

EF3441: Intermediate Macroeconomics

Lecture 8

The Main Goal of this Lecture

- ▶ Construct a consumer's lifetime budget constraint and preferences in the two-period model and solve his or her optimization problem with **with elastic labor supply**.
- ▶ Construct the labor supply curve (again) and how interest rate affects labor supply.
- ▶ Explain the decisions made by the firm in the real intertemporal model.
- ▶ Construct the investment curve.
- ▶ Construct the output supply curve.
- ▶ Construct the output demand curve.
- ▶ Understand the shifting factors.

What is the two-period model missing so far?

- ▶ Labor supply and wages
- ▶ Firms
- ▶ Investment decision

The Elastic Consumer: Household Choose to N^s

- ▶ Today's budget constraint:

$$c + s = w(h - \ell) + \pi - T$$

- ▶ Tomorrow budget constraint:

$$c' = w'(h - \ell') + \pi' - T' + (1 + r)s$$

- ▶ Lifetime budget constraint:

$$c + \frac{c'}{1 + r} = w(h - \ell) + \pi - T + \frac{w'(h - \ell') + \pi' - T'}{1 + r}$$

The Household's Problem with Elastic Labor Supply

The household maximizes lifetime utility

$$\max_{c, c', \ell, \ell'} U(c, \ell) + \beta U(c', \ell')$$

s.t.

$$c + s = w(h - \ell) + \pi - T$$

and

$$c' = w'(h - \ell') + \pi' - T' + (1 + r)s$$

Optimality and Labor Supply

Optimality conditions:

- ▶ First period consumption-leisure trade-off:

$$MRS_{\ell,c} = w$$

- ▶ Second period consumption-leisure trade-off:

$$MRS_{\ell',c'} = w'$$

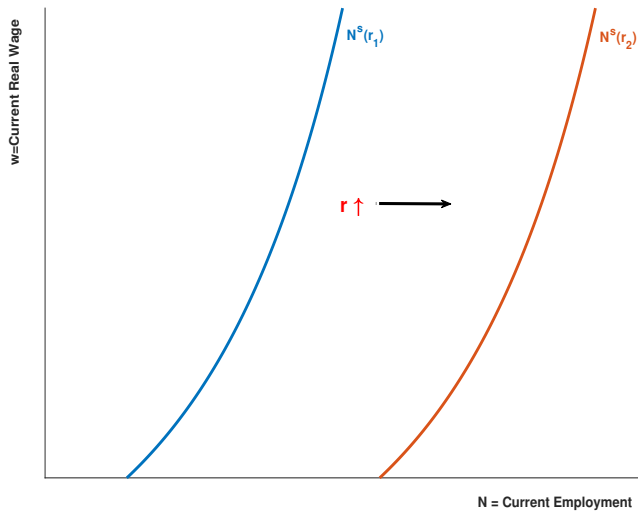
- ▶ Optimal consumption saving:

$$MRS_{c,c'} = 1 + r$$

Interest rate and Labor Supply

- ▶ Let labor supply be $N(we, r, w)$. Effects on the supply of labor:
 - ▶ Current labor supply is increasing in real wage (Assume substitution larger than income effect)
 - ▶ Current labor supply decreases if lifetime wealth increases
 - ▶ Current labor supply increases when the interest rate increases
- ▶ How interest rate affects $N(we, r, w)$?
 - ▶ Suppose r increases
 - ▶ From $MRS_{c,c'} = 1 + r$, we get c decrease (save more)
 - ▶ From $MRS_{\ell,c} = w$, we get ℓ decreases as c decreases
 - ▶ i.e. for any fixed w , as r goes up, N^d goes up.

Interest rate and Labor Supply



Introduce Investment Decision

- ▶ Output today: $Y = zF(K, N)$
- ▶ Output tomorrow: $Y' = z'F(K', N')$
- ▶ δ : capital depreciation rate over time
- ▶ Firm invests I so that

$$K' = (1 - \delta)K + I$$

The Firm

- ▶ Firm maximizes the present discounted value of the firm

$$V = \pi + \frac{\pi'}{1+r}$$

- ▶ Profits today: $\pi = Y - wN - I$
- ▶ Profits tomorrow: $\pi' = Y' - w'N' + (1 - \delta)K'$
- ▶ Liquidation value: $(1 - \delta)K'$. Note that now, the firm owns the capital and liquidates it in the last period

The Firm and Investment Decision

- ▶ The problem of the Firm is:

$$\max_{N, N', I} zF(K, N) - wN - I + \frac{z'F(K', N') - w'N' + (1 - \delta)K'}{1 + r}$$

- ▶ Optimality with respect to I gives

$$-1 + \frac{z'F_K(K', N') + (1 - \delta)}{1 + r} = 0$$

- ▶ Firm will equate the marginal benefit and marginal cost of investment.

