

EF3441: Intermediate Macroeconomics

Prepared by Dr. Tam
Lecture 3

The Main Goal of this Lecture

- ▶ Understand the firm's profit maximization problem
- ▶ To do that, we need to
 - ▶ list the properties of the firm's production function,
 - ▶ show how the firm optimizes profit given production function.

The Firm

- ▶ A firm converts inputs (factors of production) into output (consumption goods)
- ▶ Goal of the firm: maximize profits
- ▶ Assumptions:
 - ▶ Firms are very smart
 - ▶ They all have similar technology: a representative firm
 - ▶ They use only two (input) factors of productions: **capital and labor**
 - ▶ Live one period (relaxed later in the course)
 - ▶ No financing issue (relaxed later in the course)

The Production Function

- ▶ Let:
 - ▶ Capital: K
 - ▶ Labor employed: N^d
 - ▶ Total factor productivity (TFP): z
 - ▶ Output: Y
- ▶ The Ability of a firm to transform factors of production into output is summarized by the production function F

$$Y = zF(K, N^d)$$

- ▶ Properties of production function (homogeneous of degree one):

$$\lambda Y = zF(\lambda K, \lambda N^d) = \lambda zF(K, N^d)$$

Properties of the Production Function

- ▶ Monotonicity:

- ▶ Marginal product of capital

$$\frac{\partial Y}{\partial K} = MP_K > 0$$

- ▶ Marginal product of labor

$$\frac{\partial Y}{\partial N} = MP_N > 0$$

- ▶ Concavity:

