

Block A: Unit 1**The 3 Basic Laws in Circuits and Resistive Networks***G. Rizzoni Ch 2, Ch 3.1***Summary to Block A**

Block A is your introduction and gateway into electrical engineering. It focuses on the most basic concepts that form the “common sense” of electrical engineering. All modern day gadgets operate on these principles, thus it is important that one has a strong grasp from the start so that one does not trip over basic concepts later on. As such, the concepts in Block A are carried over to all the other learning blocks. Hence confusion in Block A will lead to even greater confusion in the rest of this course. **So learn it as well as you can!**

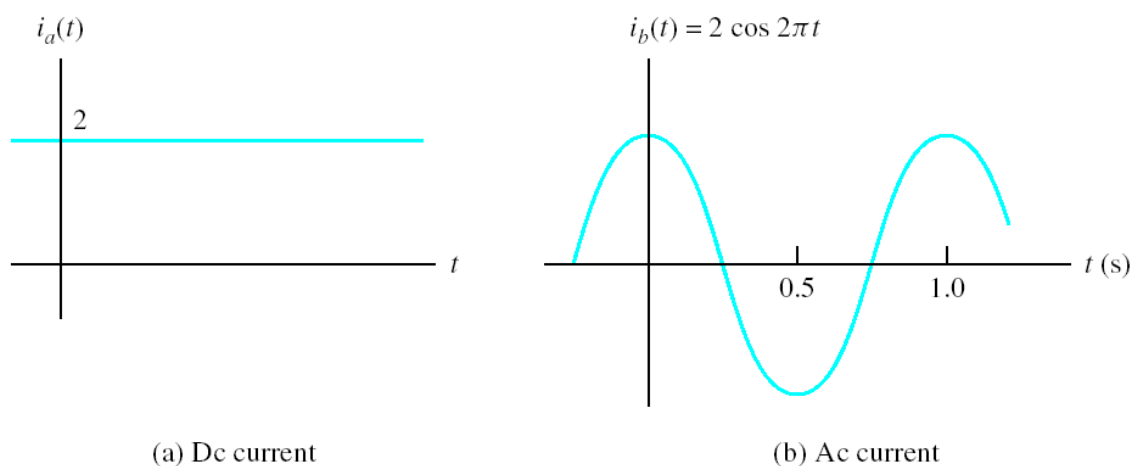


Fig 1: Illustration of difference between DC and AC

To teach the basics, we will start with the simplest case in which signals in the circuit remain constant with time as shown in Fig 1a. The signal can thus be described by

just a value. This is known as Direct Current (DC). All the basic concepts are illustrated in DC in Block A. In Block B, we will take these same concepts to a higher level in the case of Alternating Current (AC). In the case of AC as shown in Fig 1b, the signals vary with time (and can even reverse direction) periodically.

Block A: Unit 1

This unit is organized into 7 sections. The first 3 cover the 3 basic laws in circuits:

- Kirchhoff's Current Law
- Kirchhoff's Voltage Law
- Ohm's Law

The remaining sections of this unit are just applications of these 3 laws together.

I. Current and Kirchhoff's Current Law

Charge

When thinking about *electricity*, it is convenient to start with the concept of charge. Charge is a fundamental (or the most basic) electrical quality. It is typically denoted by the symbol q or Q , and its SI unit is the Coulomb (C). The smallest amount of charge that exists is the charge carried by an electron:

$$Q_e = 1.602 \times 10^{-19} \text{ C}$$

Always include units, otherwise there is no meaning

Current

Current comes simply from moving charges. Therefore, an important criterion for current to flow is for there to be charges that are free to move – mobile charges. To move the charges, a voltage needs to be applied across the conductor as shown in Fig 2a. Electrons, which are negatively charged move away from the negative end of the conductor and towards the positive end. Movement of the electrons gives rise to current.

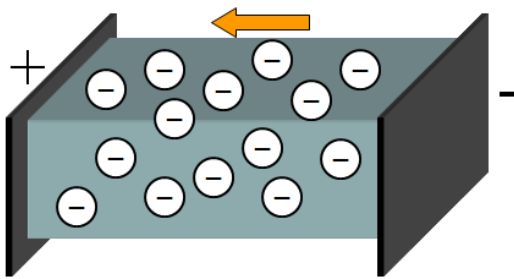


Fig 2a: Current flowing through a conductor arises from flow of free charges due to an applied voltage

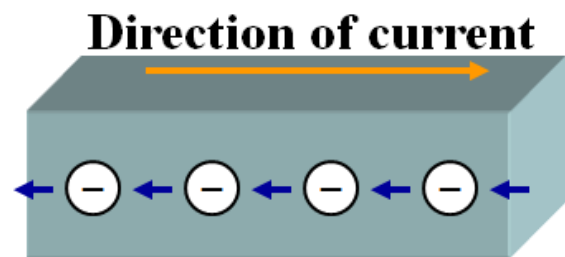


Fig 2b: Direction of conventional current opposite to electron flow

Key Concept 1

For current to flow, there must be mobile charges

In a conductor where the mobile charges are electrons, the current is due to the flow of electrons. The direction of this flow is from negative to positive. However, by convention, we think about current as flowing from positive to negative due to the movement of positive

charges. This is the convention we will use for the rest of this course: Current flows from positive to negative terminal (in the direction of positive charge flow). It is typically denoted by the symbol i or I and its SI unit is the Ampere (A)

An important property about charge is that it can neither be created nor destroyed. If current flows out of a given point (say point A), then it must return to point A with the same amount. If current does not return, then the original charge must have vanished into nothing; which cannot happen. If more current returns to point A, then charge must have been created which cannot happen. The consequence of this, as shown in Fig 3, is current must flow in loops. **What goes around comes around.**

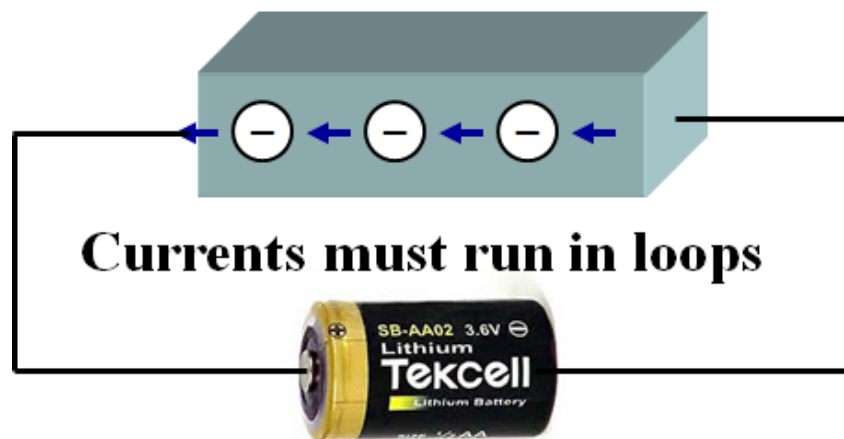


Fig 3: Currents must run in loops

Key Concept 2

Currents must run in loops (otherwise charge is either destroyed or created which cannot happen)

This property that charge cannot be created or destroyed then leads us to Kirchhoff's Current Law.

Kirchhoff's Current Law (KCL)

KCL states that the sum of currents at a point (called a **node**) is equal to zero. Fig 4 shows 4 currents arrive at the same node from 4 paths (branches).

