

EF3441: Intermediate Macroeconomics

Lecture 7

The Main Goal of this Lecture

- ▶ Construct a consumer's lifetime budget constraint and preferences in the two-period model and solve his or her optimization problem.
- ▶ Show how the consumer responds to changes in his or her current income, future income, and the market real interest rate.
- ▶ Construct the government's present-value budget constraint.
- ▶ Show how a competitive equilibrium is constructed in a two-period model.
- ▶ Explain the Ricardian Equivalence Theorem.
- ▶ Discuss how the Ricardian Equivalence Theorem helps us understand the burden of the government debt.

Two Period Model: Consumer

- ▶ Consumers live in two periods.
- ▶ New trade-off: consumption today vs consumption tomorrow.
- ▶ New instrument: bonds b .
 - ▶ All bonds are identical
 - ▶ All bonds are safe
 - ▶ No intermediaries (Borrowers and lenders face the same interest rate r)
- ▶ New price: interest rate r (1 bond today, pays $1 + r$ tomorrow)

Two Period Model: Budget Constraints

Budget constraint. Let:

- ▶ s = saving
- ▶ y = income endowment (for now)
- ▶ l = tomorrow's variable

Two Period Model: Budget Constraints

- ▶ Budget constraint today:

$$c + s = y - t$$

- ▶ Budget constraint tomorrow:

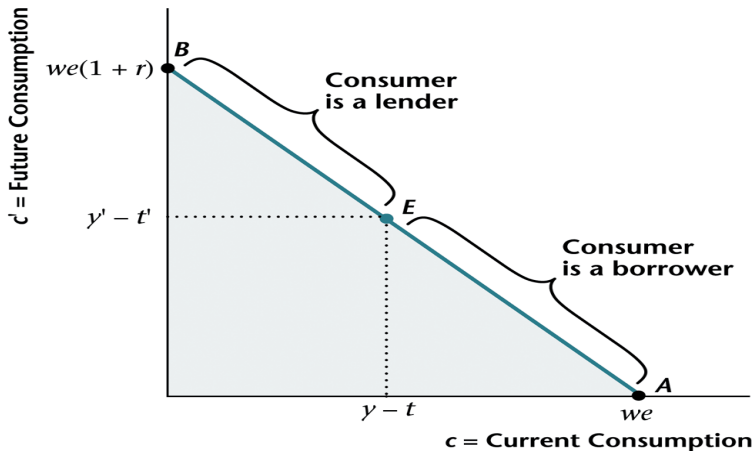
$$c' = y' - t' + (1 + r)s$$

- ▶ Define lifetime wealth (today) as:

$$we = y + \frac{y'}{1 + r} - t - \frac{t'}{1 + r}$$

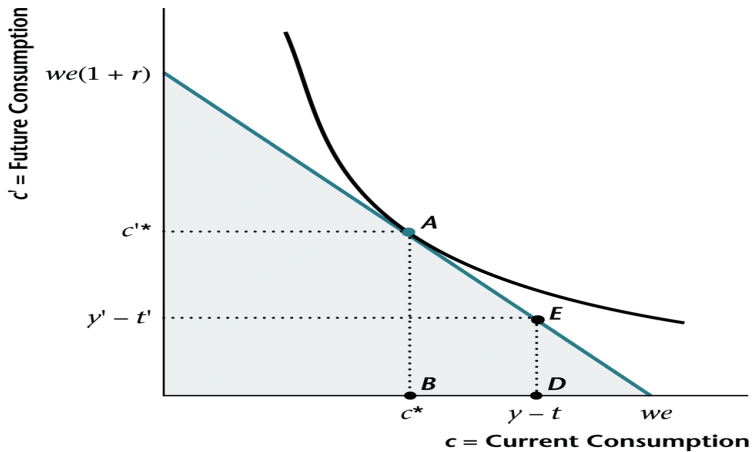
$$\text{then } c + \frac{c'}{1 + r} = we$$

Two Period Model: Budget Constraints

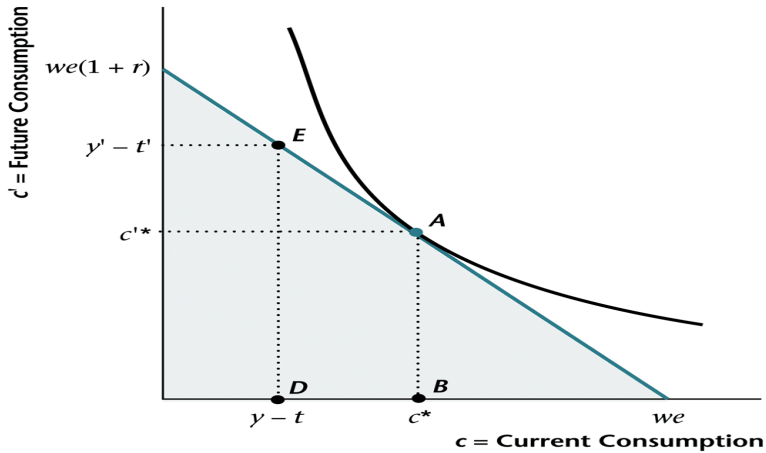


$E = (y - t, y' - t')$ used to determine how income is distributed during lifetime.