- ▶ Declining MP_K

$$\frac{\partial MP_K}{\partial K} < 0$$

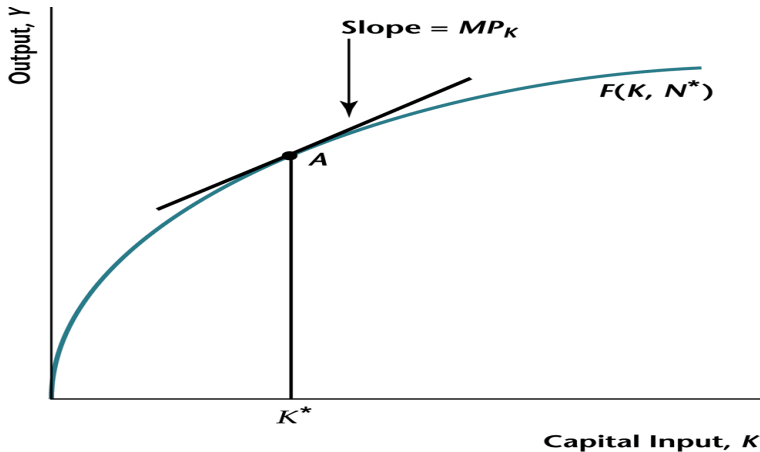
- ▶ Declining MP_N

$$\frac{\partial MP_N}{\partial N} < 0$$

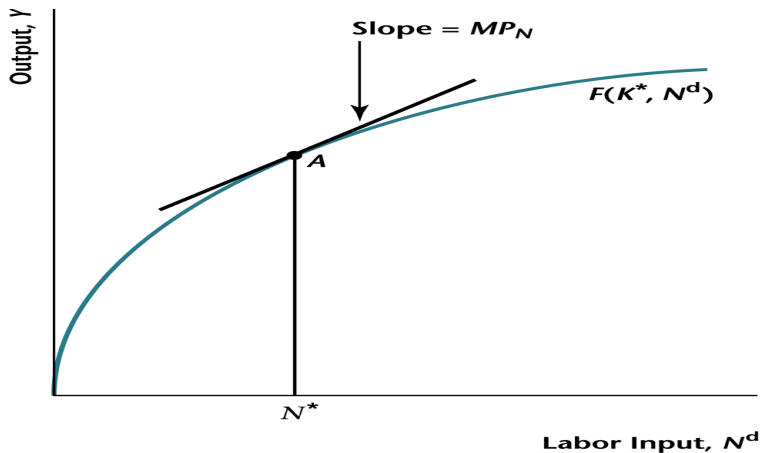
- ▶ Change in capital and MP_N

$$\frac{\partial MP_N}{\partial K} > 0$$

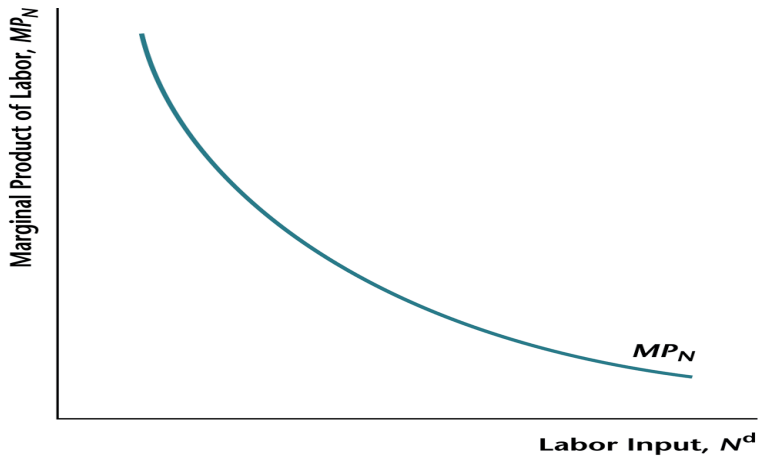
Production Function, **Fixing** the Quantity of Labor and **Varying** the Quantity of Capital



Production Function, **Fixing** the Quantity of Capital and **Varying** the Quantity of Labor

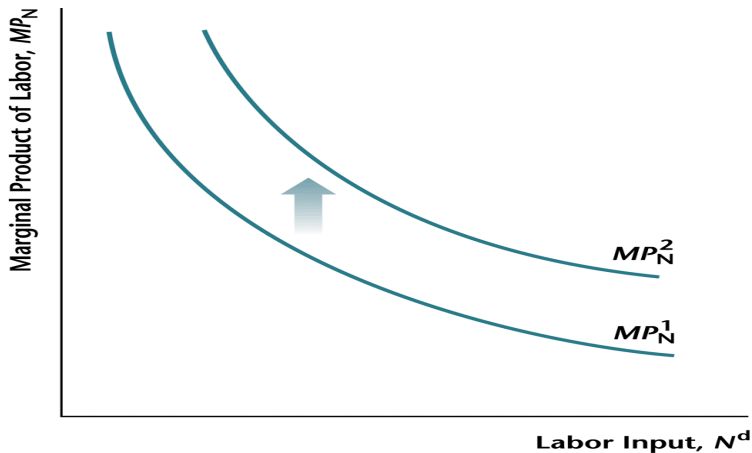


Marginal Product of Labor Curve for the Representative Firm



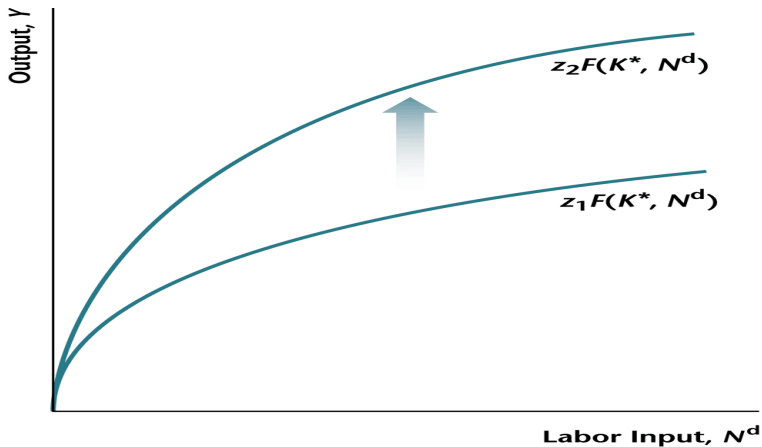
Adding Capital Increases the Marginal Product of Labor

Output increases, given everything else is fixed. That causes the MP_N curve to shift up.



Total Factor Productivity Increases

Output increases, given everything else is fixed. That causes the output curve to rotate (because $Y = 0$ when $N^d = 0$) upward.