Intuition: Investment Decision

- ▶ The marginal cost of investment **=1**. Because when I increases one unit, the present value of profits V decreases one unit accordingly. Hence, 1 is the marginal cost of investment.
- ▶ The marginal benefit from investment

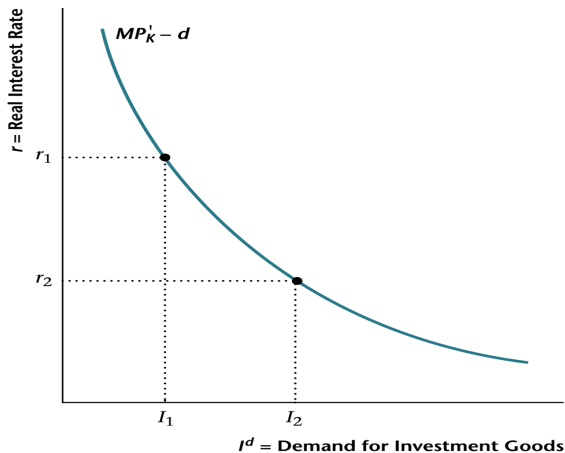
$$MB(I) = \frac{z' F_K(K', N') + (1 - \delta)}{1 + r}$$

- ▶ So that

$$z' F_K(K', N') - \delta = r$$

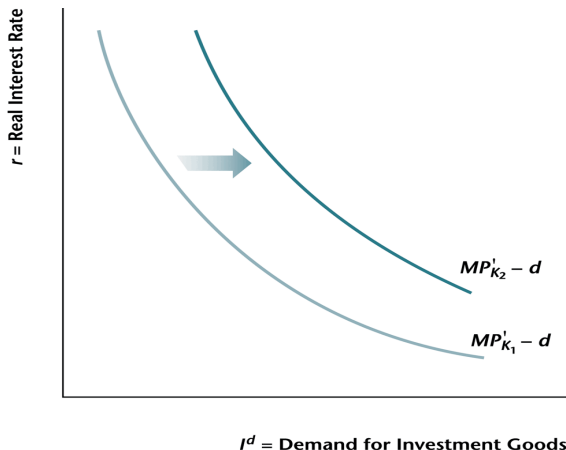
The Firm and Investment Decision

- Suppose $r_1 > r_2$ then investment $I_1 < I_2$ since $z' F_K((1 - \delta)K + I, N') - \delta = r$



The Firm and Investment Decision

- ▶ If z' tomorrow increases or if K decreases, investment curve shifts to the right (marginal benefit increases since MP'_K increases)



The Firm with Cobb-Douglas Production Function

- Suppose that the production function is Cobb-Douglas:

$$ZF(K, N) = zK^{\alpha}N^{1-\alpha}$$

where $0 < \alpha < 1$.

- Then optimality implies

$$z' \alpha \left(\frac{K'}{N'} \right)^{\alpha-1} = r + \delta$$

- substituting $K' = (1 - \delta)K + I$ we get

$$I = \left(\frac{z' \alpha}{r + \delta} \right)^{\frac{1}{1-\alpha}} N' - (1 - \delta)K$$

Note that I is increasing in z' and α and decreasing in r , δ , and K .

Competitive Equilibrium: Dynamic

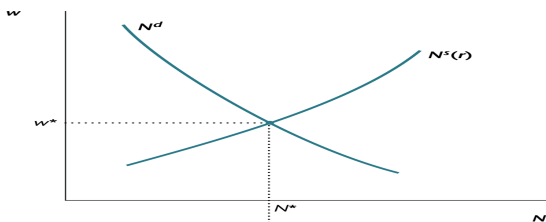
- ▶ Dynamic: more than one period. *i.e.*, it is the two-period model.
- ▶ **Competitive Equilibrium:** For a set of exogenous variables (K, G, G', z, z') , a competitive equilibrium is a set of endogenous variables $(C, N^s, S, I, N^d, B, T, Y, r, w)$ for both today (without prime) and tomorrow (with prime), so that:
 - ▶ The consumer chooses consumption, savings, and labor supply optimally, taking wages, interest rates, taxes, and dividends as given.
 - ▶ The firm chooses labor demand and investment to maximize profits, taking as given wages, interest rate, and current/future productivity.
 - ▶ All markets cleared
 - ▶ Goods market clears: $Y = C + I + G$; $Y' = C' + I' + G'$
 - ▶ Financial market clears: $B + I = S$

