

Block A: Unit 3

One port networks and Dependent Sources

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Block A: Unit 3

To see the motivation for this unit, let us first refer to the circuit shown below:

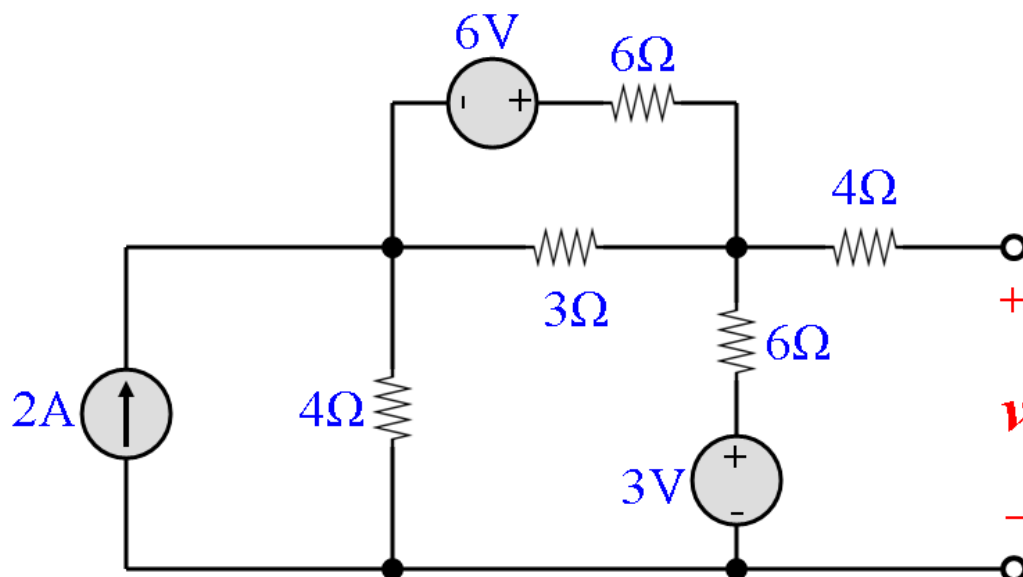


Fig 1: An example of a one port network

If you were given a number of different resistor loads to be connected to the terminal of this device, you would have to recalculate the whole circuit to find the current and voltage at the terminals for each load. A different load will give you difference output current & voltage.

If it was possible to transform any two terminal circuit into a circuit that is as simple as having one resistor and one source, that will make the analysis a lot simpler. If the transformed circuit was like the one below, then to

find the voltage for different loads, we just need to use voltage divider rule each time. Finding the current then requires simple Ohm's law.

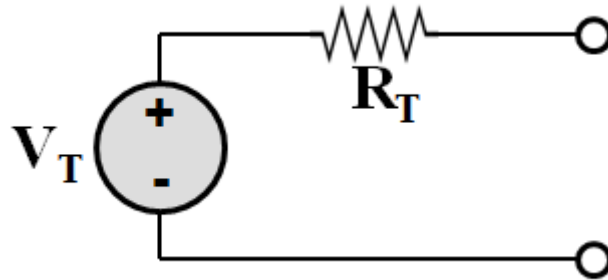


Fig 2: Transformed Thevenin equivalent circuit

It turns out this is indeed possible and you will learn how to do this in this unit. The first part of this unit is organized into the following 3 sections:

- Thevenin equivalent
- Norton equivalent
- *Source transformation

In the last part of this unit, we will take a slight detour and revisit all the concepts in Block A and apply them to dependent sources.

*Extension of the first two

I. Thevenin

Definition

Any one port network composed of **ideal** voltage and current sources, and **linear** resistors, can be represented by an equivalent circuit consisting of an ideal **voltage source** V_T in series with an equivalent resistance R_T . An example of the transformed circuit is shown in Fig 2.

Deriving the Thevenin equivalent circuit

Step 1: Remove the load from the rest of the one port network. This is the first and most fundamental step.

Step 2: Find the equivalent resistance R_T

To do this, first disable all ideal sources (refer back to Section III of the previous unit) then find the resistance across the terminals.

For voltage sources – replace with short circuit

For current sources – replace with open

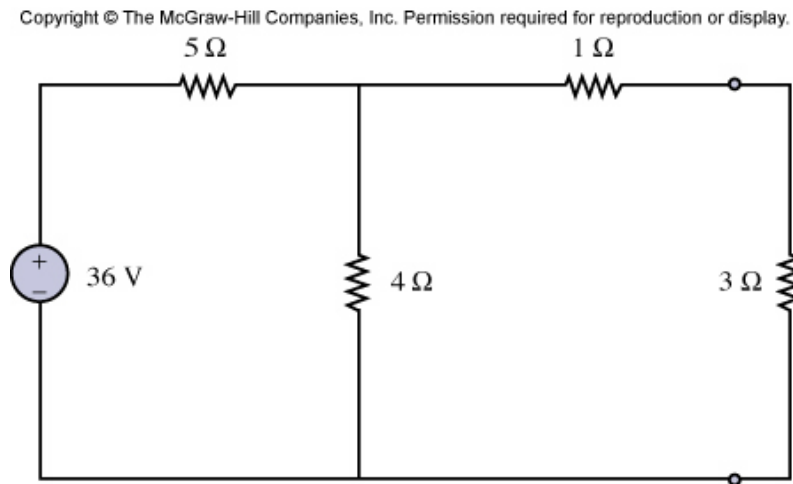
Step 3: Find the Thevenin voltage source

The Thevenin voltage source is equal to the open circuit voltage seen across the terminals (with no load). Solve for this voltage using any preferred method (refer back to previous unit).

