + 90A + 120M + 110C \leq 500$
- Calories from fat
 $15B + 80P + 60M \leq 30\% * (80B + 100P + 70J + 90A + 120M + 110C)$
 $\rightarrow -9B + 50P - 21J - 27A + 24M - 33C \leq 0$ [Rewrite the equation]
- Vitamin C
 $4J + 6A + 2M + 80C \geq 60$
- Fibber
 $4B + 3J + 10A + C \geq 10$
- Elizabeth requirement
 $B \geq 2$
 $P \geq 1$
 $J \geq 1$
 $A \geq 0$
 $M + C \geq 1$
- Non-negativity
 $B, P, J, A, M, C \geq 0$

(b) Using Excel's Solver:

	B Bread	P Peanut butter	J Jelly	A Apple	M Milk	C Cranberry juice			
Selected	2.00	1.00	1.00	0.00	0.31	0.69	Total Cost		
Cost	6	5	8	35	20	40	58.85	Object	
Constraints									
Total calories	80	100	70	90	120	110	443.08	>=	300
	80	100	70	90	120	110	443.08	<=	500
From fat	-9	50	-21	-27	24	-33	-4.46	<=	0
VC	0	0	4	6	2	80	60.00	>=	60
Fibber	4	0	3	10	0	1	11.69	>=	10
	1						2.00	>=	2
		1					1.00	>=	1
Requirement			1				1.00	>=	1
				1			0.00	>=	0
					1	1	1.00	>=	1
							LHS	Sign	RHS

- Total costs = ₱58.85
- 2 slices of bread selected
- 1 table spoons of the peanut butter selected
- 1 table spoons of the jelly selected

CB2203 Data-Driven Business Modeling

- 0 apple selected
- 0.31 cups of milk selected
- 0.69 cups of cranberry juice selected