Equilibrium: Two Markets

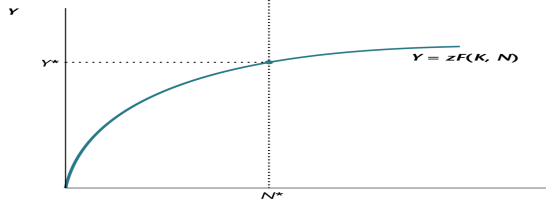
- ▶ Analytical or graphical characterization of equilibrium is hard.
- ▶ We focus on the equilibrium of two markets:
 - ▶ Today's labor market (relating wages and employment)
 - ▶ Today's goods market (relating interest rate and output)

Equilibrium in the Labor Market

We equate the demand for labor (firm) and supply of labor (household):



(a)



(b)

Equilibrium in the Labor Market

- ▶ We determine employment level: N^*
- ▶ We determine wage rate: w^*
- ▶ Recall: supply of labor depends on r^*

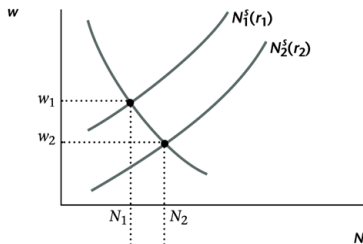
Equilibrium in the Goods Market

We equate demand of goods (Y^d) and supply of goods (Y^s), where:

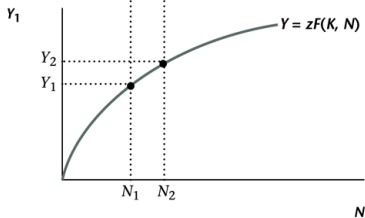
- ▶ $Y^s = zF(K, N^d)$
- ▶ $Y^d = C(w, r) + I(r) + G$
- ▶ Goal: determine the relationship between Y^s , Y^d and r

The Output Supply Curve

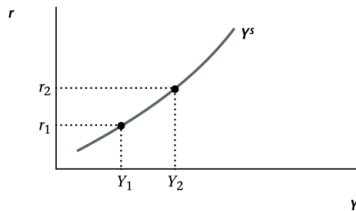
- ▶ Y^s as a function of r is called output supply curve.
- ▶ How does Y^s vary with respect to r ?



(a)



(b)



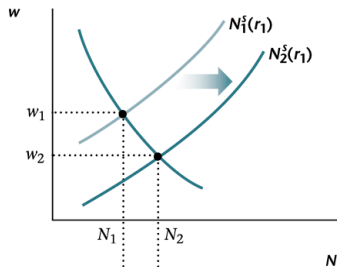
(c)

The Output Supply Curve

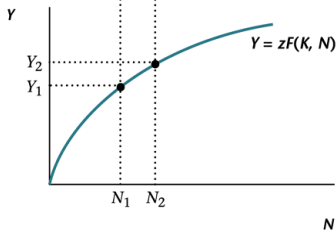
- ▶ Y^s is an increasing (and concave) function of r .
- ▶ Shifts in Y^s
 - ▶ Lifetime wealth, e.g., an increase in G or G' will shift labor supply curve to the right and shifts the output supply curve to the right due to the income effect on labor supply. (less income, less leisure).
 - ▶ TFP. An increase in z will shift the output supply curve to the right.
 - ▶ Capital Stock K . Same as z .

The Output Supply Curve

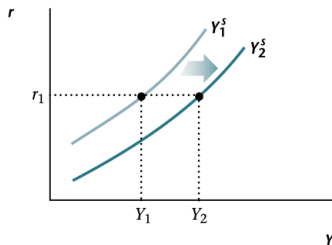
An Increase in Current or Future Government Spending Shifts the Y^s Curve.