The best way to see how to actually do this is to work through some problems.

Worked Example 1 on Thevenin

Derive the Thevenin equivalent circuit seen by the 3Ω load



$$R_{th} = 29/9\ \Omega, V_{th} = 16V$$

To find the equivalent resistance: Disable all sources and remove the 3Ω load, and then redraw the remaining resistive network

Notice that 5Ω and 4Ω are now in parallel.

$$\Rightarrow R_{th} = (5\parallel 4) + 1 = 29/9\ \Omega$$

The voltage source of the Thevenin equivalent circuit is simply the voltage across the terminals with the 3Ω load removed.

No current flows through the 1Ω resistor

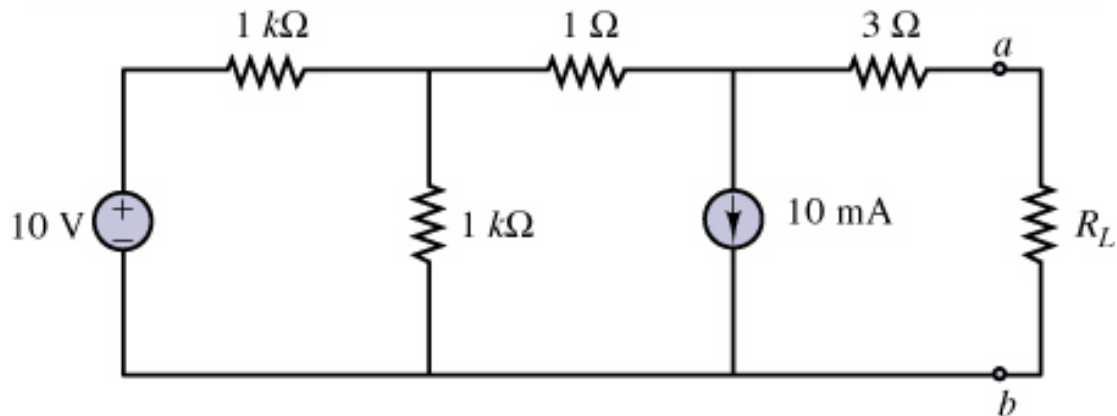
- $\Rightarrow$ No voltage difference across 1Ω resistor
- $\Rightarrow$ Terminal voltage = voltage across 4Ω
- $\Rightarrow$ We can find voltage across 4Ω by voltage divider rule

$$V_{th} = \left(\frac{4}{4 + 5} \right) 36 = 16V$$

Worked Example 2 on Thevenin

Derive the Thevenin equivalent circuit seen by the load R_L .

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$$V_{th} = -10\text{mV}, R_{th} = 504\Omega$$

Equivalent resistance: Disable all sources and redraw the circuit

10V source → _____, 10 mA source → _____

The two 1 kΩ resistors are now in parallel = 500 Ω

1 Ω and 3 Ω are now in series = 4Ω.

$$\Rightarrow R_{th} = 500 + 4 = 504 \Omega$$

Find the voltage across the terminals with R_L removed.

No current running through 3Ω resistor

$\Rightarrow V_2 =$ voltage difference across terminals

Use MCA: 1 mesh current

KVL around the unknown mesh:

$$10 = 1000 \cdot I + 1000 \cdot (I - 0.01)$$

$$I = 10 \text{ mA}$$

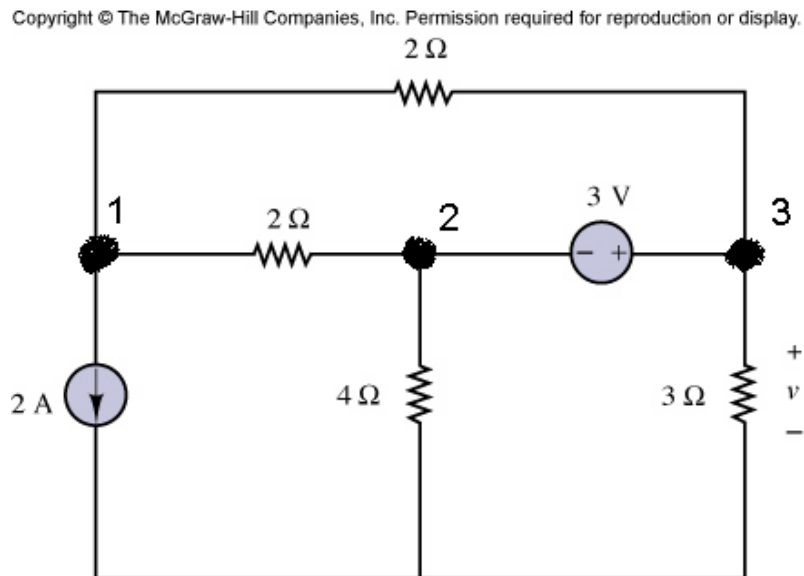
$$V_1 = 10 - 0.01 \cdot 1000 = 0 \text{ V}$$

$$V_2 = V_1 - 0.01 \cdot 1 = -0.01 \text{ V}$$

$$V_{th} = -10 \text{ mV}$$

Worked Example 3 on Thevenin

Derive the Thevenin equivalent circuit seen by the 3Ω load



$$V_{th} = -5V, R_{th} = 4\Omega$$

Equivalent resistance: Disable all sources

3 V source $\rightarrow$ _____, 2 A source $\rightarrow$ _____

Pair of 2Ω resistor form a closed loop (NO EFFECT)

$$\Rightarrow R_{th} = 4\Omega$$

Find the voltage across the terminals with $3\ \Omega$ removed.