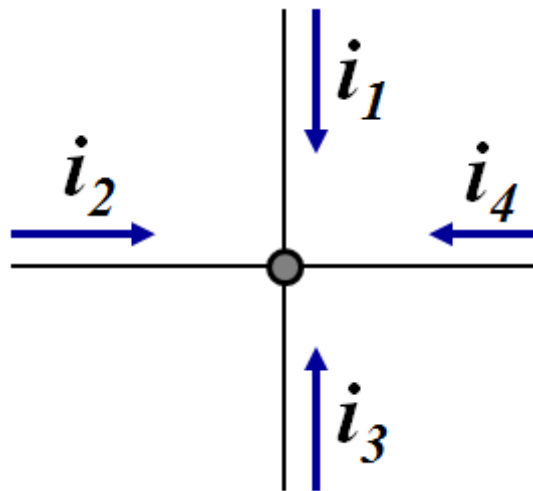


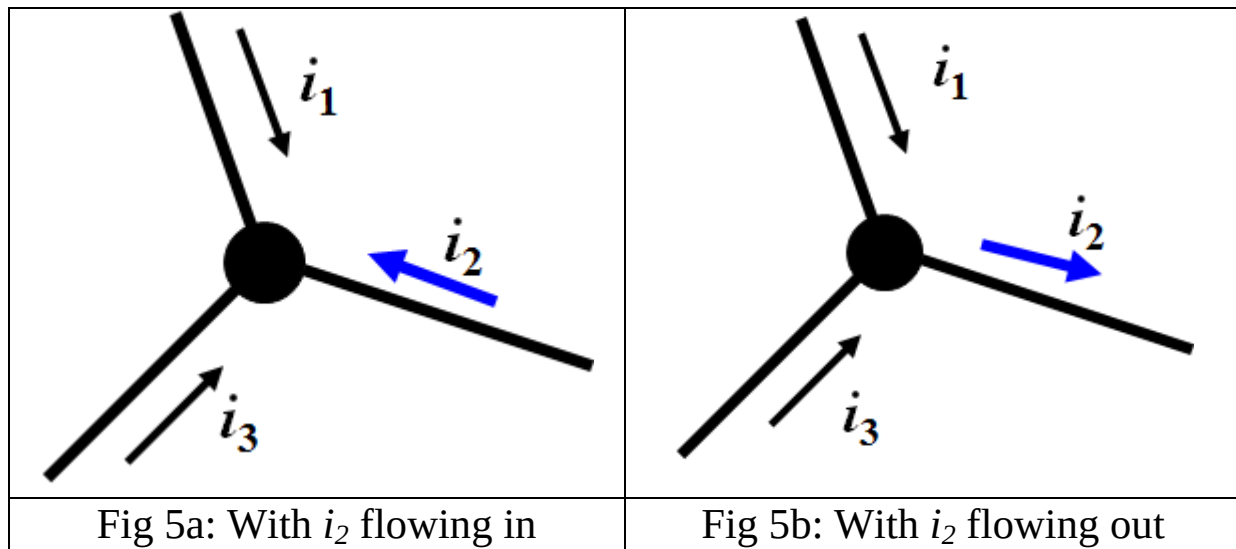
Fig 4: Illustration of 4 branch current arriving at a node

Applying KCL to the example in Fig 4, this means that:

$$I_1 + I_2 + I_3 + I_4 = 0$$

A commonsensical way to look at KCL is that total sum of current flowing into a node must equal the total sum flowing out (otherwise charge is created or destroyed). **In other words: “what goes in must also come out”.**

Whenever applying KCL, the direction of the currents matter a big deal. To illustrate why this is obviously so, refer to Fig 5.



If direction of currents did not matter and we applied KCL blindly to each example, we get the same result for both Fig 5a and 5b:

$$i_1 + i_2 + i_3 = 0$$

If we use common sense with Fig 5b (what goes in must also come out), then it should be:

$$\text{Fig 5b: } i_1 + i_3 = i_2 \Rightarrow i_1 - i_2 + i_3 = 0$$

The only difference between the two is the direction of i_2 . Obviously, the directions of the currents matter. So we define a convention to avoid this kind of confusion:

Sign convention for KCL

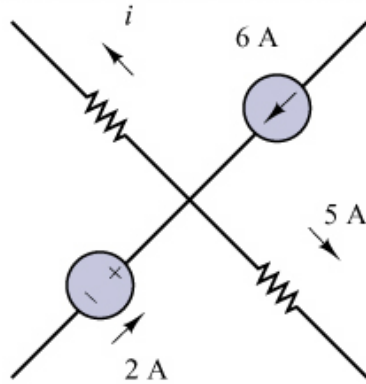
Currents flowing IN – Positive

Currents flowing OUT – Negative

Worked Example on KCL

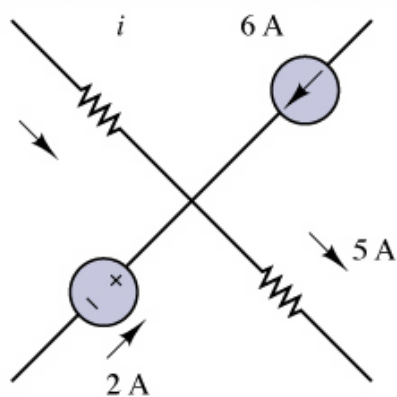
Find the unknown currents within each of the following circuit networks.

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3A

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-3A

Voltage and Kirchhoff's Voltage Law

Voltage and Potential Difference

Even if electrons are free to move, energy is still needed to move them. This energy could come from a battery cell connected to the circuit. As the current goes across the cell, energy is pumped in. And as it goes through a light bulb, energy is consumed. The amount of energy required to move a unit charge is given by the voltage. The unit of voltage is the volt (V) and is represented by the symbols v or V typically.

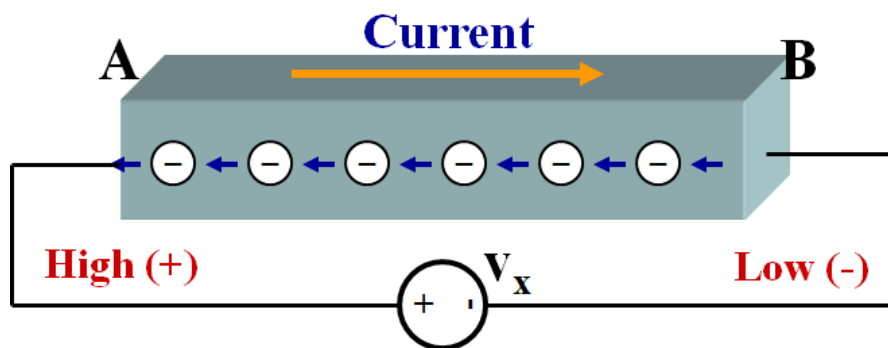


Fig 6: Potential difference and current flow

Voltage is also known as potential difference (PD). It always refers to the **difference** between 2 points. In Fig 6, V_X is the voltage across A and B. Hence when we talk about the voltage at point A, it must be referenced to another point.

Sign Convention

Note that from Fig 6, the current flows from negative (low) to positive (high) in an element that is generating power. Conversely, current flows from higher to lower potential in an element that is consuming power.

Sign Convention for Voltage

Voltage rises along the direction of the current for an element that is generating power

Voltage drops along the direction of the current for an element that is consuming power

Kirchhoff's Voltage Law (KVL)

KVL states that the sum of voltages around a loop must equal to zero. What this means is if you start at point A and when you return to point A, you do not gain or loss any voltage. Whatever energy is pumped into the loop must also be consumed.