(a)



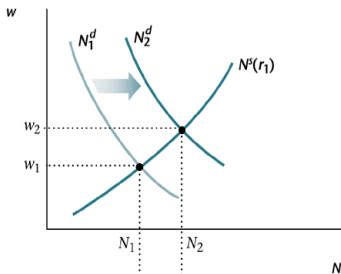
(b)



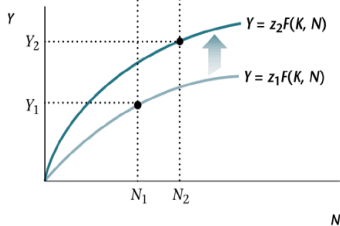
(c)

The Output Supply Curve

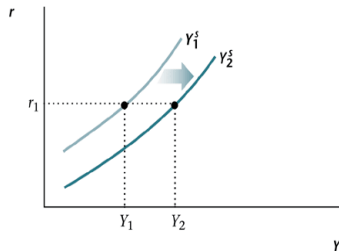
An Increase in Current Total Factor Productivity Shifts the Y^S Curve.



(a)



(b)



(c)

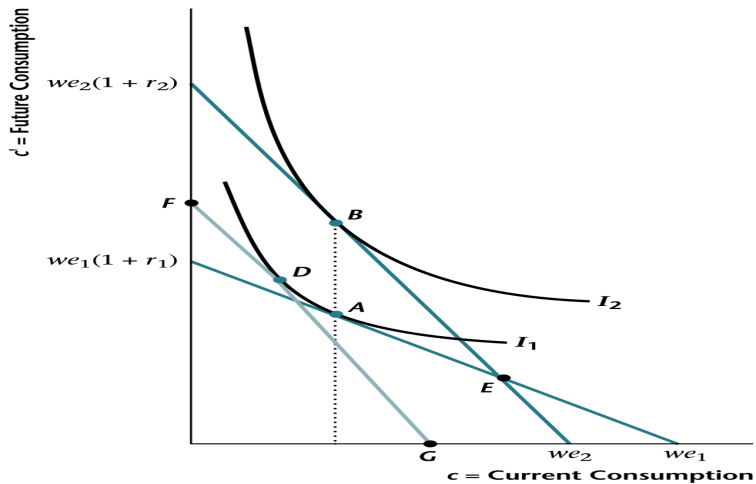
The Output Demand Curve

- ▶ Y^d as a function of r is called output demand curve.

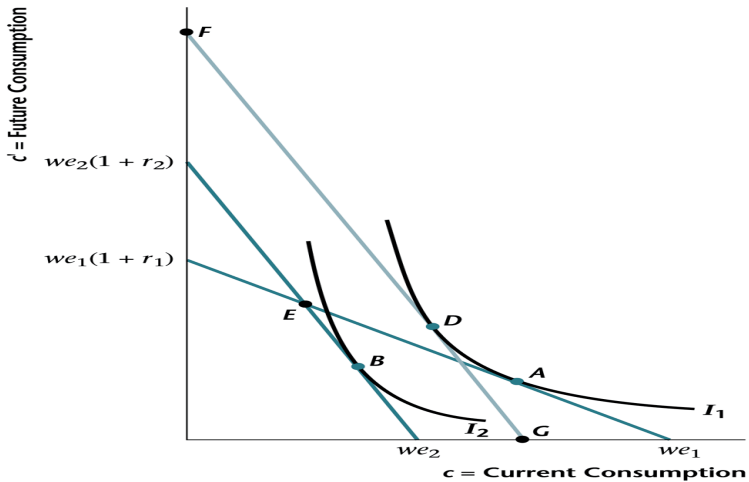
$$Y^d(r) = C(we, r) + I(r) + G$$

- ▶ The slope of Y^d is negative. Recall: the slope of $C(we, r)$ and I are negative. The assumption on $C(we, r)$: substitution effect is greater than the income effect.

