

EE2104 Introduction to Electromagnetics
Assignment 3

Deadline (for uploading): 5:00 pm, 2 Nov 2021
Please put down your name and university ID.

1. Consider a copper wire with a cross-sectional area $S = 2 \text{ mm}^2$. Copper has $n = 10^{29}$ electrons in every 1 m^3 . It carries a current of $I = 4 \text{ A}$ for driving a light bulb $L = 50 \text{ m}$ away from a voltage source. The conductivity is $\sigma = 10^8 \Omega^{-1}\text{m}^{-1}$.
 - (a) Find the time needed for an electron to drift from the source to the light bulb.
 - (b) Find the power dissipated in every 1 m^3 of the copper.

2. In a region of permittivity ϵ_0 , the potential field is given in cylindrical coordinates by

$$V(\rho, \phi, z) = V_0 \frac{\rho}{a} \cos \phi$$

where V_0 is a constant potential and a is a constant length.
Find the electric field.

3. Given a field $\mathbf{G} = y\hat{\mathbf{x}} + x\hat{\mathbf{y}}$. There are points $A(1, 1)$ and $B(-1, 1)$.
 - (a) Find

$$\int_{\text{Path } S} \mathbf{G} \cdot d\mathbf{l}$$

where the path S is a straight line from point A to point B .

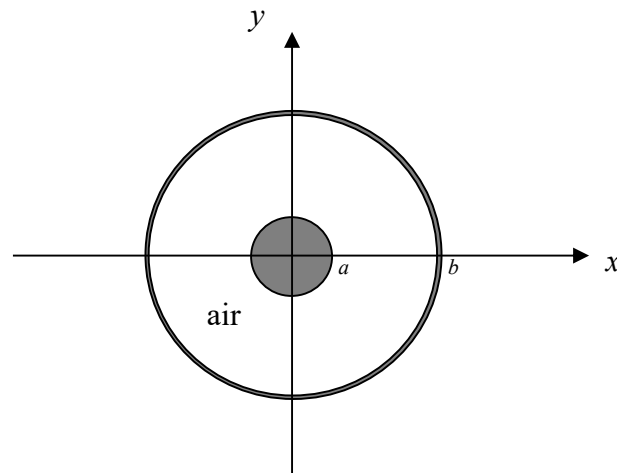
- (b) Find

$$\int_{\text{Path } C} \mathbf{G} \cdot d\mathbf{l}$$

where the path C is the circular arc of radius $\sqrt{2}$ connecting point A and point B .

- (c) Is $\mathbf{G}$ conservative? Why?

4. Consider a coaxial cable below consisting of cylinders of perfect conductors of length $l = 7$ m. In the cylindrical coordinate system, the inner conductor is at $0 < \rho < a$ and the outer conductor is at $\rho = b$, where $a = 3$ mm and $b = 8$ mm. The outer conductor is grounded. The region between the conductors is filled with air.



- (a) Find the maximal $|\mathbf{E}|$ in the structure.
 - (b) Find the capacitance of the whole cable.
 - (c) Suggest one way to double the capacitance.
5. A concentric spherical structure is placed at the origin of a spherical coordinate system (r, θ, ϕ) as shown by the figure below. The different layers contain the following materials: (i) inner conductor for $a < r < b$, (ii) outer conductor for $c < r < d$, (iii) dielectric with relative permittivity ϵ_r for $r < a$, and (iv) air for any other r . The inner conductor has potential $+V_0$ while the outer conductor is grounded.
- (a) Using Laplace equation, find the expressions for the potential V for $0 < r < a$ and $b < r < c$. Retain the constants of integration as unknowns.
 - (b) Solve the unknown constants for V .
 - (c) Find the electric field $\mathbf{E}$ for $b < r < c$.
 - (d) Find the capacitance of the concentric conducting spheres.