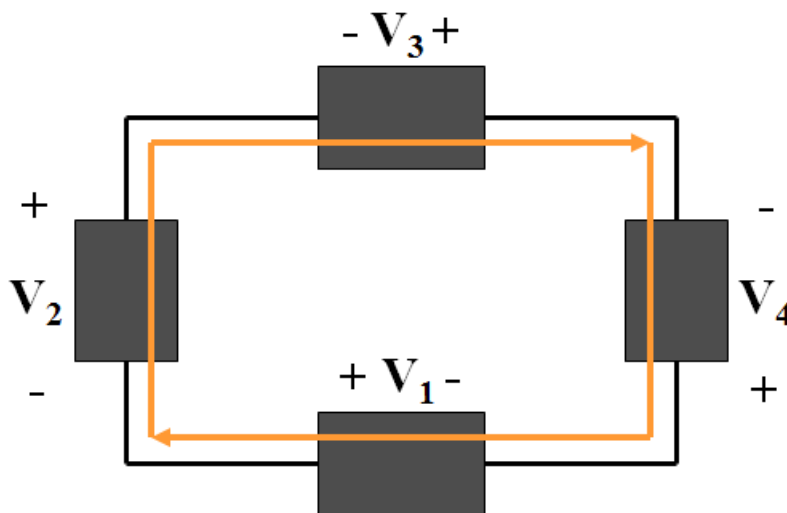


Fig 7: Voltages around a closed loop

With reference to Fig 7, applying KVL we get:

$$V_1 + V_2 + V_3 + V_4 = 0$$

If the above statement sounds suspicious, consider the following: V_1 and V_2 are voltage rises, while V_3 and V_4 are voltage drops. What it then means is that the total voltage rise of V_1 and V_2 equals the total voltage drop

of V_3 and V_4 . In simple words, **“what goes up must come down”**.

As with KCL, in KVL whether the voltage is a rise or drop makes a difference. Once again we need to define a convention to avoid confusions.

Sign convention for KVL

Voltage RISE – Positive

Voltage DROP – Negative

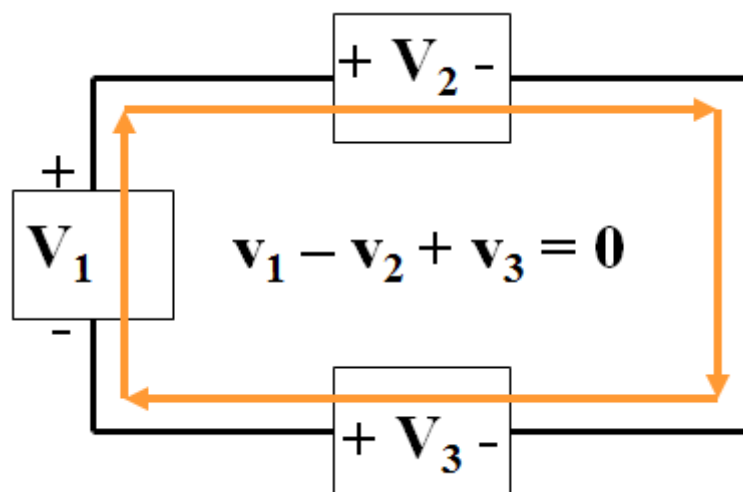


Fig 8: Defined voltage drops and rises

In Fig 8, note that in the defined direction, V_1 and V_3 are voltage rises (positive) while V_2 is a voltage drop (negative). If we went in the other direction instead, V_2 would now be a voltage rise (positive), while V_1 and V_3 are now voltage drops (negative). This would yield:

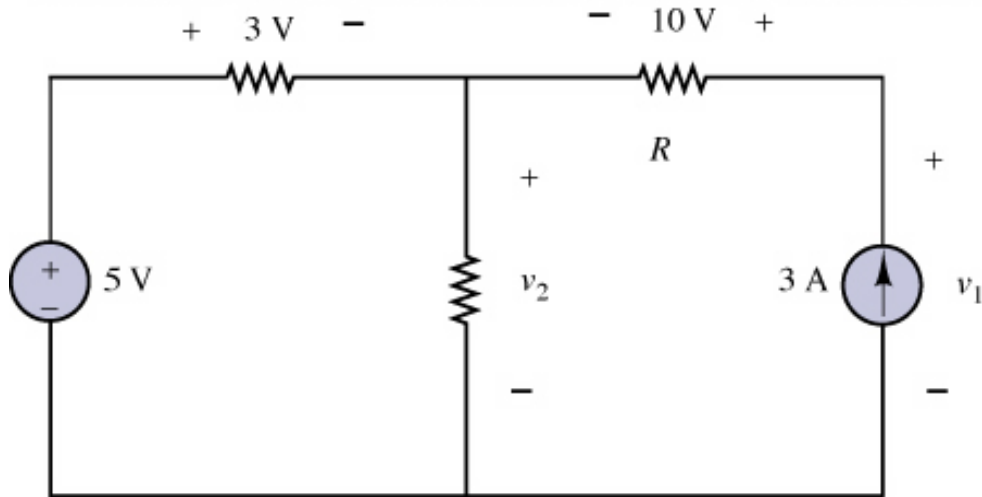
$$V_2 - V_1 - V_3 = 0$$

Hence the direction you follow does not matter but the sign convention does.

Worked Example on KVL

Apply KVL to find V_1 and V_2 in the following circuit

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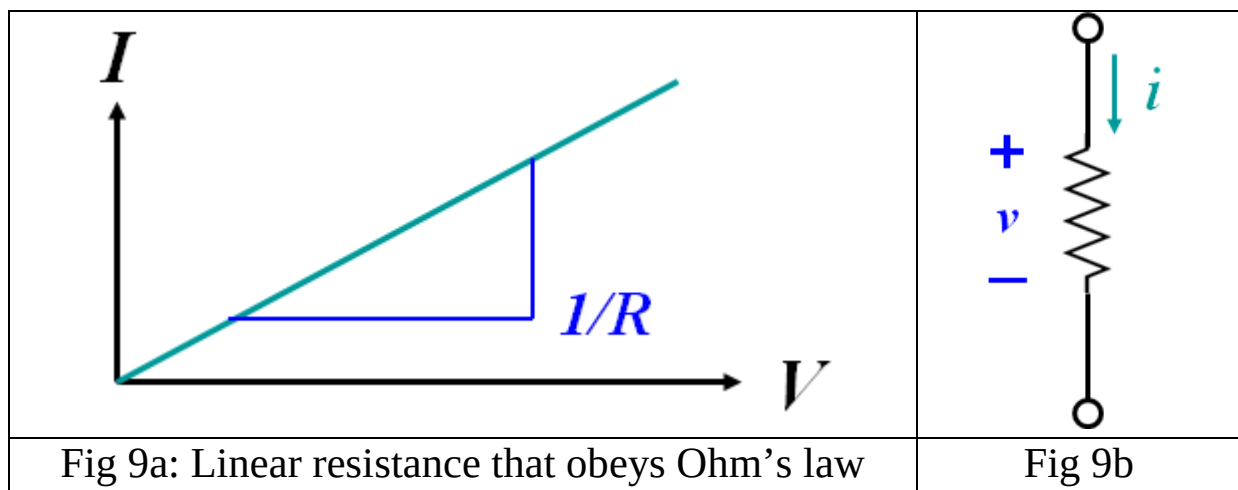


$$V_2 = 2V$$

$$V_1 = 12V$$

II. Resistance and Ohm's Law

When current flows through a conductor, it will always experience some resistance. Resistance is given by the change in voltage over change in current. Resistance is typically denoted by the symbol R and whose SI unit is the Ohm (Ω). If the voltage is linear with current, then resistance is said to be linear and obeys Ohm's law as is shown in Fig 9a.



