
EE1001

Foundations of Digital Techniques

Sequences and Series

Exercise 3, Q+A

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Sequences and series
Recurrence relations of sequences

Question 1

- Determine the following statements whether they are true or false

$$\sum_{n=k}^N a_n \times \sum_{n=k}^N b_n = \sum_{n=k}^N (a_n \times b_n)$$

$$c \times \prod_{n=k}^N a_n = \prod_{n=k}^N ca_n$$

$$\prod_{n=k}^N a_n + \prod_{n=k}^N b_n = \prod_{n=k}^N (a_n + b_n)$$

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Ans: False

$$c \times \prod_{n=k}^N a_n = \prod_{n=k}^N ca_n$$

Ans: False

$$\prod_{n=k}^N a_n + \prod_{n=k}^N b_n = \prod_{n=k}^N (a_n + b_n)$$

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Question 2

- Find the general formula of the following sequences
 - 1) $\{0, 3, 2, 5, 4, \dots\}$
 - 2) $\{-1, 5, -7, 17, -31, \dots\}$

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Ans:

1) $a_k = (-1)^k + k$

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2) $a_k = (-2)^k + 1$

Question 3

- Find the first 3 terms and the 11th term of the sequence whose general term is given by $a_n = 4(-2)^n$

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Ans:

$$a_n = 4(-2)^n$$

$$a_1 = 4(-2)^1 = -8$$

$$a_2 = 4(-2)^2 = 16$$

$$a_3 = 4(-2)^3 = -32$$

$$a_{11} = 4(-2)^{11} = -8192$$

Question 4

- Use summation notation and product notation to denote the following formulas

1) $0 + 3 + 2 + 5 + 4 + \dots$

2) $-1 \times 5 \times -7 \times 17 \times -31 \times \dots \times 1025$

Question 4

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1) $0 + 3 + 2 + 5 + 4 + \dots$

2) $-1 \times 5 \times -7 \times 17 \times -31 \times \dots \times 1025$

Ans:

1) The general term is $a_k = (-1)^k + k$

2) The general term is $a_k = (-2)^k + 1$

$$\sum_{k=1}^{\infty} [(-1)^k + k]$$

$$\prod_{k=1}^{10} [(-2)^k + 1]$$

Question 5

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Ans:

$$\begin{aligned}a_n &= 1 + 2a_{n-1} \\&= 1 + 2(1 + 2a_{n-2}) = 1 + 2 + 2^2 \times a_{n-2} \\&= 1 + 2 + 2^2 \times (1 + 2a_{n-3}) = 1 + 2 + 4 + 2^3 \times a_{n-3} = 2^0 + 2^1 + 2^2 + 2^3 \times a_{n-3} \\&= \dots \\&= 2^0 + \dots + 2^{n-3} + 2^{n-2} \times a_2 \\&= 2^0 + \dots + 2^{n-3} + 2^{n-2} + 2^{n-1} \times a_1 \\&= 2^0 + \dots + 2^{n-2} + 2^{n-1} \times 3 \\&= (2^0 + \dots + 2^{n-2}) + 2^{n-1} \times 3 \\&= \sum_{k=1}^{n-1} 2^{k-1} + 3 \times 2^{n-1}\end{aligned}$$

Question 6

- The device population increases 20% from time $n-1$ to time n . Given the initial condition of $a_1 = 1000$, estimate the number of devices at the time instant 38.

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$$\begin{aligned}a_n &= 1.2a_{n-1} \\&= (1.2)^2 \times a_{n-2} \\&= \dots \\&= (1.2)^{n-1} \times a_1 \\&= 1000(1.2)^{n-1}\end{aligned}$$

So, at the time instant 38, there are $1000(1.2)^{38-1}$ devices.

END