

Practice Problem Set 4

Notes: Please note that the questions are based on the material in Lecture 6. There are five questions in this practice problem set. You do not need to submit this problem set; this is only for your practice; the solutions to the problems will be posted on Canvas after the deadline for HW 3, i.e., Nov 6.

Question 1: The function f is given by

$$f(x, y) = xy \ln \left(\frac{x}{y} \right)$$

for $x, y > 0$. Find the partial derivatives $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.

Verify that $x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y} = 2f(x, y)$.

Question 2: Show that the function $f(x, y) = (x^2 + y^2)^{3/2} x^{1/2} y^{1/2}$ is homogeneous of degree 4, and verify that

$$x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y} = 4f$$

Question 3: The notation $\min(x, y)$ stands for ‘the smaller of the values of x and y ’. Sketch typical contours for the function $m(x, y) = \min(x, y)$ in $\mathbb{R}_+^2$.

Question 4: The function f is given, for $x, y \neq 0$ by

$$f(x, y) = \frac{4}{x} - \frac{2x}{y} + y$$

Show that f has one critical point and determine what type of critical point this is.

Question 5: A monopoly manufactures two commodities, X and Y , the markets for which interact, such that the demand functions are given by (x and y are quantities of X and Y , respectively, and p^X and p^Y are prices of X and Y , respectively)

$$x = 800(p^Y - p^X) \qquad y = 400(9 + 2p^X - 4p^Y)$$

How much of each commodity should be manufactured to maximize the profit, given that it costs \$1 to produce one unit of X and \$1.50 to produce one unit of Y ?