

EE3008 Quiz 2

(12:05-1:45 pm, Apr. 5, 2024)

Question 1 (21 marks)

Consider the following three random signals:

- $X(t)=X$, and X is a Gaussian random variable with mean 0 and variance 1.
- $X(t) = \sum_{n=-\infty}^{\infty} Z_n \cdot v(t-nT_0)$, which is shown in Fig. 1, where $Z_n = Z$ is a discrete random variable with $\Pr\{Z=1\} = 1/2$ and $\Pr\{Z=0\} = 1/2$, and $v(t)$ is a rectangular pulse.

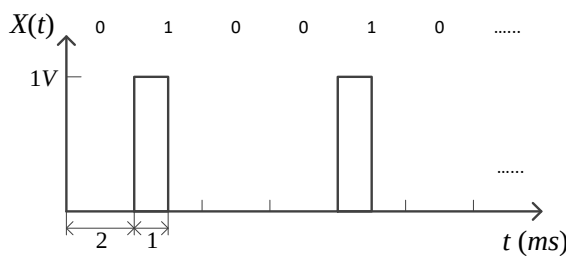


Fig. 1

- $X(t)$ has a constant power spectral density of 0.01 W/Hz in the whole spectrum.

For each of the above random signals:

- Determine its power. (15 marks)
- Suppose that $X(t)$ passes through a low-pass filter. Would the output signal be a White process? State your reason. (6 marks)

Solution:

- For signal (a), $X(t)$ is a **WSS** process because its mean is independent of time t and its autocorrelation function only depends on the time difference (See Q2.1, Tutorial 4 for more details). Its power is given by $P_X = R_X(0) = E[X^2] = 1$ W. 1'

For signal (b), $X(t)$ is a **cyclostationary** process because it is a digital amplitude modulated signal whose mean and autocorrelation function are both **periodic** (See Q2.2, Tutorial 4 for more details). 2'

Its power is given by $P_X = \frac{E[Z^2]}{T_0} \int_{-T_0/2}^{T_0/2} v^2(t) dt$ where $E[Z^2] = 1/2$, $T_0 = 0.002$, and $\int_{-T_0/2}^{T_0/2} v^2(t) dt = 0.001$. We can then obtain that $P_X = 1/4$ W. 1'

For signal (c), $X(t)$ is a **white** process because it has a constant power spectral density over the whole spectrum. Its power is **infinite**. 3'

2. None of it is a white process because the power spectrum of the output signal is $G_Y(f) = G_X(f) |H(f)|^2$, which cannot be a constant with $H(f) \neq 0$ only for a certain range.

Question 2 (37 marks)

A random signal $x(t)$ with power spectrum $G_X(f)$ shown in Fig. 2 is sampled at the Nyquist sampling rate and then applied to a 4-bit Midriser with dynamic range from -2 V to 2 V.

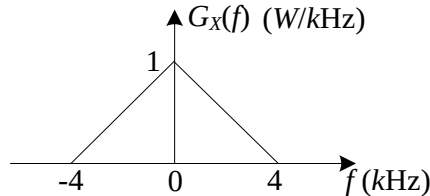


Fig. 2

1. Determine the sampling rate. (4 marks)
2. Determine the step size and the maximum quantization error. (6 marks)
3. Determine the signal-to-quantization-noise ratio (SQNR) in the form of dB. (8 marks)
4. To increase the SQNR by 3 dB, how many additional bits per sample are required? (6 marks)
5. If a 2-bit μ -law quantizer ($\mu=255$) with the same dynamic range is adopted, determine the output sequence of sampled values (0.01 V, -0.1 V, -0.3 V, -0.7 V) using Gray coding. (13 marks)

Hint: the compression curve of μ -law is given by

$$y = y_{\max} \frac{\ln[1 + \mu(|x|/x_{\max})]}{\ln(1 + \mu)} \text{sgn}(x), \text{ where } \text{sgn}(x)=1 \text{ if } x>0 \text{ and } \text{sgn}(x)=-1 \text{ if } x<0.$$

Solution:

1. As $x(t)$ is a baseband signal with bandwidth 4 kHz, we can obtain the Nyquist sampling rate as 8k samples/s.
2. The step size is $\Delta = \frac{2 - (-2)}{2^4} = 0.25 \text{ V}$. The maximum quantization error is $\frac{\Delta}{2} = 0.125 \text{ V}$.
3. As the power of $x(t)$ is $P_X = \int_{-\infty}^{\infty} G_X(f) df = 4 \text{ W}$, and the quantization error power is $\sigma_e^2 = \Delta^2 / 12 = 1/192 \text{ W}$, we can obtain that $\text{SQNR} = P_X / \sigma_e^2 = 768 = 28.85 \text{ dB}$.
4. Suppose that b' bits are required to represent each sample. According to $P_X = 4 \text{ W}$,

$$\sigma_e^2 = \frac{\Delta^2}{12} = \frac{\left(\frac{2 - (-2)}{2^{b'}}\right)^2}{12} = \frac{4^2 / 12}{2^{2b'}} \text{ W}, \text{ and } \text{SQNR} = P_X / \sigma_e^2 \geq 31.83 \text{ dB} = 1543.6, \text{ we have } b' \geq 4.5.$$

Therefore, additional 1 bit per sample is required to increase the SQNR by 3dB.