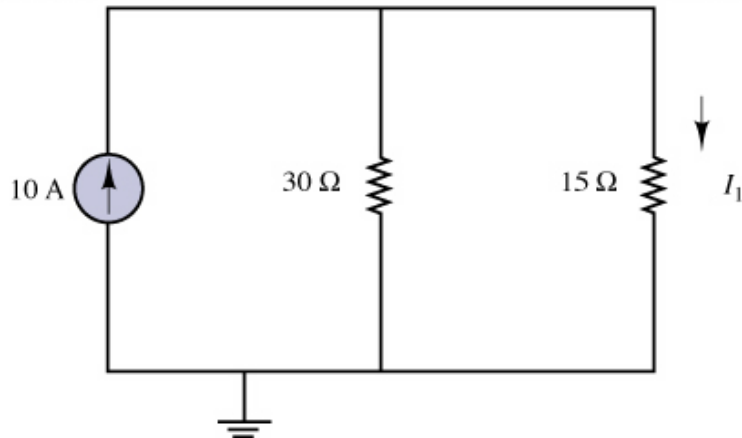
Ohm's law is given as: $V = IR$. Fig 9b shows the sign convention to be applied always when using Ohm's law. Current I in the equation is defined as flowing across a voltage V from higher to lower. **This is a simple error that is very easily and commonly overlooked!**

What this means is if you define an unknown current to flow from A to B, you are assuming the voltage at A to be higher than B.

Worked example applying the 3 laws

Find the current through the 15Ω resistor

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Voltage is the same across all 3 branches

$$I_1 = 6.67\text{A}$$

$$\text{And through the } 30\Omega = 3.33\text{A}$$

III. Electrical Power

In Section II, we have already seen that a voltage drop in the direction of the current indicates that power is consumed. Conversely, a voltage rise in the direction of the current indicates that power is generated.

If the voltage across a resistor is V , and current through it is I , then the power consumed is given by:

$$P = VI$$

$$\text{Ohm's law} \Rightarrow P = I^2 R$$

$$\text{Ohm's law} \Rightarrow P = V^2 / R$$

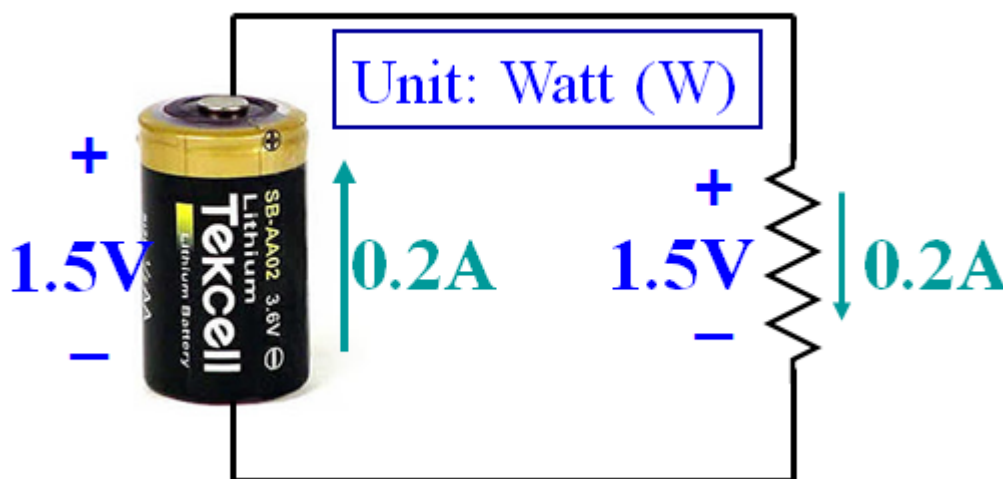


Fig 11: Power generated and consumed in a circuit

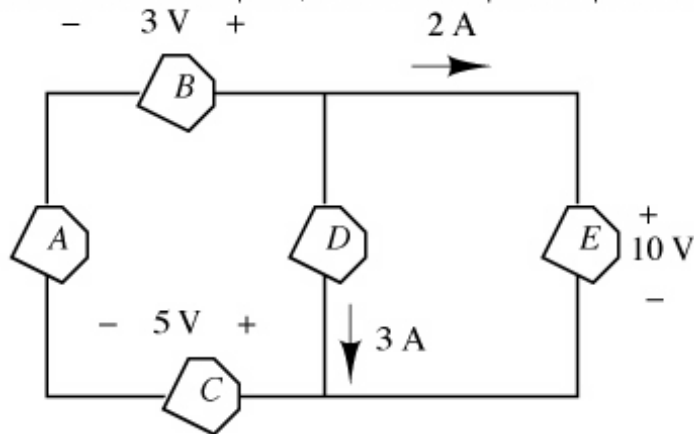
Because KVL is based on physical laws which state that energy cannot be created or destroyed, the power that is generated in a circuit must equal the power consumed.

This can be a simple but powerful tool for checking the validity of calculated values for I and V .

Example on Electrical Power

Determine which components are absorbing power and which are delivering power

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A: 60W (generating)

B: 15W (generating)

C: 25W (dissipating)

D: 30W (dissipating)

E: 20W (dissipating)

The following are common terms of referring to parts of a circuit. They will be used throughout this course.

Branches and Nodes

A **branch** is any path of a circuit that has 2 terminals connected to it. In Fig 12a, 3 branches are shown.

A **node** is a junction of 2 or more branches. In Fig 12b, 1 node is shown, which is connected to 4 branches.

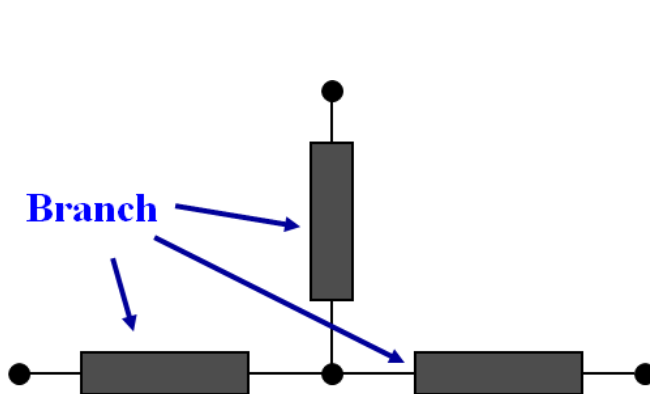


Fig 12a: 3 branches

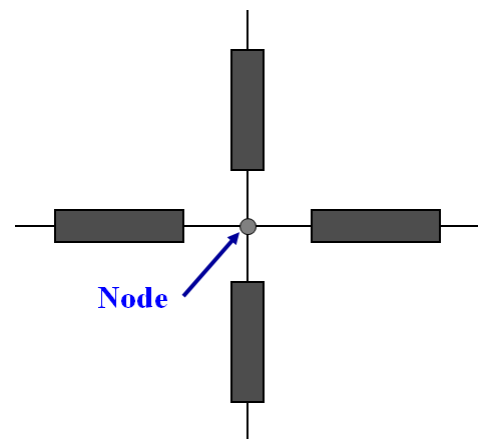


Fig 12b: 1 node

Loops and Meshes

A **loop** is any closed connection of branches

A **mesh** is a loop that does not contain other loops

In Fig 13, there are 6 loops and 3 meshes. We will use the term mesh more than loop in this course.

