

EE3123 Tutorial 3 (Solution)

Three-phase power circuit analysis

Name: _____

Student No.: _____

Q1

A Y-connected balanced three-phase 240-V source supplies a balanced three-phase load. If the line current I_A is measured to be 15 A and is in phase with the line-to-line voltage V_{BC} , find the per-phase load impedance if the load is (a) Y-connected, (b) Δ -connected.

Solution

$$(a) \bar{V}_{AN} = \frac{240}{\sqrt{3}} \angle 0^\circ = 138.56 \angle 0^\circ \text{ V (Assumed as Reference)}$$

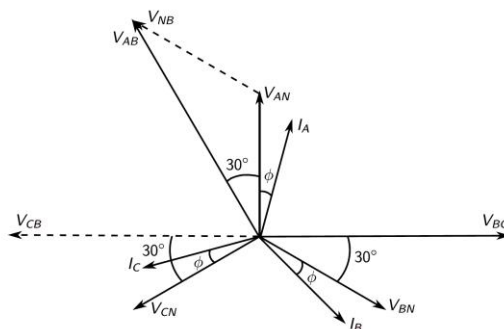
$$\bar{V}_{AB} = 240 \angle 30^\circ \text{ V}; \bar{V}_{BC} = 240 \angle -90^\circ \text{ V}; \bar{I}_A = 15 \angle -90^\circ \text{ A}$$

$$\bar{Z}_Y = \frac{\bar{V}_{AN}}{\bar{I}_A} = \frac{138.56 \angle 0^\circ}{15 \angle -90^\circ} = 9.24 \angle 90^\circ = (0 + j9.24) \Omega$$

$$(b) \bar{I}_{AB} = \frac{\bar{I}_A}{\sqrt{3}} \angle 30^\circ = \frac{15}{\sqrt{3}} \angle -90^\circ + 30^\circ = 8.66 \angle -60^\circ \text{ A}$$

$$\bar{Z}_\Delta = \frac{\bar{V}_{AB}}{\bar{I}_{AB}} = \frac{240 \angle 30^\circ}{8.66 \angle -60^\circ} = 27.71 \angle 90^\circ = (0 + j27.71) \Omega$$

$$\text{Note: } \bar{Z}_Y = \bar{Z}_\Delta / 3$$



Q2

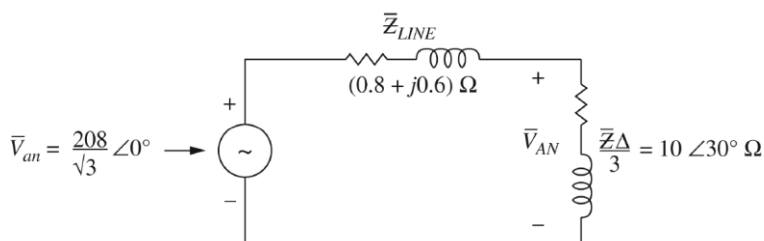
Three identical impedances $Z_\Delta = 30 \angle 30^\circ \Omega$ are connected in Δ to a balanced three-phase 208-V source by three identical line conductors with impedance $Z_L = (0.8 + j0.6) \Omega$ per line.

(a) Calculate the line-to-line voltage at the load terminals.

(b) Repeat part (a) when a Δ -connected capacitor bank with reactance $(-j60) \Omega$ per phase is connected in parallel with the load.

Solution

(a)



Using voltage division: $\bar{V}_{AN} = \bar{V}_{an} \frac{\bar{Z}_{\Delta}/3}{(\bar{Z}_{\Delta}/3) + \bar{Z}_{LINE}}$