(You can also obtain b' according to $\frac{2^{2b'}}{2^{2b}} = \frac{SQNR'}{SQNR} = 3 \text{ dB} = 2 \Rightarrow b' - b = 0.5$) 6'

5. The quantization thresholds at the output are given by $y_{th} = [-2, -1, 0, 1, 2]$. According to 1'

$$y = y_{\max} \frac{\ln[1 + \mu(|x|/x_{\max})]}{\ln(1 + \mu)} \text{sgn}(x),$$

we have

$$x = (x_{\max} / \mu) \left[(1 + \mu)^{y/y_{\max}} - 1 \right] \text{ when } y > 0 \quad 2'$$

and

$$x = -(x_{\max} / \mu) \left[(1 + \mu)^{-y/y_{\max}} - 1 \right] \text{ when } y < 0. \quad 2'$$

The quantization thresholds at the input can be then obtained as $x_{th} = [-2, -0.1176, 0, 0.1176, 2]$. 5'

For (0.01 V, -0.1 V, -0.3 V, -0.7 V), the first input value falls into the range of $[0, 0.1176 \text{ V}]$ with the assigned codeword 11, 1' the second input value falls into the range of $[-0.1176 \text{ V}, 0]$ with the assigned codeword 01, 1' and last two input values fall into the range of $[-2 \text{ V}, -0.1176 \text{ V}]$ with the assigned codeword 00. Therefore, the output bit sequence is 11 01 00 00. 1'

Question 3 (42 marks)

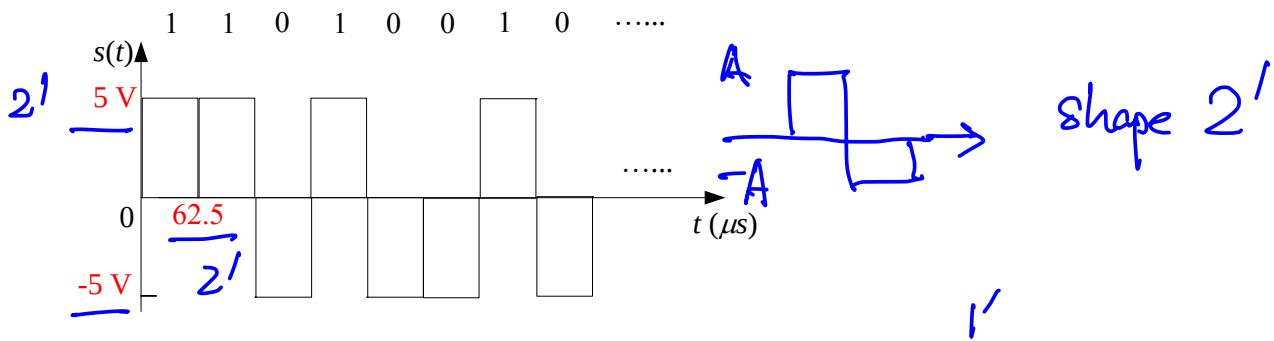
An audio signal is sampled at the Nyquist rate, quantized into 256 levels, modulated by using binary PAM with power 25 W, and transmitted over a channel with the frequency range of [0, 16 kHz]. It is observed that 90% of the modulated signal can pass through the channel.

1. Determine the bit rate of the modulated signal. (4 marks)
2. Determine the bandwidth of the original audio signal. (6 marks)
3. Plot and label the modulated signal for bit sequence "11010010". (12 marks)
4. If the bit stream is modulated by using binary Sinc-shaped pulse modulation, determine the minimum required channel bandwidth for including 100% in-band power. (4 marks)
5. If a 16-bit quantizer is adopted, and the bit stream is modulated by using M -ary PAM with the same power over the same channel.
 - a. Determine the minimum value of M for including 90% in-band power. (4 marks)
 - b. Plot and label the modulated signal for bit sequence "11010010". (12 marks)

Solution:

1. For binary PAM, to include 90% in-band power, the required channel bandwidth is equal to the bit rate. Therefore, the bit rate is $R_b = B_n = 16 \text{ kbit/s}$. 4'

2. Assume that the bandwidth of the original audio signal is B_s . The Nyquist sampling rate is then $f_s = 2B_s$. According to $R_b = bf_s$ where $b = \log_2 256 = 8$, we can obtain that $B_s = 1 \text{ kHz}$.
3. For binary PAM signal, assume that the amplitude of the rectangular pulse is A . The signal power is then given by A^2 (see Q1, Tutorial 6 for details). We can then obtain that $A = 5 \text{ V}$.
The modulated signal for bit sequence "11010010" can be plotted as follows.



4. For binary Sinc-shaped pulse modulation, the bandwidth efficiency is 2 with 100% in-band power. Therefore, with the bit rate 16 kbit/s , the required channel bandwidth is 8 kHz .
- 5a. With 90% in-band power, the channel bandwidth should be $B_h \geq R_s = R_b / \log_2 M$. With $B_h = 16 \text{ kHz}$ and $R_b = bf_s = 32 \text{ kbit/s}$, we can obtain that $M \geq 4$.
- 5b. For 4-ary PAM, the signal power is $\frac{1}{4} \left(A^2 + \left(\frac{A}{3} \right)^2 + \left(-\frac{A}{3} \right)^2 + (-A)^2 \right) = \frac{5}{9} A^2$ (see Q2, Tutorial 6 for details). We can then obtain that $A = 3\sqrt{5} \text{ V}$.

The modulated signal for bit sequence "11010010" can be plotted as follows.