Consider the various options:

NVA: 2 nodes unknown

MCA: 1 mesh current unknown, but you will need to add up all the voltage drops to find the terminal voltage

3rd option: consider the current through $4\ \Omega$ resistor

Current through $4\ \Omega$ resistor = _____ (_____)

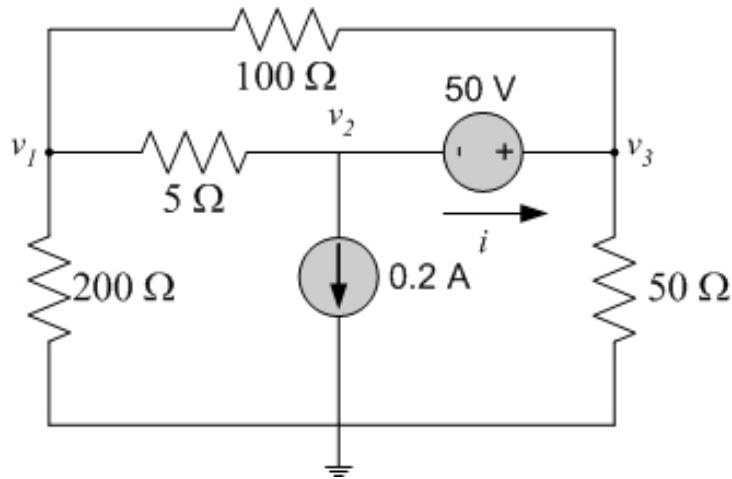
Voltage across $4\ \Omega$ resistor = _____

What is the direction of the voltage drop?

$V_{th} = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$

Worked Example 4 on Thevenin

Derive the Thevenin equivalent circuit seen by the 50Ω load



$$V_{\text{th}} = 20/3 \text{ V}, R_{\text{th}} = 4300/21 \Omega$$

Equivalent resistance: Disable all sources

50 V source $\rightarrow$ _____, 0.2 A source $\rightarrow$ _____

$$100\Omega \text{ and } 5\Omega \text{ in parallel} = 100/21 \Omega$$

$$\Rightarrow R_{\text{th}} = 100/21 + 200 = 4300/21$$

Find the voltage across the terminals with 50Ω removed.

MCA: 1 mesh current but you will need to add up all the voltage differences to find the terminal voltage

NVA: 2 nodes unknown

Apply MCA

KVL around only unknown mesh:

$$50 = 100I + (I + 0.2)(5)$$

$$\Rightarrow I = 7/15 \text{ A}$$

$$\text{Voltage across } 100 \Omega = \underline{\hspace{2cm}} * 100 \Omega = 700/15 \text{ V}$$

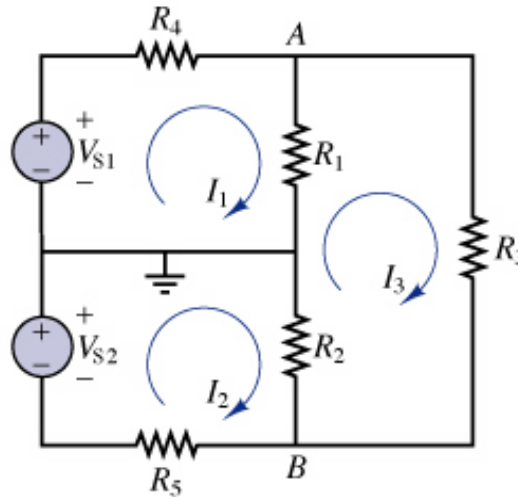
$$\text{Voltage across } 200 \Omega = \underline{\hspace{2cm}} * 200 \Omega = 40 \text{ V}$$

$$V_{th} = \underline{\hspace{1cm}} 40 \underline{\hspace{1cm}} 700/15 = 20/3$$

KEYPOINT: You should always be re-drawing the circuit when finding the equivalent circuit!

Worked Example 5 on Thevenin

Derive the Thevenin equivalent circuit seen across A-B. All the resistors have the same value of $5\text{k}\Omega$ and both supply voltages are 10V .



V_{th} is given by the voltage across A-B. Recall that we have previously solved this in Worked Example 5 on NVA in A.2.