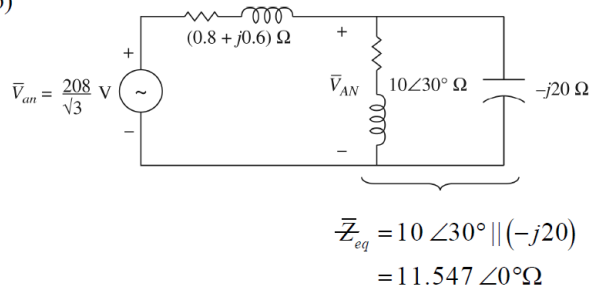
$$= \frac{208}{\sqrt{3}} \angle 0^\circ \frac{10 \angle 30^\circ}{10 \angle 30^\circ + (0.8 + j0.6)}$$

$$= \frac{(120.09)(10 \angle 30^\circ)}{9.46 + j5.6} = \frac{1200.9 \angle 30^\circ}{10.99 \angle 30.62^\circ}$$

$$= 109.3 \angle -0.62^\circ \text{ V}$$

Load voltage = $V_{AB} = \sqrt{3} (109.3) = 189.3 \text{ V Line-to-Line}$

(b)



$$\bar{V}_{AN} = \bar{V}_{an} \frac{\bar{Z}_{eq}}{\bar{Z}_{eq} + \bar{Z}_{LINE}}$$

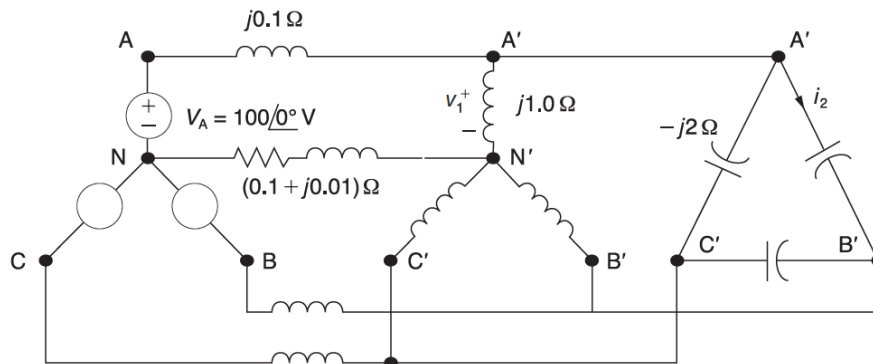
$$= \left(\frac{208}{\sqrt{3}} \right) \frac{11.547}{(11.547 + 0.8 + j0.6)}$$

$$= \frac{1386.7}{12.362 \angle 2.78^\circ} = 112.2 \angle -2.78^\circ \text{ V}$$

Load voltage Line-to-Line $V_{AB} = \sqrt{3} (112.2) = 194.3 \text{ V}$

Q3

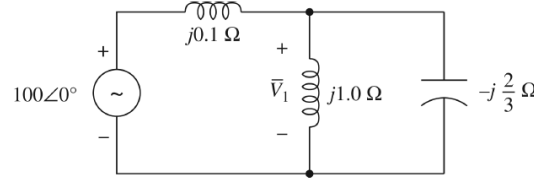
Consider the balanced three-phase system shown below. Determine $v_1(t)$ and $i_2(t)$. Assume positive phase sequence.



Solution

Replace delta by the equivalent WYE: $\bar{Z}_Y = -j\frac{2}{3} \Omega$

Per-phase equivalent circuit is shown below:



Noting that $\left(j1.0 \parallel -j\frac{2}{3} \right) = -j2$, by voltage-divider law,

$$\bar{V}_1 = \frac{-j2}{-j2 + j0.1} (100 \angle 0^\circ) = 105 \angle 0^\circ$$

$$\therefore v_1(t) = 105\sqrt{2} \cos(\omega t + 0^\circ) = 148.5 \cos \omega t \text{ V} \leftarrow$$

In order to find $i_2(t)$ in the original circuit, let us calculate $\bar{V}_{A'B'}$

$$\bar{V}_{A'B'} = \bar{V}_{A'N'} - \bar{V}_{B'N'} = \sqrt{3} e^{j30^\circ} \bar{V}_{A'N'} = 173.2 \angle 30^\circ$$

Then
$$\bar{I}_{A'B'} = \frac{173.2 \angle 30^\circ}{-j2} = 86.6 \angle 120^\circ$$