Recall: An Increase in r , Lender

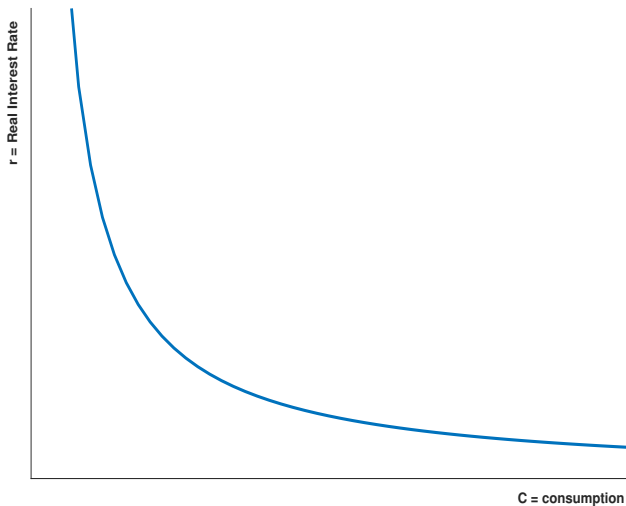


Recall: An Increase in r , Borrower

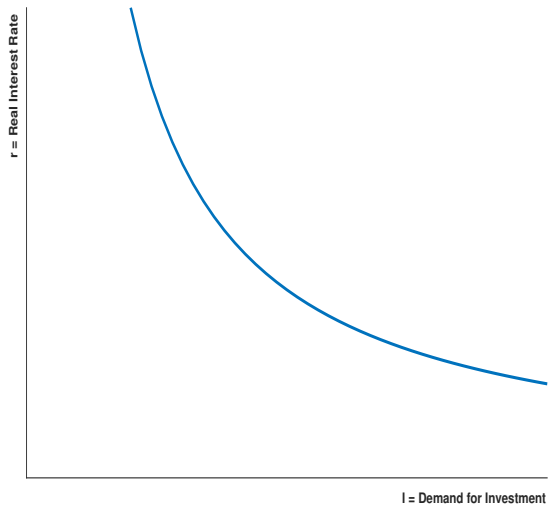


Aggregate Consumption Curve

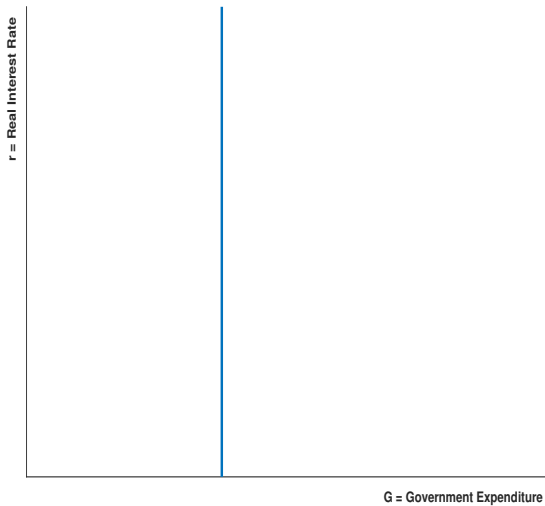
substitution effect is greater than income effect.



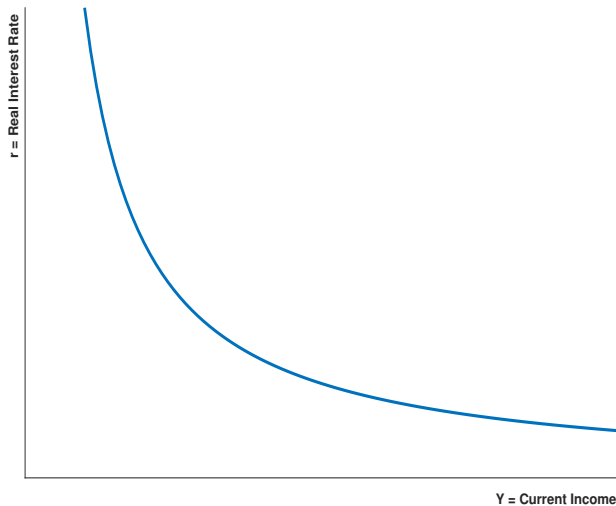
Recall: Investment Curve



Government Expenditure



The Output Demand Curve



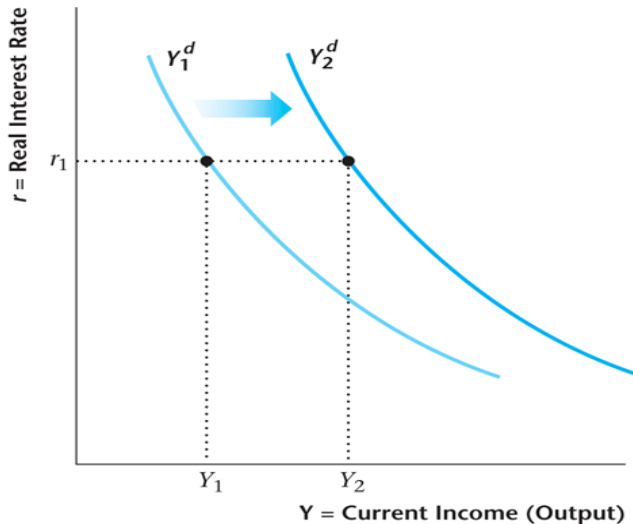
The Output Demand Curve

Shifts in Output Demand:

- ▶ Current government spending (Effects G)
 - ▶ Present value of taxes (Effects C)
 - ▶ Future income (Effects C)
 - ▶ Future total factor productivity (Effects I)
 - ▶ Current capital stock (Effects I)
- Will shift the output demand curve ~~to the right~~ .