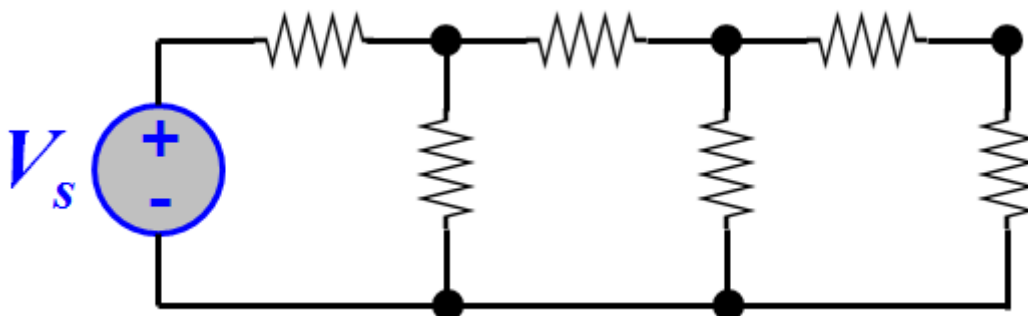


Fig 13: Loops (6) and meshes (3)

IV. Sources, Short circuit, Open circuit

We have seen so far that loads (e.g. resistors) consume power. Sources on the other hand deliver power. It is useful to describe sources as either a voltage source or a current source. The function of each is to maintain the value of the quantity they provide.

Ideal voltage source

The purpose of a voltage source is to keep the voltage across its terminals unchanged regardless of any change across its terminals. Current through a voltage source is allowed to change in order to maintain the voltage at V with reference to Fig 14a.

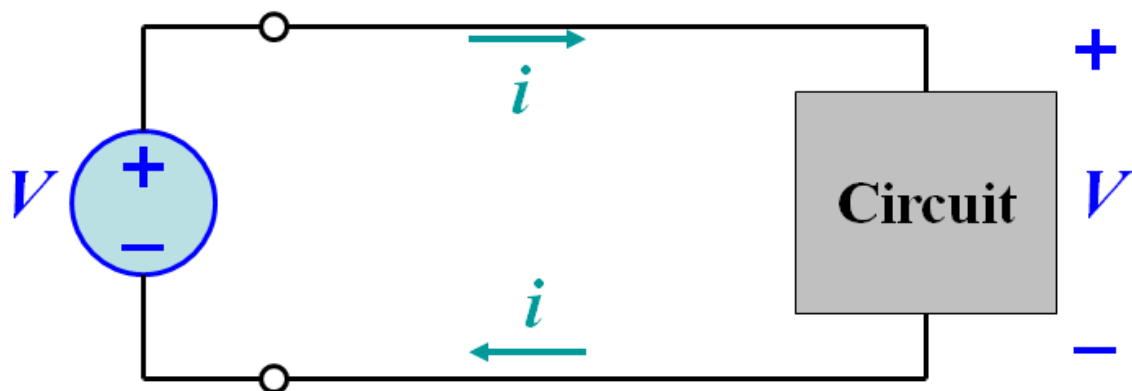


Fig 14a: Ideal voltage source in a circuit

Ideal current source

The purpose of a current source is to keep the current flowing through it unchanged regardless of any change across its terminals. Voltage across a current source is allowed to change in order to maintain the current at I with reference to Fig 14b.

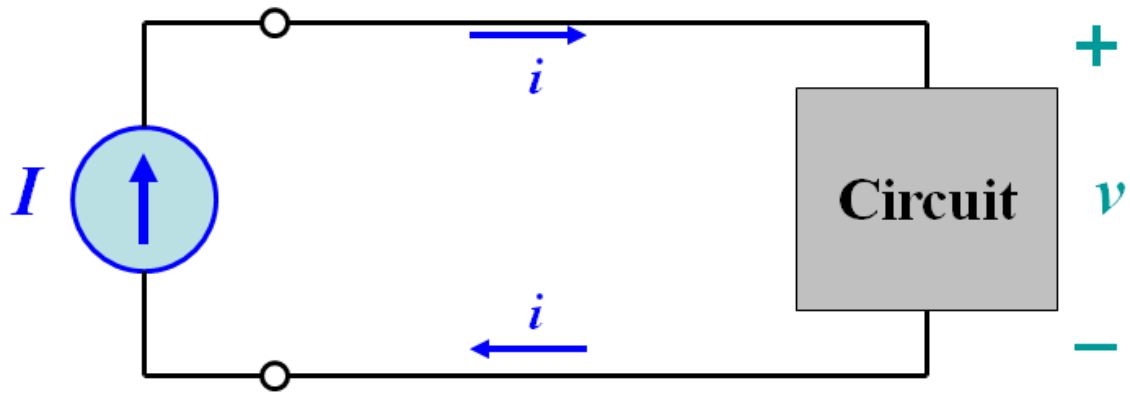


Fig 14b: Ideal current source in a circuit

Recognise the differences between an ideal voltage and current source in their function and symbols.

Short Circuit

A short circuit means we connect 2 or more terminals together so that the voltage between them is zero. It is typically associated with currents rather than voltage, e.g. short circuit current. We use Fig 15a as an example. If we short circuit A and B, we connect A and B so that the voltage across A and B is zero. We can see now that all the current from the source flows through the short circuit, bypassing the resistor. The short circuit current is then: $I_{SC} = 0.1A$

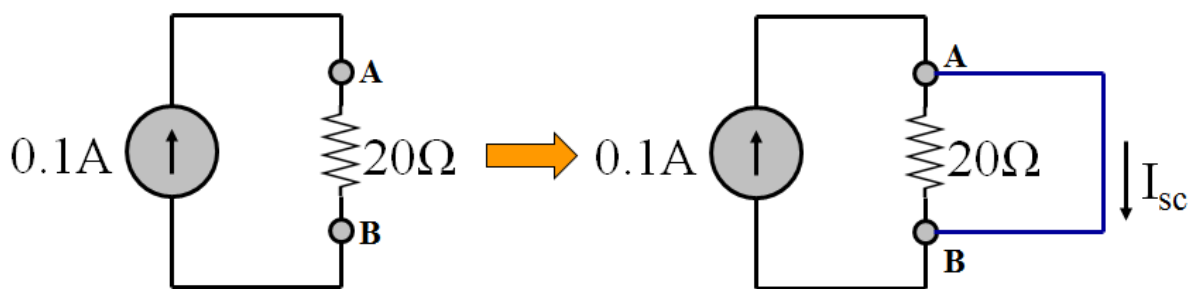


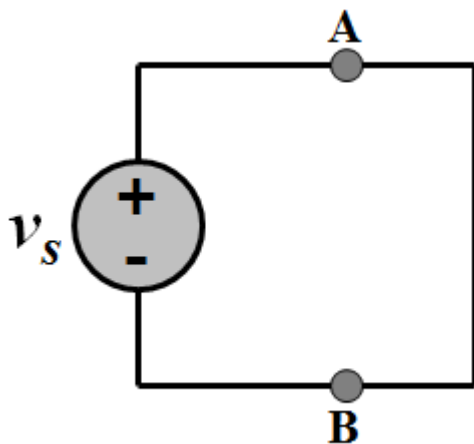
Fig 15a: Short circuiting the resistor

Open Circuit

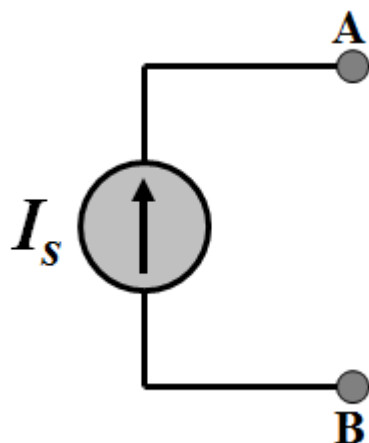
An open circuit means that no extra connections are imposed across two terminals. It is typically associated with voltage rather than current. Using Fig 15b as an example, say if we want to find the open circuit voltage across the resistor, this is simply the voltage across the resistor: $V_{OC} = 2V$.

Examples of self-contradicting circuits

What is the voltage across A and B?



What is the current through the source?