A Consumer Who Is a Lender



A Consumer Who Is a Borrower



Two Period Model: Optimality

- ▶ Households enjoy consumption today and tomorrow
- ▶ Households enjoy more the present
- ▶ Preferences :

$$U(c) + \beta U(c')$$

$0 < \beta < 1$: time discount factor; $U(\cdot)$

- ▶ Problem of the household:

$$\max_{c, c'} U(c) + \beta U(c')$$

$$\text{s.t.} \quad c + \frac{c'}{1+r} = we$$

Two Period Model: Optimality

- Rewrite the household's problem:

$$\max_c U(c) + \beta U((1+r)we - (1+r)c)$$

- Taking the first-order derivative respect to c , we get

$$U_c(c) - \beta U_{c'}(c')(1+r) = 0$$

- Rearrange we get

$$\frac{U_c(c)}{\beta U_{c'}(c')} = 1+r$$

- This expression is called the Euler equation for consumption.

Two Period Model: Optimality

- ▶ Example: log utility

$$\max_{c, c'} \log(c) + \beta \log(c')$$

$$s.t. \quad c + \frac{c'}{1+r} = we$$

- ▶ If $U(c) = \log(c)$, then the marginal utility of consumption is $U'(c) = \frac{1}{c}$.
- ▶ Using this fact, we get the Euler equation:

$$\frac{1}{c} = \beta(1+r) \frac{1}{c'}$$

or

$$\frac{c'}{c} = \beta(1+r)$$

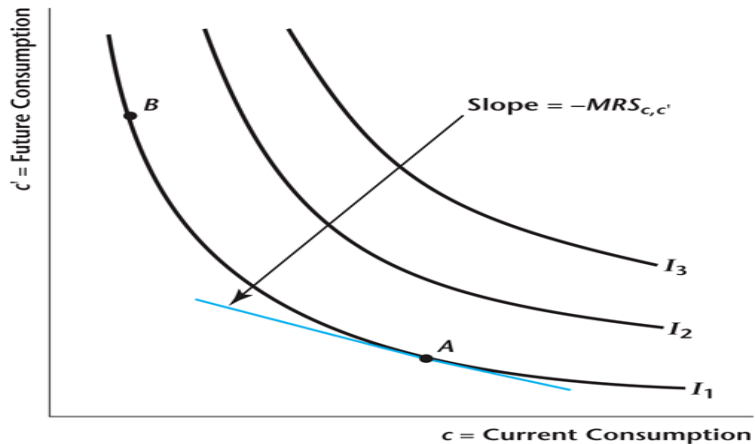
Two Period Model: Optimality

- ▶ The optimization condition

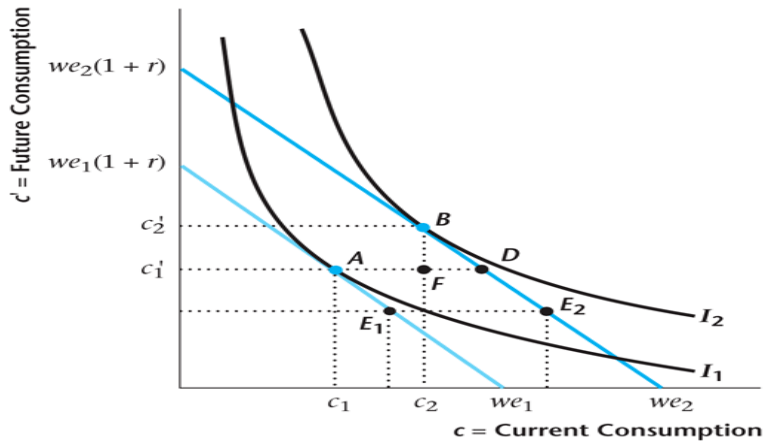
$$MRS_{c,c'} = \frac{U_c(c)}{\beta U_{c'}(c')} = 1 + r$$