The Problem of the Firm

Some additional assumptions:

- ▶ The objective of a firm is to maximize profits.
- ▶ A firm owns the capital (for now) from an endowment. *i.e.*, capital is given.
- ▶ Firms take the wage as given by the market.
- ▶ No taxes in the baseline environment.
- ▶ The firm chooses the demand for labor: N^d .

Profit Function

- ▶ Let:
 - ▶ Revenue: $zF(K, N^d)$
 - ▶ Labor cost: wN^d
 - ▶ Profit: $\pi = zF(K, N^d) - wN^d$
- ▶ The firm maximizes profit by choosing N^d :

$$\max_{N^d} zF(K, N^d) - wN^d$$

Profit Maximization

$$\max_{N^d} zF(K, N^d) - wN^d$$

FOC:

$$\frac{\partial zF(K, N^d)}{\partial N^d} - w = 0$$

$$\Rightarrow \frac{\partial zF(K, N^d)}{\partial N^d} = MP_N = w$$

Profit Maximization, Con't

$$\max_N zK^{0.3}N^{0.7} - wN$$

FOC:

$$\frac{\partial zF(K, N)}{\partial N} - w = 0$$

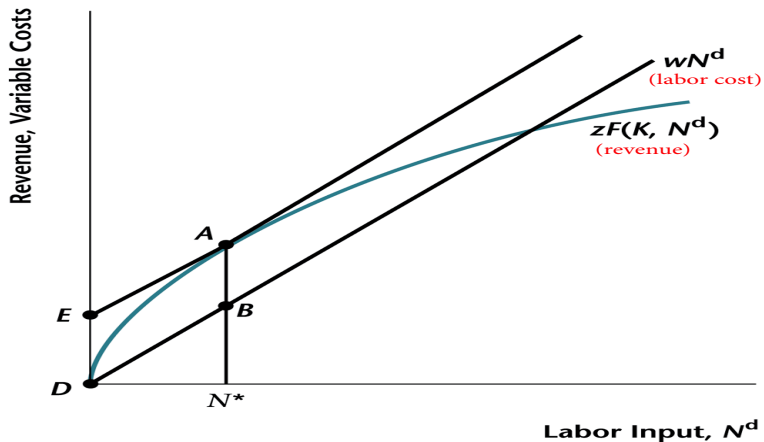
and

$$\frac{\partial zF(K, N)}{\partial N} = 0.7zK^{0.3}N^{-0.3}$$

So we get $0.7zK^{0.3}N^{-0.3} - w = 0$. Solve for N ,

$$N = \left(\frac{0.7z}{w} \right)^{10/3} K$$

Optimal Labor Choice



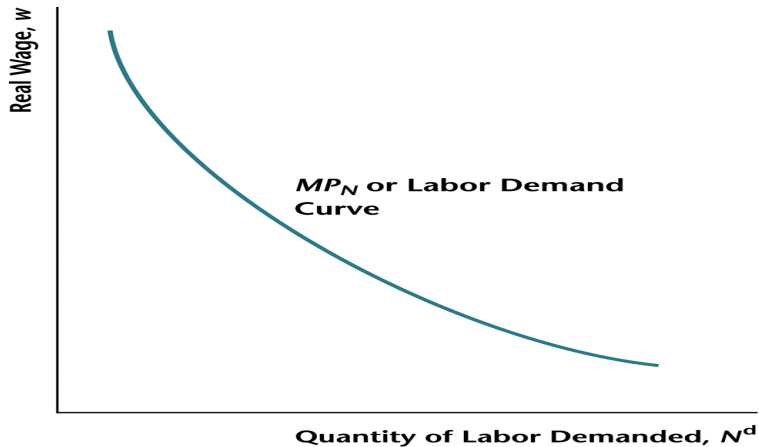
Choose N^d so that $MP_N = w$.

Solution: Intuition

- ▶ Suppose $MP_N > w$, then raise N^d : revenue raise faster than the costs.
- ▶ Suppose $MP_N < w$, then lower N^d : revenue decrease faster than the costs
- ▶ In both cases, we reach a contradiction so that the only alternative is:

$$MP_N = w$$

Labor Demand and Wage



Question: how is w determined?

Labor Demand and Wage, Con't