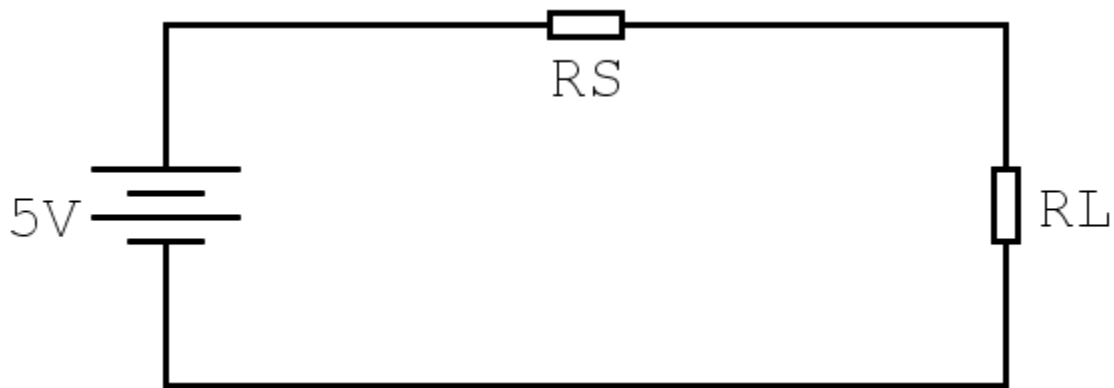
$$V_{\text{th}} = V_{AB} = 5\text{V}$$

Thevenin Resistance

$$R_{\text{th}} = 2.5\text{ k}\Omega$$

Application of Thevenin equivalent circuits: Maximum Power Transfer

The concept of deriving a Thevenin equivalent circuit is particularly powerful when trying to predict the value of the load to obtain the maximum transfer of power.



In the above circuit, the 5V voltage source and R_S are the Thevenin equivalent circuit and R_L is the load.

It can be proven that the power consumed by R_L reaches a maximum when $R_S = R_L$ assuming R_S and the voltage source are fixed.

You will get a chance to prove this in Lab 2.

II. Norton

Definition

Any one-port network composed of **ideal** voltage and current sources, and **linear** resistors, can be represented by an equivalent circuit consisting of an ideal **current source** I_N in parallel with an equivalent resistance R_N . An example of the transformed circuit is shown in Fig 3.

The Norton equivalent circuit is simply a current source version of the Thevenin equivalent circuit.

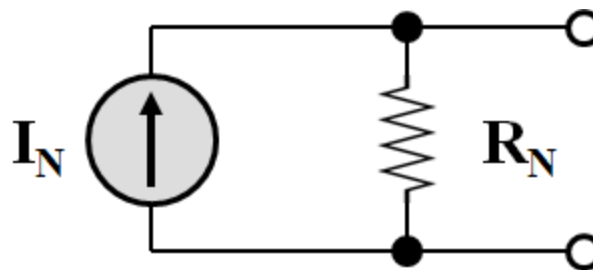


Fig 3: Transformed Thevenin equivalent circuit

Deriving the Norton equivalent circuit

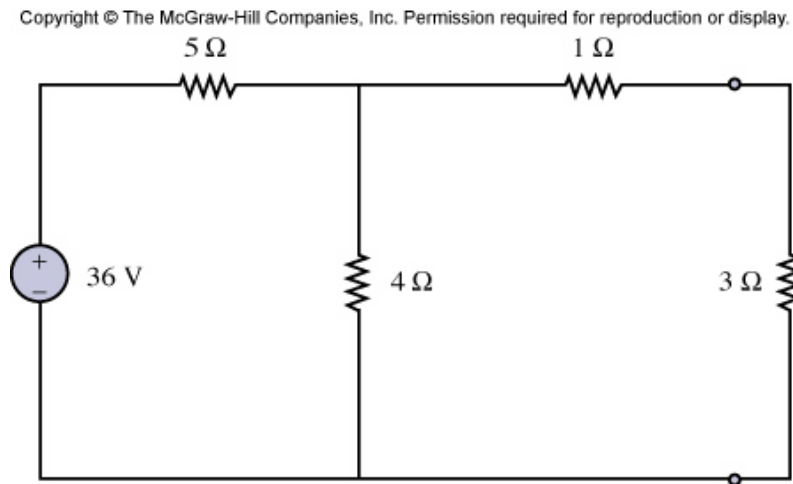
Apart from the last step of deriving the Norton current source, the steps for deriving the Norton equivalent circuit are exactly the same as that for Thevenin.

To find the Norton current source:

- (1) Short circuit the terminals to the load.
- (2) Find the current through the short circuit. This is the Norton current source.

Worked Example 1 on Norton

Derive the Norton equivalent circuit seen by the 3Ω load



$$(V_{th} = 16V, R = 29/9\ \Omega), I_N = 29/144\ A$$

Replace 3Ω with a short circuit and redraw the circuit to find the current through the short circuit (I_N).