- ▶ Intuition: Give up one consumption today (valued at $U_c(c)$) for $(1 + r)$ units of consumption tomorrow (valued at $U_{c'}(c')$) and discounted by β)

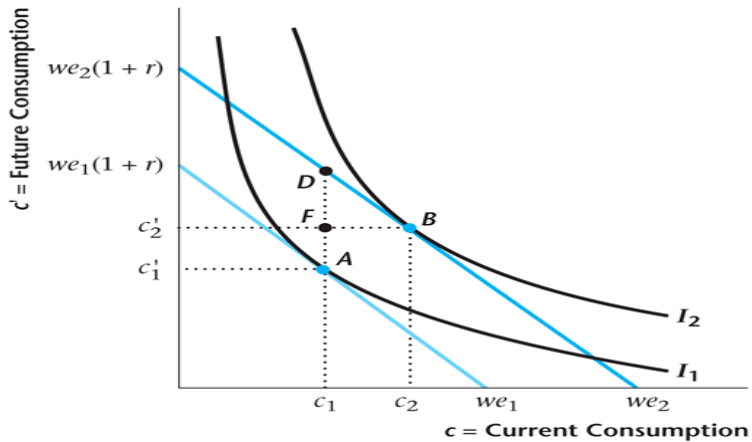
A Consumer's Indifference Curves



An Increase in Current Income for a Lender



An Increase in Future Income



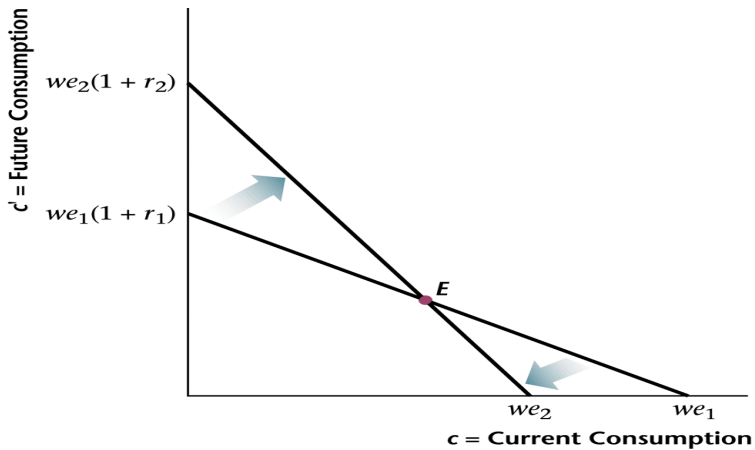
Temporary and Permanent Increases in Income

- ▶ As a permanent increase in income will have a larger effect on lifetime wealth than a temporary increase, there will be a larger effect on current consumption.
- ▶ A consumer will tend to save most of a purely temporary income increase.

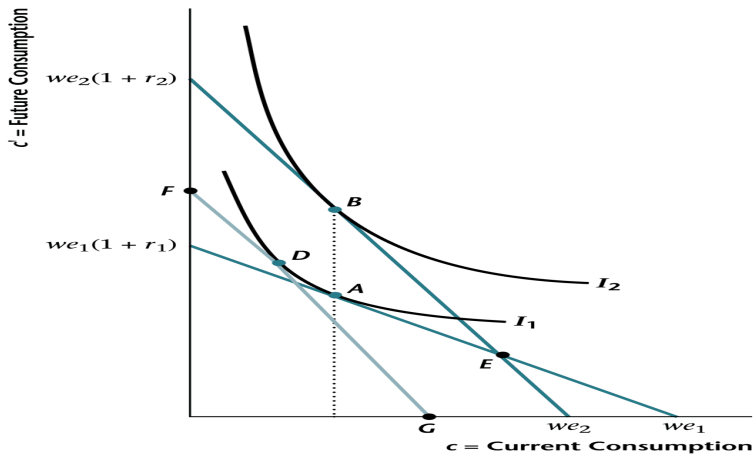
An Increase in r

- ▶ Budget constraint pivots around the endowment point.
- ▶ Income effects vs. substitution effects.