Prefixes

As engineers, it is important that we replace exponents with prefixes. For example, if you find the current in a circuit is 0.00215A, you should express it as 2.15mA; never 2.15×10^{-3} A. Memorize and use them well!

tera	T	10^{12}
giga	G	10^9
mega	M	10^6
kilo	k	10^3
milli	m	10^{-3}
micro	μ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}
femto	f	10^{-15}

In this course you are likely to encounter quantities that range from pico (p) to mega (M). It is quite common to mix up milli (m $\sim 10^{-3}$) and mega (M $\sim 10^6$), but you can see there is a HUGE difference between them.

Make sure you know the prefixes for at least this range. It is good practice to use prefixes, so start early.

V. Resistive Networks

Resistors are either arranged in parallel or in series or as a combination of both. This section is about how any network can be replaced by a single equivalent resistor.

Parallel network

Fig 16a shows an example of a parallel network of N resistors. These can be represented by a single resistor R_P whose value is given by:

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots + \frac{1}{R_N}$$

Hence now if you want to find the voltage across the current source, this is simply $= R_P I_S$

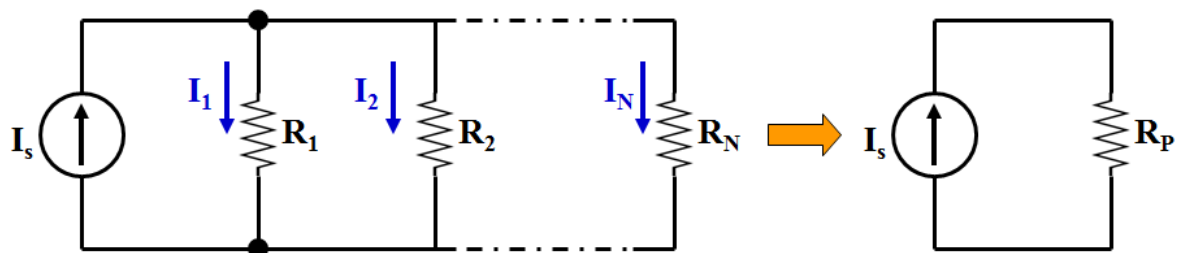


Fig 16a: Parallel resistive network

Current divider rule

One can see that the current from the source will have to be shared between the resistors in each branch. The current will divide itself up according to the following proportion (through resistor R_k):

$$I_k = \frac{R_P}{R_k} I_S$$

Series network

Fig 16a shows an example of a parallel network of N resistors. These can be represented by a single resistor R_p whose value is given by:

$$R_S = R_1 + R_2 + \dots + R_N$$

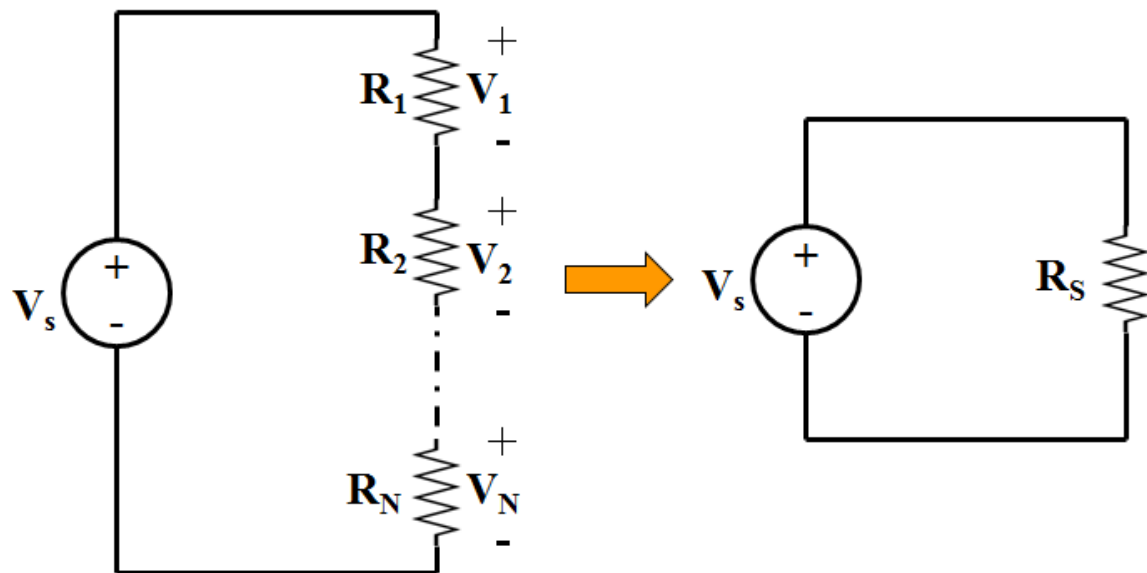


Fig 16b: Series resistive network

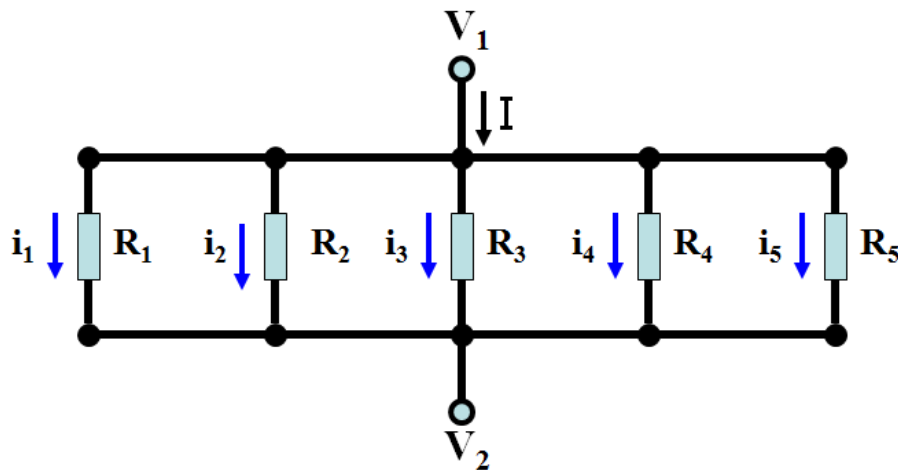
Voltage divider rule

One can see that the total voltage drop across all the resistors in series is made up of the sum total of voltage drops across each resistor. The voltage drop across a given resistor (V_k) as a fraction of the total resistance is:

$$V_k = \frac{R_k}{R_S} V_S$$

Worked Example on current divider rule

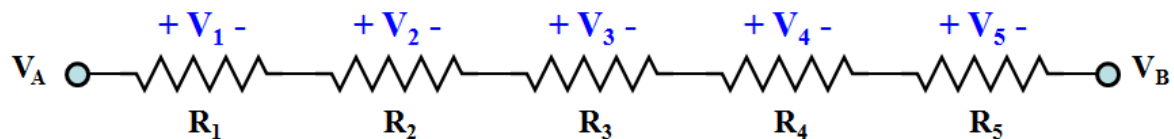
Rank the currents from largest to smallest