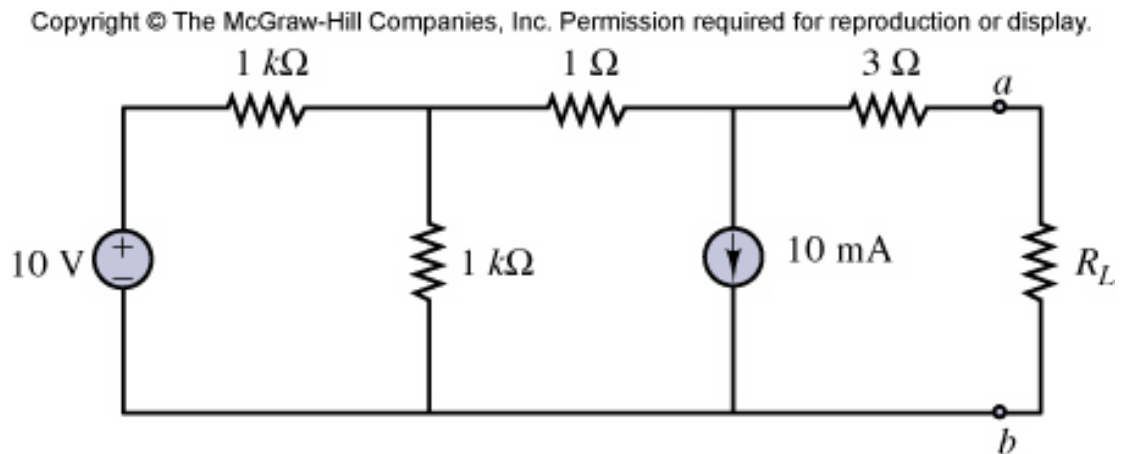
Current through short circuit = current through $1\ \Omega$
 $4\ \Omega$ and $1\ \Omega$ now in parallel (same voltage difference)

$$V_{4\Omega\parallel 1\Omega} = \left[\frac{4 \parallel 1}{(4 \parallel 1) + 5} \right] 36 = \frac{144}{29} V$$

$$\Rightarrow I_N = 144/29\ A$$

Worked Example 2 on Norton

Derive the Norton equivalent circuit seen by the load R_L



$$(V_{\text{th}} = -10\text{mV}, R = 504\Omega), I_N = -10/504\text{ mA}$$

Replace 3Ω with a short circuit and redraw the circuit to find the current through the short circuit.

Apply nodal voltage analysis (2 nodes) instead of mesh current analysis (3 mesh currents)

KCL @ node 1:

$$\frac{10 - V_1}{1000} = \frac{V_1}{1000} + \frac{V_1 - V_2}{1}$$
$$10 - V_1 = V_1 + 1000V_1 - 1000V_2 \quad (1)$$

KCL @ node 2:

$$\frac{V_1 - V_2}{1} = \frac{V_2}{3} + 0.01$$
$$3V_1 - 3V_2 = V_2 + 0.03 \quad (2)$$

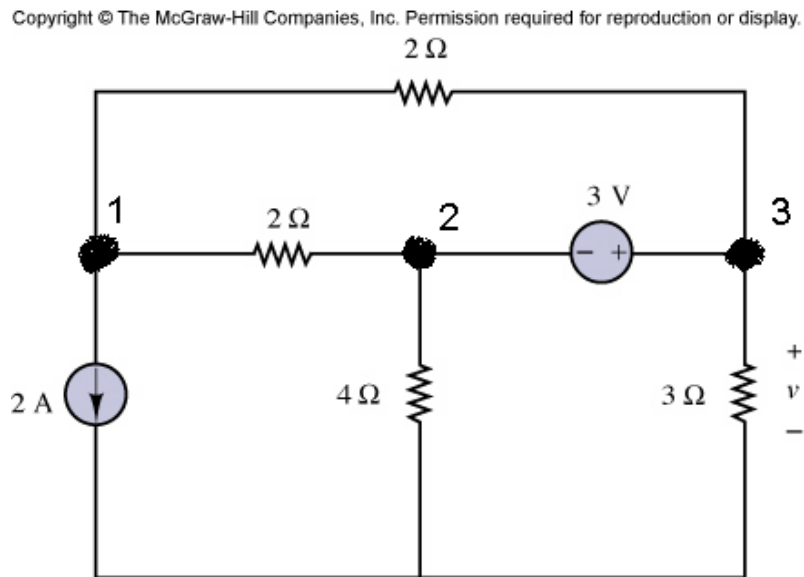
Solving (1) and (2):

$$V_2 = -10/168 \text{ mV}$$

$$\Rightarrow I_N = V_2/3 = -10/504 \text{ mA}$$

Worked Example 3 on Norton

Derive the Norton equivalent circuit seen by the 3Ω load



$$(V_{\text{th}} = -5\text{ V}, R = 4\Omega), I_{\text{N}} = -5/4\text{ A}$$

Replace 3Ω with a short circuit. Redraw the circuit and find the current through the short circuit.

NVA: 1 node, but you will need to add up currents to find I_{N} later

MCA: 2 mesh currents, but one that directly gives I_{N}

By MCA

KVL around Mesh 1:

$$3 = (I_1 + 2) * 4 \quad (1)$$

KVL around Mesh 2:

$$3 = I_2 * 2 + (I_2 - 2) * 2 \quad (2)$$

Solving (1) and (2): $I_1 = -5/4A$

$$\Rightarrow I_N = -5/4A$$

By NVA

Node 3: 0V $\Rightarrow$ Node 2: _____ V $\Rightarrow$ only node 1 is unknown

KCL @ node 1:

$$\frac{-3 - V_1}{2} = 2 + \frac{V_1}{2}$$
$$-3 - V_1 = 4 + V_1 \quad \Rightarrow V_1 = -3.5V$$

So how do you now find I_N ? I_N is the combination of the current source and the current through 4Ω

Current through $4\Omega = -3/4 A$ (going _____)

