

EE3109 Applied Electromagnetics
2022/23 Semester A
Test 1

Name: _____

Student #: _____

Date: _____

1. **(27 Marks)** The electromagnetic field of a wave propagating in an optical fiber can be approximated by a plane wave. A plane wave has a wavelength (in free-space) of 1550 nm and the fiber can be approximated by a material with $\epsilon_r = 2.25$ and $\mu_r = 1$. The wave travels in the +z-direction and the electric field points in the $(\hat{x} + \hat{y})$ -direction. Assume the optical fiber is lossless, answer the following questions:

- (a) What is the frequency of the wave? (2)
- (b) What is the propagation velocity of the plane wave inside the fiber? (2)
- (c) The plane wave has a maximum magnetic flux density of $B_0 = 1$ [μT]. What is the peak electric field amplitude? (3)
- (d) Write the electric field phasor of this plane wave. Assume the field has a phase of -45° at $t = 0$ [s] and $z = 0$ [m]. (4)
- (e) Write the corresponding magnetic field phasor. (4)
- (f) Write the time-domain electric field. (3)

You are now given in the information that this plane wave has a loss of 0.18 [dB/km].

- (g) Is the glass making up the fiber a good dielectric, a good conductor, or neither? (3)
- (h) Find the attenuation constant (α) and propagation constant (β). (3)
- (i) By how much (how many percent) does the electric field decay after a propagation of 100 km? (3)

Solution:

$$(a) f = \frac{c}{\lambda_0} = \frac{3 \times 10^8}{1550 \times 10^{-9}} = 193.5 \text{ [THz]}$$

$$(b) v_p = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = \frac{c}{\sqrt{\mu_r \epsilon_r}} = \frac{3 \times 10^8}{\sqrt{(1)(2.25)}} = \frac{3 \times 10^8}{1.5} = 2.0 \times 10^8 \text{ [m/s]}$$

- (c) First find the intrinsic impedance of fiber:

$$\eta_{fiber} = \sqrt{\frac{\mu_{fiber}}{\epsilon_{fiber}}} = \sqrt{\frac{\mu_0(1)}{\epsilon_0(2.25)}} = \sqrt{\frac{\mu_0}{\epsilon_0}} \sqrt{\frac{1}{2.25}} = \frac{120\pi}{1.5} = 80\pi \text{ } [\Omega]$$

Then the electric field amplitude is:

$$E_0 = H_0 \eta_{fiber} = \frac{B_0 \eta_{fiber}}{\mu_{fiber}} = \frac{(1 \times 10^{-6})(80\pi)}{4\pi \times 10^{-7}} = 200 \text{ [V/m]}$$

- (d) We first find the wavenumber k and the phase shift ϕ .

$$k = \frac{\omega}{v_p} = \frac{2\pi f}{\frac{c}{\sqrt{\epsilon_r}}} = \frac{2\pi f \sqrt{\epsilon_r}}{c} = \frac{2\pi \sqrt{\epsilon_r}}{\lambda_0} = \frac{(2\pi)(1.5)}{1550 \times 10^{-9}} = 6.081 \times 10^6 \text{ [rad/m]}$$

It is known that the phase of the electric field at $t = 0$ and $z = 0$ is -45° , so

$$\Phi|_{t=0, z=0} = \omega(0) - k(0) + \phi = -45^\circ \Rightarrow \phi = -45^\circ$$

We now write the phasor in general form and substitute in the variables:

$$\mathbf{E} = E_0 e^{j\phi} e^{-jkz} \left(\frac{1}{\sqrt{2}} (\hat{x} + \hat{y}) \right) = 100\sqrt{2} e^{-j\frac{\pi}{4}} e^{-j6.081 \times 10^6 z} (\hat{x} + \hat{y}) \text{ [V/m]}$$

- (e) We first write the entire H field then separately find its direction and magnitude.

$$\mathbf{H} = -\frac{1}{\eta_{fiber}} \mathbf{E} \times \hat{\mathbf{k}} = \left(\frac{E_0}{\eta_{fiber}} \right) e^{j\phi} e^{-jkz} (-\hat{\mathbf{E}} \times \hat{\mathbf{k}})$$

Direction:

$$-\hat{\mathbf{E}} \times \hat{\mathbf{k}} = -\frac{1}{\sqrt{2}} (\hat{\mathbf{x}} + \hat{\mathbf{y}}) \times \hat{\mathbf{z}} = -\frac{1}{\sqrt{2}} (-\hat{\mathbf{y}} + \hat{\mathbf{x}}) = \frac{1}{\sqrt{2}} (\hat{\mathbf{y}} - \hat{\mathbf{x}})$$

Magnitude:

$$\frac{E_0}{\eta_{fiber}} = \frac{200}{80\pi} = \frac{5}{2\pi} = 0.8276 \text{ [A/m]}$$

$$\Rightarrow \mathbf{H} = 827.6 e^{j\phi} e^{-jkz} \left(\frac{1}{\sqrt{2}} (\hat{\mathbf{y}} - \hat{\mathbf{x}}) \right) = 585.2 e^{-j\frac{\pi}{4}} e^{-j6.081 \times 10^6 z} (\hat{\mathbf{y}} - \hat{\mathbf{x}}) \text{ [mA/m]}$$