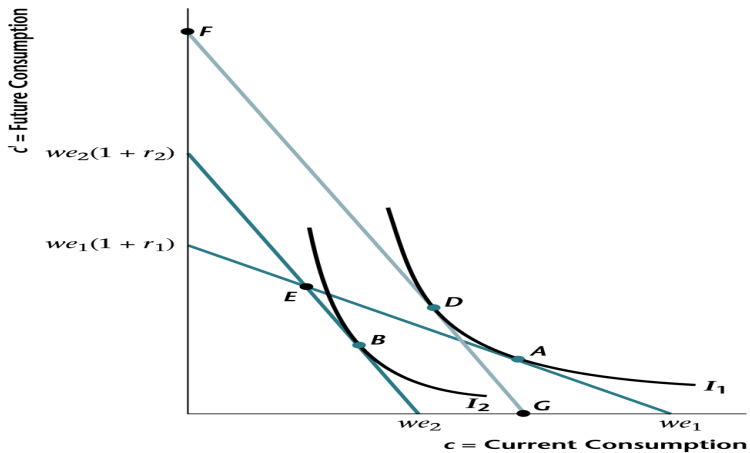
An Increase in r



An Increase in r , Lender



An Increase in r , Borrower



Consumption Today

- ▶ Let consumption depend on wealth and interest rate: $C(w, r)$
- ▶ What happens when we change wealth and interest rates?
- ▶ Effects on the consumption of goods:
 - ▶ Demand of current consumption increases when lifetime wealth increases (consumption is a normal good)
 - ▶ Current consumption decreases with interest rate increases (we assume substitution effect dominates)

The Dynamic Government

- ▶ Government spends and taxes in two periods
- ▶ **NEW:** now it can *borrow* and *save*!
- ▶ Budget constraint today

$$G = T + B$$

- ▶ Budget constraint tomorrow

$$G' + (1 + r)B = T'$$

- ▶ Life-time budget

$$G + \frac{G'}{1 + r} = T + \frac{T'}{1 + r}$$

Competitive Equilibrium

- ▶ Consumers choose first- and second-period consumption and savings optimally given the real interest rate.
- ▶ The government present-value budget constraint holds.
- ▶ The credit market clears.

Ricardian Equivalence

- ▶ **Punchline:** the timing of taxes is irrelevant
- ▶ Key equation: lifetime budget constraint (in today's \$)

$$c + \frac{c'}{1+r} = y + \frac{y'}{1+r} - \left(t + \frac{t'}{1+r} \right)$$

- ▶ Budget constraint today

$$t + \frac{t'}{1+r} = G + \frac{G'}{1+r}$$

- ▶ substituting in consumer lifetime budget

$$c + \frac{c'}{1+r} = y + \frac{y'}{1+r} - \left(G + \frac{G'}{1+r} \right)$$

Ricardian Equivalence

- ▶ Suppose the government today lowers T by 100 and raises T' to balance budget tomorrow, G and G' are fixed.
- ▶ If $r = 3\%$, tomorrow the government owes 103, so $T' = 103$.

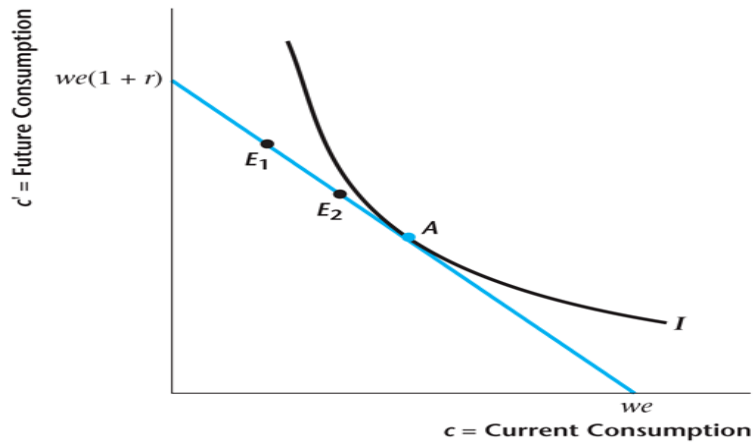
$$c + \frac{c'}{1+r} = y + \frac{y'}{1+r} - \left(t + \frac{t'}{1+r} \right)$$

- ▶ Household has two courses of actions:
 - ▶ Keep same spending on c , today save 100, tomorrow earn 103 and use to pay increased taxes.
 - ▶ Spend today the extra 100, today save 0, tomorrow lower consumption by 103 to pay for taxes.
- ▶ What would the household do?

Ricardian Equivalence

- ▶ Does r change? No: since demand and supply of assets cancels out
- ▶ Think about the previous example.

Ricardian Equivalence



Ricardian Equivalence: Assumptions

- ▶ Taxes are equal for all populations (no redistribution)
- ▶ Debt repaid while you are alive
- ▶ Lump sum taxes
- ▶ Credit markets without frictions

Example 1

A consumer can only live two periods. His real income is y in the first period and y' in the second period. He pays no taxes in each period. He chooses the period-one consumption c , the period-two consumption c' , and the period-one saving s to maximize his utility subject to the budget constraint. When the consumer saves the amount s in the first period, he can receive $(1 + r)s$ in the second period. Assume that $r > 0$, $y > 0$, and $y' > 0$. The consumer has preferences represented by downward-sloping and convex indifference curves. The consumption goods in both periods are normal goods.