$$R_2 > R_4 > R_1 > R_5 > R_3$$

Worked Example on voltage divider rule

Rank the PDs from largest to smallest

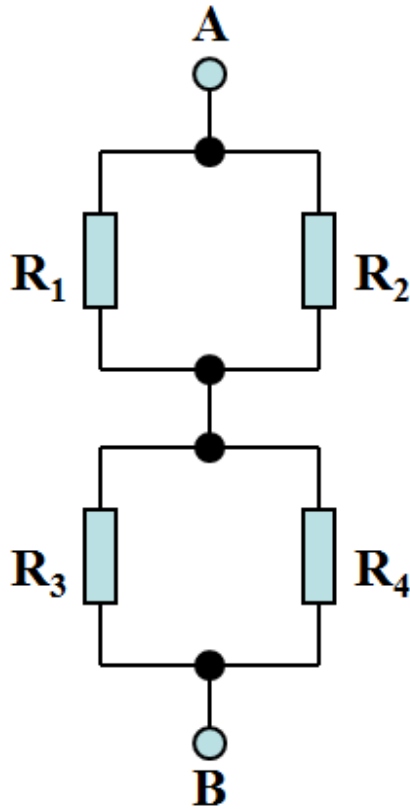


$$R_2 > R_4 > R_1 > R_5 > R_3$$

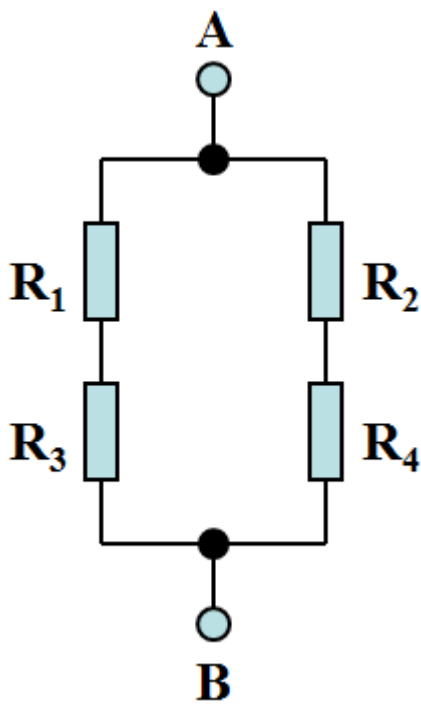
Worked Example 1 on resistive networks

Find the effective resistance across A and B

$$R_1 = R_4 = 1\Omega, R_2 = R_3 = 3\Omega$$



Equivalent $R = 1.5\Omega$



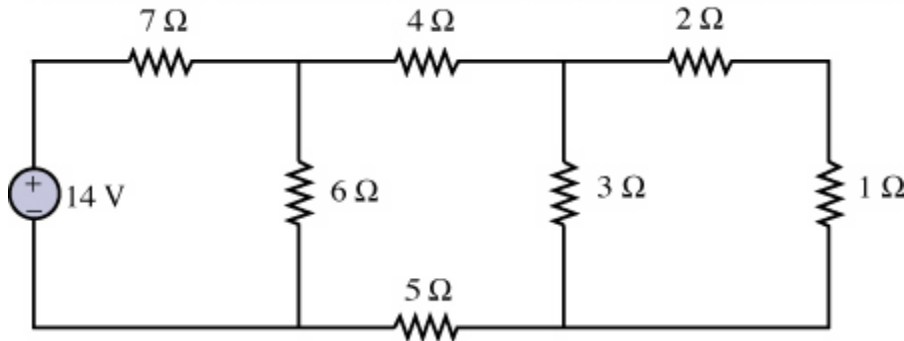
Equivalent $R = 2\Omega$

Worked Example 2 on resistive networks

What is the equivalent resistance seen by the source?

What is power supply by the source?

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Resistive network to right of 6Ω (excluding 90Ω) has an equivalent resistance of 10.5Ω

6Ω is parallel to this resistance, so after including 6Ω we have an equivalent resistance of 3.818Ω

Total equivalent resistance is 10.818Ω

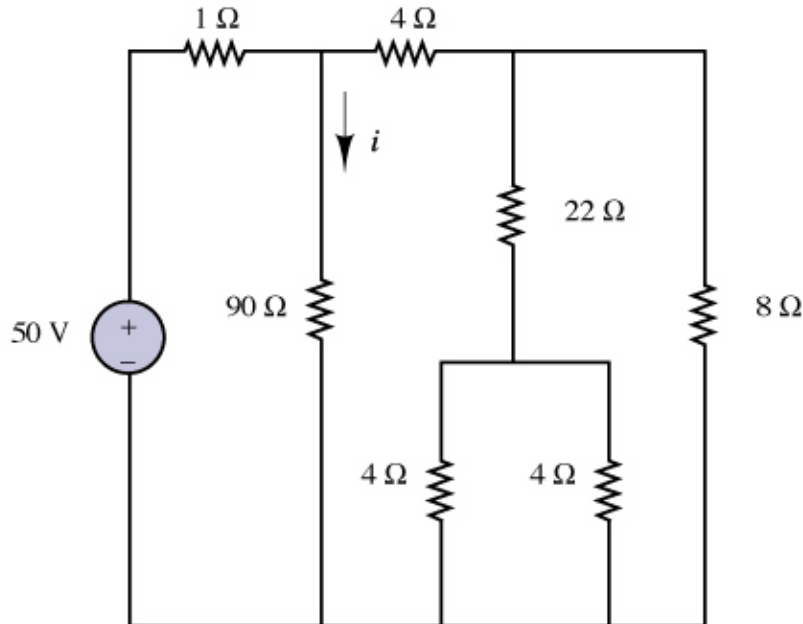
Power supplied = $14^2/10.818 = 18.1\text{W}$

Worked Example 3 on resistive networks

Find the equivalent resistance seen by the source

Find the current through the 90Ω resistor

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Resistive network to right of 90Ω (including 90Ω) has an equivalent resistance of 9Ω .

⇒ Circuit simplifies to 50V in series with 1Ω and 9Ω

⇒ Equivalent resistance across 50V source = 10Ω

Voltage across 90Ω = Voltage across equivalent 9Ω

By voltage divider rule, voltage across 90Ω = 45V

Current through 90Ω = 0.5A (ohm's law)

Worked Example 4 on resistive networks

For the following circuit, find the equivalent resistance if all resistors have the same value of 100Ω .

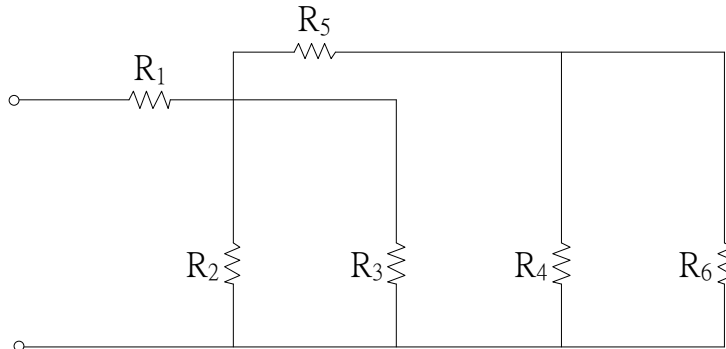


Figure P2.49

$$R4 \text{ and } R6 \text{ in parallel} \Rightarrow \text{combine} = 50\Omega \quad (1)$$

$$R2 \text{ and } R3 \text{ in parallel} \Rightarrow \text{combine} = 50\Omega \quad (2)$$

$$R5 \text{ in series with (1)} \Rightarrow \text{combine} = 150\Omega \quad (3)$$

$$(2) \text{ \& (3) in parallel} \Rightarrow \text{combine} = 37.5\Omega \quad (4)$$

$$R1 \text{ and (4) in series} \Rightarrow 137.5\Omega$$

VI. Measuring instruments

In theory, all values of voltage and current are given to you. In real life, we obtain them by measurement. One should thus at least know how to use these measuring instruments and be aware of things to look out for when using them. **You will be using them in the labs.**

Voltmeter

As the name suggests, it is obvious that the function of a voltmeter is to measure voltage or PD. Fig 17 shows what a typical voltmeter looks like. Fig 18a shows how to place a voltmeter when trying to measure the PD. As shown in Fig 18a, the PD across R_2 can be measured by placing the voltmeter across (or parallel to) R_2 . If you wanted to measure the PD across v_s , place the voltmeter across v_s .