(f) We first find the frequency: $\omega = 2\pi f = 1.216 \times 10^{15} \text{ [Hz]}$. Then the electric field is

$$\begin{aligned} \mathbf{E} &= E_0 \cos(\omega t - kz + \phi) \left(\frac{1}{\sqrt{2}} (\hat{\mathbf{x}} + \hat{\mathbf{y}}) \right) \\ &= 100\sqrt{2} \cos(1.216 \times 10^{15} t - 6.081 \times 10^6 z - 45^\circ) (\hat{\mathbf{x}} + \hat{\mathbf{y}}) \text{ [V/m]} \end{aligned}$$

(h) We find α and β as follows

$$\alpha = \frac{0.18 \text{ dB}}{1 \text{ km}} = \left(\frac{0.18 \text{ dB}}{1 \text{ km}} \right) \left(\frac{0.1151 \text{ Np}}{1 \text{ dB}} \right) \left(\frac{1000 \text{ m}}{1 \text{ km}} \right) = 2.07 \times 10^{-15} \text{ [Np/m]}$$

$$\alpha^2 - \beta^2 = -\omega^2 \mu \epsilon' = -k^2$$

Here k is the same quantity as in the previous parts of this question. Rearrangement gives

$$\beta = \sqrt{\alpha^2 + k^2} = \sqrt{(2.07 \times 10^{-15})^2 + (6.081 \times 10^6)^2} = 6.081 \times 10^6 \text{ [rad/m]}$$

(g) Since $\beta \cong k$ and $\beta \gg \alpha$ in the above, we see that the material loss is really small. Therefore the material is a good dielectric / good insulator.

$$(i) \text{ Loss in dB} = \left(0.18 \frac{\text{dB}}{\text{km}} \right) (100 \text{ km}) = 18 \text{ dB}$$

$$\begin{aligned} 20 \log \left(\frac{\mathbf{E}(z = 100 \text{ m})}{\mathbf{E}(z = 0 \text{ m})} \right) &= -18 \text{ dB} \\ \Rightarrow \frac{\mathbf{E}(z = 100 \text{ m})}{\mathbf{E}(z = 0 \text{ m})} &= 10^{-\frac{18}{20}} = 0.1259 = 12.59\% \end{aligned}$$

Hence the decay in electric field amplitude is $1 - 12.59\% = 87.41\%$.

2. (13 Marks) Consider the magnetic field of the following plane wave:

$$\mathbf{H}(z, t) = M \cos(\omega t - kz + \phi) \hat{\mathbf{x}} + N \cos(\omega t - kz) \hat{\mathbf{y}}$$

- Under what condition(s) for M, N and ϕ will the wave be linearly polarized? (3)
- Under what condition(s) for M, N and ϕ will the wave be circularly polarized? (3)
- Trace the time-evolution of the electric field when $M = N = 7.958 \text{ [mA/m]}$, $\phi = 90^\circ$. (5)
- What is the polarization of the wave in (c)? (2)

Solution:

We note that the wave travels in the $+z$ direction. We first write the E-field:

$$\mathbf{E}(z, t) = \eta_0 \mathbf{H} \times \hat{\mathbf{z}} = M\eta_0 \cos(\omega t - kz + \phi)(\hat{\mathbf{x}} \times \hat{\mathbf{z}}) + N\eta_0 \cos(\omega t - kz)(\hat{\mathbf{y}} \times \hat{\mathbf{z}})$$

We note that $\hat{\mathbf{x}} \times \hat{\mathbf{z}} = -\hat{\mathbf{y}}$ and $\hat{\mathbf{y}} \times \hat{\mathbf{z}} = \hat{\mathbf{x}}$, so the electric field becomes

$$\begin{aligned} \mathbf{E}(z, t) &= N\eta_0 \cos(\omega t - kz) \hat{\mathbf{x}} - M\eta_0 \cos(\omega t - kz + \phi) \hat{\mathbf{y}} \\ &= N\eta_0 \cos(\omega t - kz) \hat{\mathbf{x}} + M\eta_0 \cos(\omega t - kz + (\phi + 180^\circ)) \hat{\mathbf{y}} \end{aligned}$$

- Linear polarization: $\delta = \phi + 180^\circ = (180^\circ)n \Rightarrow \phi = (180^\circ)n$ where $n \in \mathbb{Z}$

- Circular polarization:

$$N\eta_0 = M\eta_0 \Rightarrow M = N, \text{ and}$$

$$\delta = \phi + 180^\circ = 90^\circ + (180^\circ)n \Rightarrow \phi = 90^\circ + (180^\circ)n \text{ where } n \in \mathbb{Z}$$

- $E_0 = N\eta_0 = M\eta_0 = (7.958 \times 10^{-3})(377) = 3.000 \text{ [V/m]}$, $\delta = \phi + 180^\circ = -90^\circ$
Since $a_x = a_y$, $\delta = -90^\circ$ and the propagation is in the $+z$ direction, this plane wave is RHC polarized.

- $\mathbf{E}(z, t) = 3.000 \cos(\omega t - kz) \hat{\mathbf{x}} + 3.000 \cos(\omega t - kz - 90^\circ) \hat{\mathbf{y}} \text{ [V/m]}$

	$\omega t - kz$	$E_x \text{ [V/m]}$	$E_y \text{ [V/m]}$
t_0	0°	3	0
t_1	90°	0	3
t_2	180°	-3	0
t_3	270°	0	-3