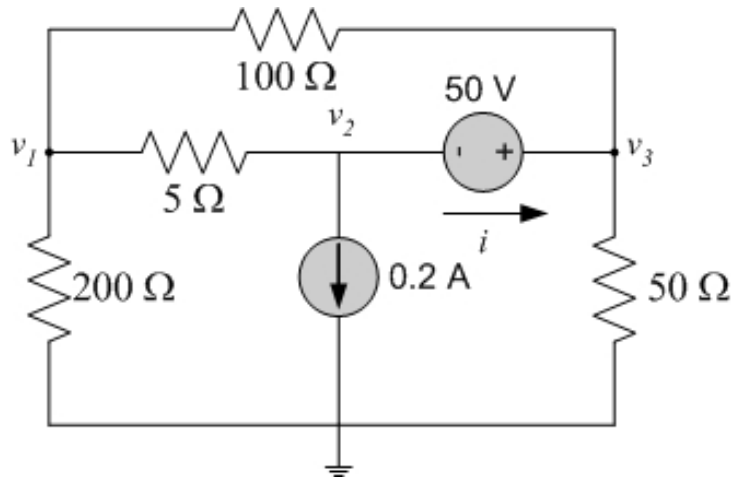
Current via short circuit = $2 - (3/4) = 5/4$ (going _____)

I_N is defined as running _____

Hence, $I_N =$

Worked Example 4 on Norton

Derive the Norton equivalent circuit seen by the 50Ω load



$$(V_{\text{th}} = 20/3 \text{ V}, R = 4300/21 \Omega), I_{\text{N}} = 7/215 \text{ A}$$

Replace 50Ω with a short circuit. Redraw the circuit and find the current through the short circuit.

NVA: 1 node, but you will need to add up currents to find I_{N} later

MCA: 3 mesh currents, but one that directly gives I_{N}

By NVA

$V_3 = 0V \Rightarrow V_2 = \underline{\hspace{2cm}} V \Rightarrow$ only V_1 is unknown

KCL @ V_1

$$\frac{-50 - V_1}{5} = \frac{V_1}{200} + \frac{V_1}{100}$$

$$-2000 - 40V_1 = V_1 + 2V_1$$

$$\Rightarrow V_1 = -2000/43 \text{ V}$$

Use the current through 200Ω and current source to find I_N .

Current through $200 \Omega = V_1/200 = -10/43 \text{ A}$

(This current is going the ground node)

0.2 A from the current source is going the ground node

Short circuit current going into the ground node
 $= 10/43 - 0.2 = 7/215 \text{ A}$

Since I_N is defined as going the ground node,
 $I_N = 7/215 \text{ A}$

Source transformation

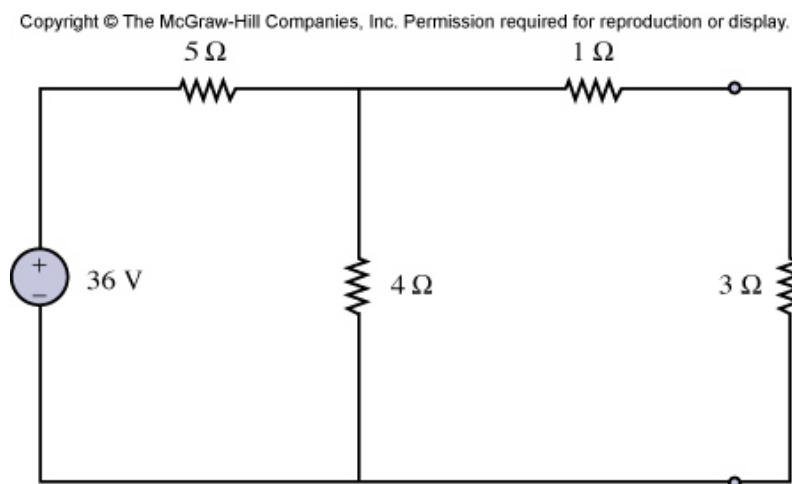
This trick rests on the fact that the Thevenin and Norton forms are equivalent to each other and therefore are also interchangeable. In other words, we switch between the 2 equivalent forms.

$$V_{th} = I_N * R$$

Rather than transform the whole circuit in one go to one of the equivalent forms, we can instead transform part of the circuit. From then on, we can attempt to repeat the process of merge transform and merge again. The best way to illustrate this is to work through a problem.

Worked Example 1 on Source transformation

Derive either of the equivalent circuit forms seen by the 3Ω load



Cut out 36 V source, 5Ω and 4Ω and transform this part into a Thevenin equivalent circuit first.