The Output Demand Curve

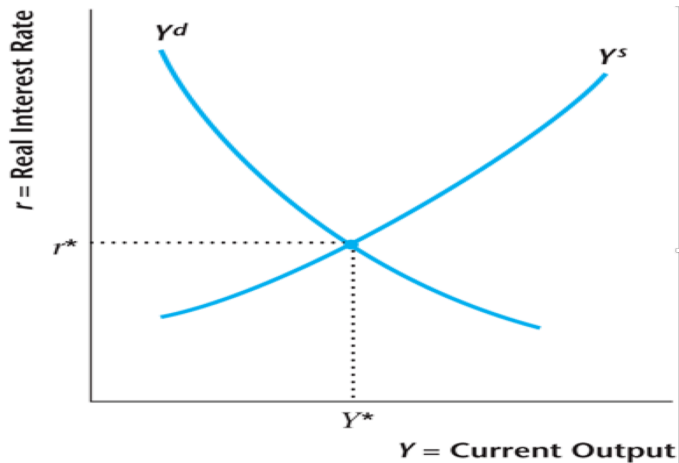
The output demand curve shifts to the right if current government spending increases



(b)

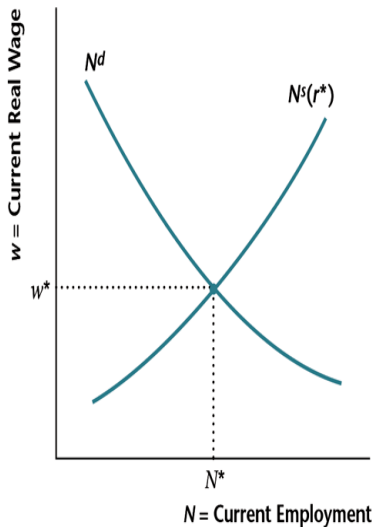
Output Supply Output Demand

Graphing together demand and supply:

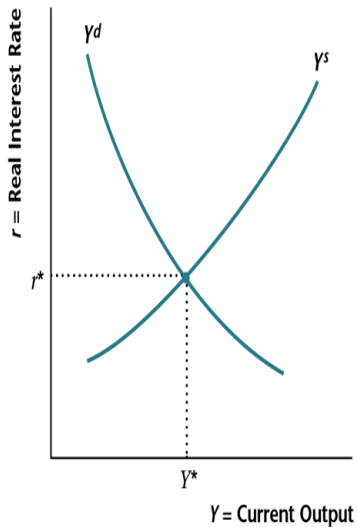


Output Supply Output Demand

Combining with the labor market equilibrium graph:



(a)



(b)

Questions

- ▶ On March 11, 2011, Japan was hit by a massive earthquake. What do you forecast will happen to GDP, wages, employment, investment, interest rates?
- ▶ On February 17, 2009, the US passed the American Recovery and Reinvestment Act. What do you forecast will happen to GDP, wages, employment, investment, interest rates?

Shifting the Output Supply Curve

- ▶ How does output supply change following a change in G ?
 - ▶ If G increases, then T or T' increases (because balance government budget), so w (lifetime wealth) decreases.
 - ▶ If w decreases, then N^s increases (because leisure is a normal good).
 - ▶ So $zF(K, N)$ goes up, supply curve shifted to the right
- ▶ How does output supply change following a change in z ?
 - ▶ If z increases, then $zF(K, N)$ increases.
 - ▶ If $zF(K, N)$ increases, then N^d increases.
 - ▶ Supply curve shifts to the right!
- ▶ Note: these are exogenous changes, not changes due to r or w .

Experiments

We next go over the following examples studying equilibrium changes:

- ▶ Temporary Increase in G
- ▶ Increase in TFP today
- ▶ Decrease In Capital Stock
- ▶ Increase in TFP tomorrow
- ▶ Permanent Increase In G

Algorithm

To solve these exercises, follow these steps:

1. Identify the effect of change on consumer, government, firm (these are exogenous changes, not changes due to r or w)
2. Identify the first tentative equilibrium in the labor market
3. Identify the effect on the output supply curve
4. Identify the effect on the output demand curve
5. Determine changes in Y and r in the goods market.
6. Using result in 2) and change in r determine change in N and w
7. In 5) and 6), specify which result are determinate and which are indeterminate