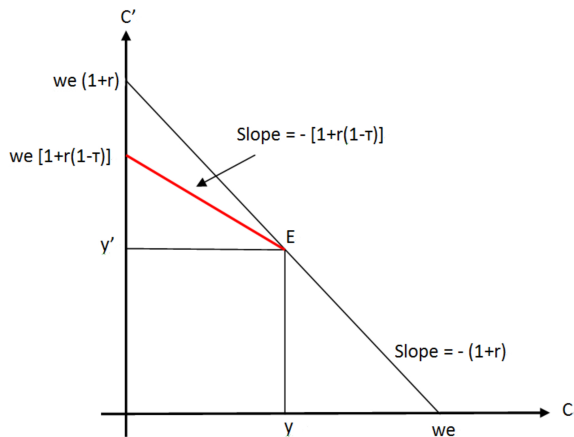
Suppose that the government introduces a tax on interest earnings. *i.e.* borrowers face a real interest rate of r before and after the tax is introduced, but lenders receive an interest rate of $(1 - \tau)r$ on their savings, where τ is the tax rate. Therefore, we are looking at the effects of having τ increase from zero to some value greater than zero, with r is assumed to remain constant.

Example 1

1. Show the effects of the increase in the tax rate on the consumer's lifetime budget constraint.
2. How does the increase in tax rate affect the optimal choice of consumption (in the current and future periods) and saving for the consumer?
3. Suppose the consumer receives y and y' and pays taxes T and T' in today and tomorrow. Use this model to explain the Ricardian equivalence of the timing of taxes

Example 1

1.



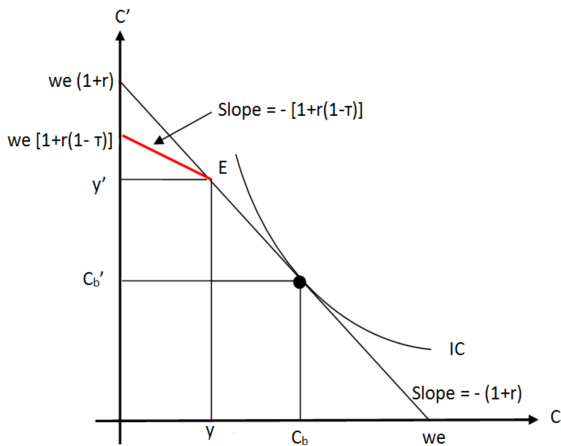
Example 1

2.

- ▶ For the borrower: faces a real interest rate of r before and after the tax is introduced. $\Rightarrow$ No impact to the borrower
- ▶ For the lender: receives an interest rate of $(1 - \tau)r$ after the tax is introduced. The return on savings is lower. $\Rightarrow$ Income and substitution effects.

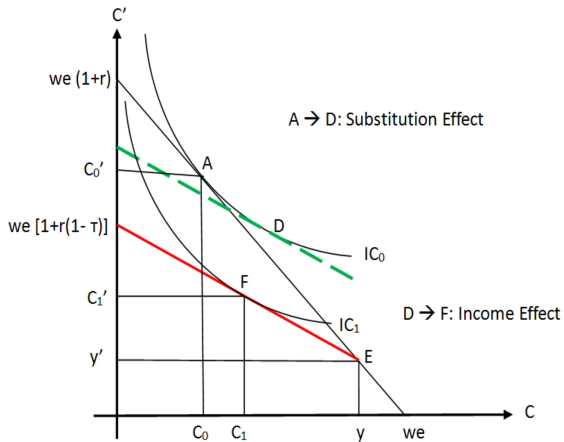
Example 1

2. for a borrower



Example 1

2. for a lender



Example 1

3.

- ▶ If the representative household is a borrower and the tax cut is **less than** he borrows Ricardian equivalent holds since this new policy does **not** change the interest rate $\Rightarrow$ do not the optimal allocation. *i.e.* c and c' are unchanged.
- ▶ Otherwise, this tax policy changes the representative household's optimal allocation. Ricardian equivalent does **not** hold. For example, for each dollar he saves, he will get $1 + (1 - \tau)r$ tomorrow. However, for each dollar of tax cut today, he has to pay $1 + r$ tomorrow.