$$V_{th} = 16V, R_{th} = 20/9 \Omega$$

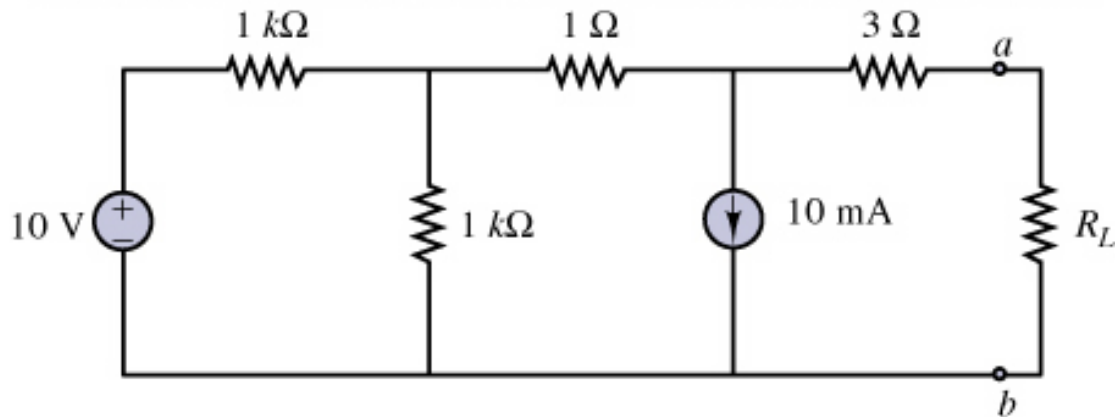
Next, reconnect this back with the rest of the circuit.

$$\text{Finally, } V_{th} = 16V, R = 29/9 \Omega$$

Worked Example 2 on Source transformation

Derive either of the equivalent circuit forms seen by the load R_L

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Cut out 10V source, and the two 1kΩ resistors. First, transform this part into a Thevenin equivalent circuit.

$$V_{th} = 5V, R_{th} = 500 \, \Omega$$

Connect this back to the rest of the circuit where it was cut off from.

1 Ω now appears in series with 500 Ω . Transform 501 Ω with 5V into a Norton equivalent circuit.

$$\Rightarrow I_N = 5/501 \text{ A}, R_N = 501 \Omega$$

The 5/501A source appears as parallel to 10 mA source.
The sources can be combined: $I = (5/501) - (1/100)$

Transform the merged current source into a Thevenin equivalent circuit once again.

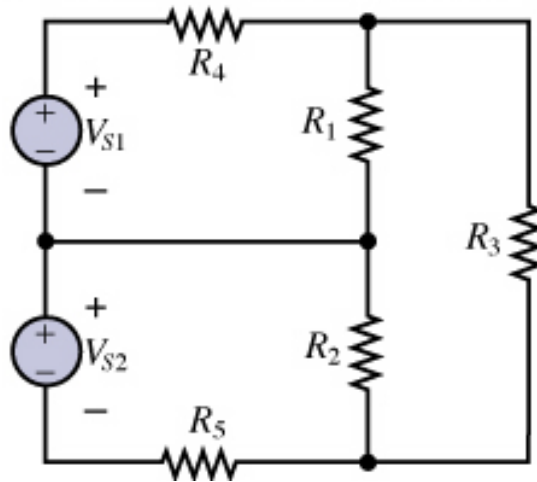
$$V_{th} = I * 501 = -10 \text{ mV}$$

Now the 501Ω appears in series with 3Ω .

Hence, $V_{th} = -10 \text{ mV}$, $R_{th} = 504 \Omega$

Worked Example 3 on Source transformation

Derive either of the equivalent circuit forms seen by the load R_3 . $V_{S1} = V_{S2} = 10V$, all resistors have the value of $5\text{ k}\Omega$.



Transform V_{S1} , R_4 and R_1 into a Thevenin equivalent circuit.

$$V_{th1} = \frac{R_1}{R_1 + R_4} V_{S1} = 5V$$

$$R_{th1} = R_1 \parallel R_4 = 2.5k\Omega$$

Then do the same for V_{S2} , R_5 and R_2 .

$$V_{th1} = \frac{R_1}{R_1 + R_4} V_{S1} = 5V$$

$$R_{th1} = R_1 \parallel R_4 = 2.5k\Omega$$

Reconnect the transformed circuits back into the circuit.

All the elements are now in series. We can add up V_{th1} and V_{th2} , as well as merge R_{th1} and R_{th2} to reduce from 5 to 3 elements: $V_{th} = 10V$, $R_{th} = 5\text{ k}\Omega$.

Finally, transform the 3 elements into a Thevenin:

$$V_{th} = 5V, R_{th} = 2.5\text{ k}\Omega$$

III. Dependent Sources

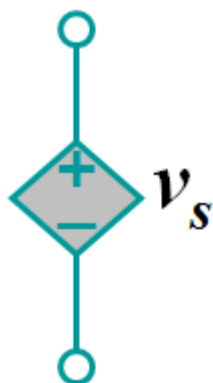
So far in Block A, all the sources we have come across are independent sources. What this means is that for a voltage source, the voltage maintained across the source is fixed. For an independent current source, this means that the current through the source is fixed. However, it is worth noting that there also exist dependent sources in circuit theory. Unlike independent sources, the value of a dependent source is not predetermined, but is set by the current or voltage through a specific branch.

Symbol

The symbol for an independent source is a circle. For a dependent source, the symbol is a diamond. There are a total of 4 basic types of dependent sources.

Dependent Voltage source

The source a voltage across the terminals of v_s . But the value of v_s could be set by either current or voltage from another branch.



Current Controlled Voltage Source

For example, $v_s = 5i_x$. The current i_x is not the current through the source but belongs to another branch. As i_x changes, so also v_s according to the above relation.

Voltage Controlled Voltage Source

For example, $v_s = 7v_x$. The voltage v_x is not the source voltage. As v_x changes, so also v_s according to the above relation.

Dependent Current source

The current through the source is kept at i_s , but the value of i_s is set by either current or voltage from another branch.