Fig 17: Image of typical voltage with a measuring range up to 30V.

How would you place the voltmeter in the circuits of the previous worked examples?

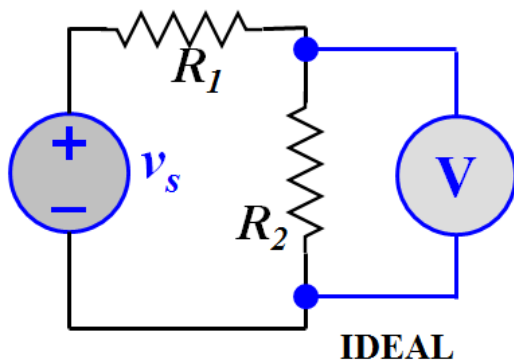


Fig 18a: Placement of an ideal voltmeter to measure the PD across R_2

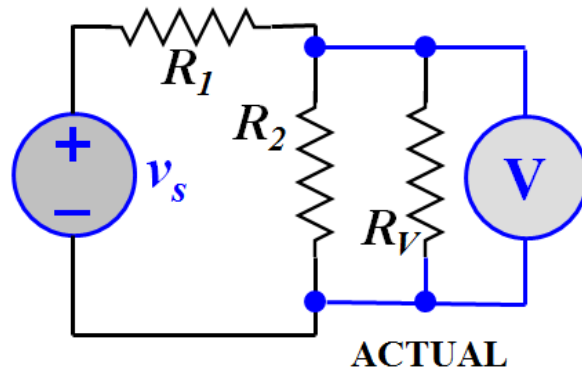


Fig 18: Circuit model of how a real voltmeter behaves, which looks like a large parallel resistor (R_V)

Fig 18a represents the ideal case. Notice here that the voltmeter has no effect on the original circuit (it does not change any voltage or current). When one actually connects a voltmeter like in Fig 18a, the voltmeter will draw some current. Fig 18b illustrates this effect. R_V in the voltmeter steals current from R_2 . This will change the voltage measured across R_2 . So in real life, the PD that you measure with a voltmeter is lower than the true value.

The question is: how big is this difference? The worked example hopefully helps to illustrate this point.

Worked example on Voltmeters

Referring to Fig 18b, if $V_s = 5V$, and $R_V = 1M\Omega$, find the voltage across R_2 when (i) $R_1 = R_2 = 1k\Omega$; (ii) $R_1 = R_2 = 1M\Omega$

When resistors = $1k\Omega$

Without voltmeter: $V = 2.5V$

With voltmeter: $V = 2.499V$ (not much difference)

When resistors = $1M\Omega$

Without voltmeter: $V = 2.5V$

With voltmeter: $V = 1.67V$ (much more obvious difference)

Ammeter

Just as obvious, an ammeter measures current. Fig 19 shows what a typical ammeter. Fig 20a shows how to place an ammeter when trying to measure current. As shown in Fig 20a, the current through across R_1 can be measured by placing the ammeter in series with R_1 .

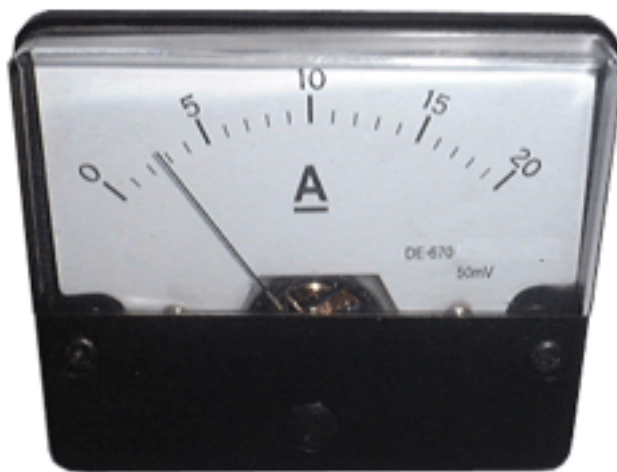


Fig 19: Image of typical voltage with a measuring range up to 20A.

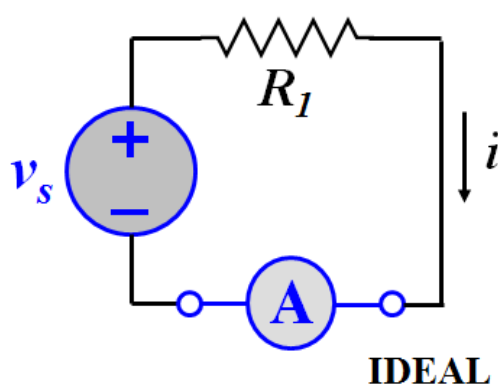


Fig 20a: Placement of an ideal ammeter to measure current through R_1

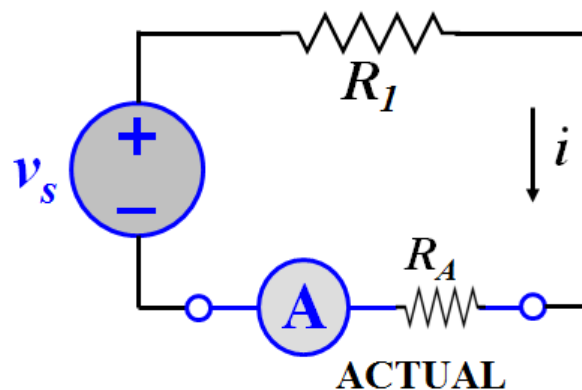
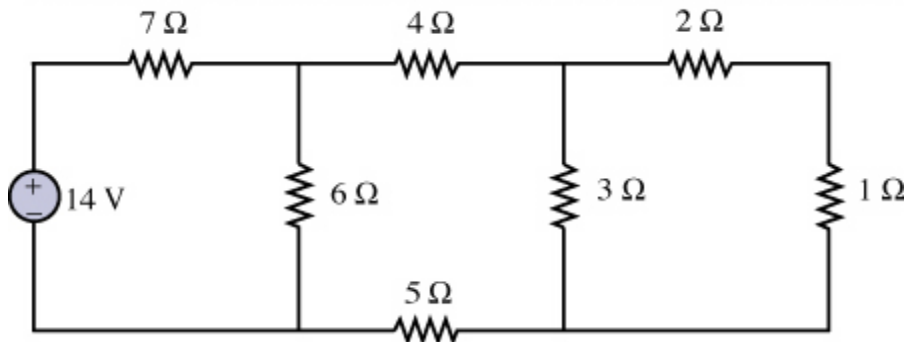


Fig 20b: Circuit model of how a real ammeter behaves, which looks like a small series resistor (R_A)

How would you place the ammeter in the circuits of the previous worked examples?

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Once again, Fig 20a represents the ideal case. Notice here that the ammeter has no effect on the circuit. In reality, the ammeter contains some resistance as shown in Fig 20b (R_A). In other words, the ammeter will add to the total resistance of the branch it is connected to. This will lower the measured current. So in reality, the current that you measure with an ammeter is lower than the true value.

The question is: *how big is this difference?* The worked example hopefully helps to illustrate this point.

Worked example on Ammeters

Referring to Fig 20b, if $V_S = 5V$, and $R_A = 1\Omega$, find the loop current when (i) $R_1 = 1k\Omega$; (ii) $R_1 = 2\Omega$

When $R_1 = 1k\Omega$:

Without ammeter, $I = 5mA$

With ammeter, $I = 4.995mA$ (very little difference)

When $R_1 = 2\Omega$:

Without ammeter, $I = 2.5A$

With ammeter, $I = 1.33A$ (error much more obvious)