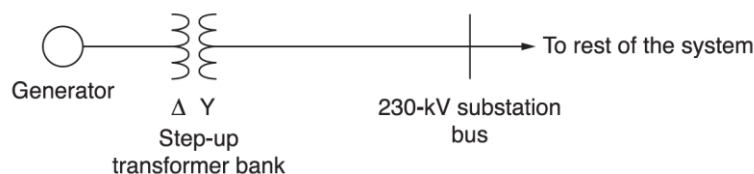
$$\begin{aligned} \therefore i_2(t) &= 86.6\sqrt{2} \cos(\omega t + 120^\circ) \\ &= 122.5 \cos(\omega t + 120^\circ) \text{ A} \leftarrow \end{aligned}$$

Q4

Consider the one-line diagram shown below. The three-phase transformer bank is made up of three identical single-phase transformers, each specified by $X_1 = 0.24 \Omega$ (on the low-voltage side), negligible resistance and magnetizing current, and turns ratio $n = N_2/N_1 = 10$. The transformer bank is delivering 100 MW at 0.8 p.f. lagging to a substation bus whose voltage is 230 kV (line voltage).

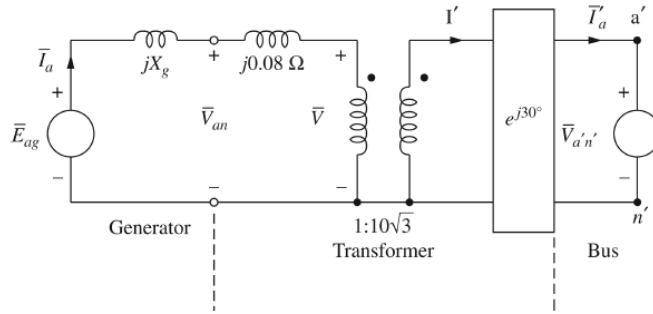
(a) Determine the primary current magnitude, primary voltage (line-to-line) magnitude, and the three-phase complex power supplied by the generator. Choose the line-to-neutral voltage at the bus, $V_{a'n'}$ as the reference. Account for the phase shift, and assume positive-sequence operation.

(b) Find the phase shift between the primary and secondary voltages.



Solution

- (a) For positive sequence operation and standard Δ -Y connection, the per-phase diagram is shown below:



$$\bar{V}_{a'n'} = \frac{230}{\sqrt{3}} \angle 0^\circ \text{ kV, choosing that as a reference.}$$

$$\bar{S}' = \frac{100 \times 10^6}{0.8 \times 3} \angle \cos^{-1} 0.8 = 41.67 \angle 36.87^\circ \text{ MVA}$$

$$\bar{I}_a^* = \frac{\bar{S}'}{\bar{V}_{a'n'}} = \frac{41.67 \times 10^6 \angle 36.87^\circ}{132.8 \times 10^3} = 313.8 \angle 36.87^\circ \text{ A}$$

$$\therefore \bar{I}'_a = 313.8 \angle -36.87^\circ \text{ A}$$

$$\bar{I}_a = 10\sqrt{3} e^{-j30^\circ} \bar{I}'_a = 5435 \angle -66.87^\circ \text{ A}$$

The primary current magnitude is 5435A. ←

$$\begin{aligned} \bar{V}_{an} &= \bar{V} + j0.08 \bar{I}_a = \left(\frac{1}{10\sqrt{3}} 132.8 \times 10^3 \angle -30^\circ \right) + j0.08 (5435 \angle -66.87^\circ) \\ &= 7667.4 \angle -30^\circ + 434.8 \angle 23.13^\circ \\ &= 7253.3 \angle -13.93^\circ \text{ V} \end{aligned}$$

$$\text{Line-to-line primary voltage magnitude} = \sqrt{3} (7253.3) = 12.56 \text{ kV} \leftarrow$$

Three-phase complex power supplied by the generator is

$$\begin{aligned} \bar{S}_{3\phi} &= 3 \bar{V}_{an} \bar{I}_a^* = 3 (7253.3 \angle -13.93^\circ) (5435 \angle 66.87^\circ) \\ &= 118.27 \angle 52.94^\circ \text{ MVA} \leftarrow \end{aligned}$$

- (b) The secondary phase leads the primary by 13.93° ; this phase shift applies to line-to-neutral (phase) as well as line-to-line voltages. ←