Current Controlled Voltage Source

For example, $i_s = 5i_x$. The current i_x is not the current through the source but belongs to another branch. As i_x changes, so also i_s according to the above relation.

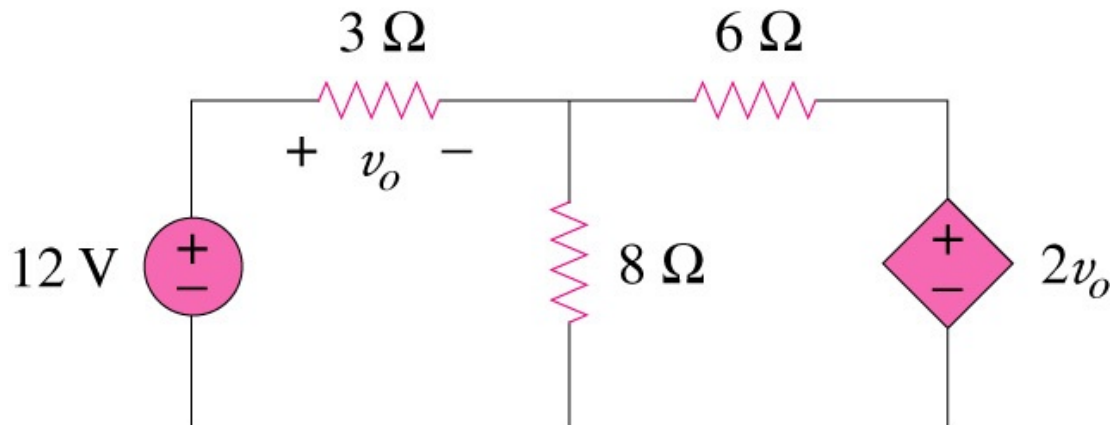
Voltage Controlled Current Source

For example, $i_s = 7v_x$. The voltage v_x is not the source voltage but from another branch. As v_x changes, so also will i_s according to the above relation.

Once again, working through a few problems will help to illustrate how these dependent sources work.

Worked Example 1 on Dependent Sources

Calculate v_o in the circuit



The dependent source here in this example is a voltage controlled voltage source

Apply KCL at node X:

$$\frac{v_o}{3} + \frac{2v_o - v_x}{6} = \frac{v_x}{8}$$

But note that $v_x = 12 - v_o$ (Sub into equation)

$$\frac{v_o}{3} + \frac{2v_o - (12 - v_o)}{6} = \frac{12 - v_o}{8}$$

$$\frac{v_o}{3} + \frac{3v_o - 12}{6} = \frac{12 - v_o}{8}$$

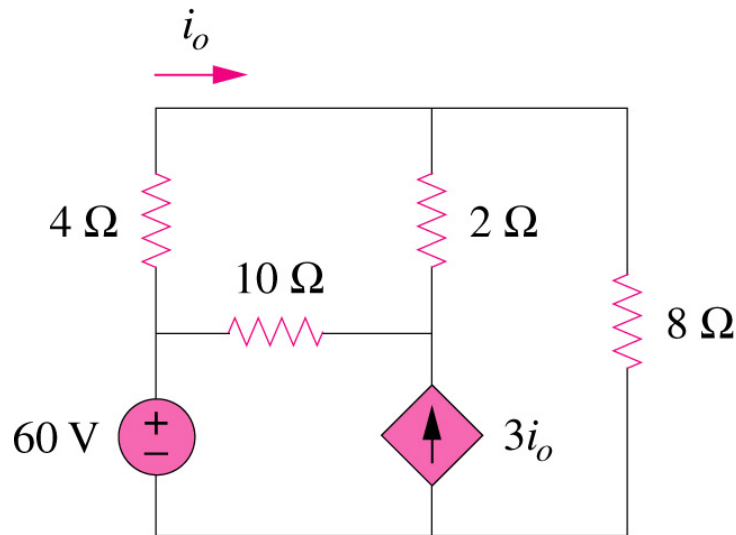
$$8v_o + 4(3v_o - 12) = 3(12 - v_o)$$

$$23v_o = 84$$

$$v_o = 3.65V$$

Worked Example 2 on Dependent Sources

Calculate i_o in the circuit



The dependent source here in this example is a current controlled current source

Identify the unknown node voltages

Apply KCL at node A:

$$\frac{60 - v_A}{4} + \frac{v_B - v_A}{2} = \frac{v_A}{8} \quad (1)$$

Apply KCL at node B:

$$\frac{60 - v_B}{10} + 3i_o = \frac{v_B - v_A}{2} \quad (2)$$

$$\text{But } i_o = \frac{60 - v_A}{4} \quad (3): \text{ sub into (2) then solve for } v_A$$

Simplify (1):

$$2(60 - v_A) + 4(v_B - v_A) = v_A$$

$$7v_A - 4v_B = 120$$

Simplify (2):

$$2(60 - v_B) + 15(60 - v_A) = 10(v_B - v_A)$$

$$5v_A + 12v_B = 1020$$

Solving for v_A :

$$26v_A = 1380$$

$$v_A = 53.077V$$

Finally, use v_A to find i_o :

$$\begin{aligned} i_o &= \frac{60 - 53.077}{4} \\ &= 1.73A \end{aligned}$$