

EE 3210 Signals and Systems

Semester B 2023 – 2024

Answer **ALL SIX** questions:

Question 1 (12 marks)

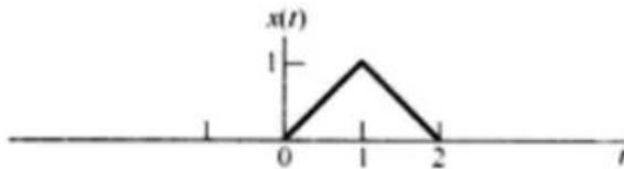
Consider a continuous-time signal $x(t)$, which is expressed as:

$$x(t) = \begin{cases} t, & 0 \leq t \leq 1 \\ 2-t, & 1 < t \leq 2 \\ 0, & \text{else} \end{cases}$$

- (a) Plot the signal $x(t)$.
- (b) Determine and plot the even and odd components of the signal $x(t)$, $x_e(t)$ and $x_o(t)$, respectively.

Solution:

(a)



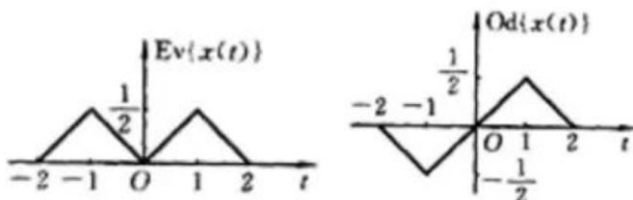
(b) The even and odd components can be determined by:

$$x_e(t) = \frac{1}{2}[x(t) + x(-t)] \quad \text{and} \quad x_o(t) = \frac{1}{2}[x(t) - x(-t)]$$

Hence we have

$$x_e(t) = \begin{cases} 1+0.5t, & -2 < t \leq -1 \\ -0.5t, & -1 < t \leq 0 \\ 0.5t, & 0 < t \leq 1 \\ 1-0.5t, & 1 < t \leq 2 \\ 0, & \text{else} \end{cases}, \quad \text{and} \quad x_o(t) = \begin{cases} -1-0.5t, & -2 < t \leq -1 \\ 0.5t, & -1 < t \leq 1 \\ 1-0.5t, & 1 < t \leq 2 \\ 0, & \text{else} \end{cases}$$

Their graphic representations are given as



Relevant contents: LT2, P8-P9, Eq. (2.12)

Similar questions: Example 2.3(d)

Question 2 (14 marks)

Given a signal discrete-time signal $x[n] = (\frac{1}{2})^n u[n]$.

- (a) Is $x[n]$ an energy signal?
- (b) Compute the power and energy of $x[n]$.

Solution:

(a) Energy signal.

(b) The power:

$$\begin{aligned} P_x &= \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{n=-N}^N [(\frac{1}{2})^n u(n)]^2 \\ &= \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{n=0}^N (\frac{1}{2})^{2n} = \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{n=0}^N (\frac{1}{4})^n = \lim_{N \rightarrow \infty} \frac{1}{2N+1} \frac{1 - (1/4)^{N+1}}{1 - 1/4} = 0 \end{aligned}$$

The energy:

$$\begin{aligned} E_x &= \lim_{N \rightarrow \infty} \sum_{n=-N}^N [(\frac{1}{2})^n u(n)]^2 = \lim_{N \rightarrow \infty} \sum_{n=0}^N [\frac{1}{4}]^n \\ &= \lim_{N \rightarrow \infty} \frac{1 - (1/4)^{N+1}}{1 - 1/4} = \frac{4}{3} \end{aligned}$$

Relevant contents: LT2, P11-12, Eq. (2.13) & Eq. (2.14)

Similar questions: Example 2.4; Exercise1 Q5

Question 3 (19 marks)

Determine the following signals are periodic signals or not. If yes, determine their fundamental period and compute their Fourier series.

(a) $x(t) = 2 \cos(\pi t) + 4 \sin(3t)$

(b) $x(t) = 1 + \cos(t) + 2 \sin(3t)$

Solution:

(a) The signal is not a periodic signal. $T_1 = 2\pi / \pi = 2$, $T_2 = 2\pi / 3$, T_1 and T_2 do not have lowest common multiple.

(b) The signal is a periodic signal.

The fundamental period is $T = 2\pi$.

Using the Euler formulas, we have

$$\cos(t) = \frac{1}{2}(e^{jt} + e^{-jt})$$

$$\sin(3t) = \frac{1}{2j}(e^{j3t} - e^{-j3t})$$

$x(t)$ can be written as

$$x(t) = je^{-j3t} + \frac{1}{2}e^{-jt} + 1 + \frac{1}{2}e^{jt} - je^{j3t}$$

Therefore, the Fourier series is

$$a_{-3} = j, a_{-1} = \frac{1}{2}, a_0 = 1, a_1 = \frac{1}{2}, a_3 = -j, a_k = 0(\text{others})$$

Relevant contents: LT2, P6 (for periodic signal);
LT4, P3-P4 (for Fourier Series)

Similar questions: Example 2.2; Exercise1, Q9 (for periodic signal);
Example 4.1; Example 4.2 (for Fourier Series)

Question 4 (15 marks)

Determine $y[n] = x[n] \otimes h[n]$, where $x[n]$ and $h[n]$ are

$$x[n] = \begin{cases} 1, n = 1 \\ 2, n = 2 \\ 1, n = 3 \\ 0, \text{else} \end{cases}, \quad \text{and} \quad h[n] = \begin{cases} 1, n = 2 \\ 2, n = 3 \\ 3, n = 4 \\ 0, \text{else} \end{cases}.$$

Solution:

$$\begin{aligned} y[n] &= \sum_{m=-\infty}^{\infty} h[m]x[n-m] \\ &= h[2]x[n-2] + h[3]x[n-3] + h[4]x[n-4] \\ &= x[n-2] + 2x[n-3] + 3x[n-4] \end{aligned}$$

$$y[3] = x[1] + 2x[0] + 3x[-1] = 1$$

$$y[4] = x[2] + 2x[1] + 3x[0] = 4$$

$$y[5] = x[3] + 2x[2] + 3x[1] = 8$$

$$y[6] = x[4] + 2x[3] + 3x[2] = 8$$

$$y[7] = x[5] + 2x[4] + 3x[3] = 3$$

$$\text{Therefore, } y[n] = \begin{cases} 1, n = 3 \\ 4, n = 4 \\ 8, n = 5 \\ 8, n = 6 \\ 3, n = 7 \\ 0, \text{else} \end{cases}$$

Relevant contents: LT3, P21-22

Similar questions: Example 3.15; Exercise2 Q6

Question 5 (20 marks)

Determine whether the discrete-time system $y[n] = x[n]x[n-1]$ is memoryless, stable, causal, linear, and time-invariant. Please explain your answer.

Solution:

Memory. $y[0] = x[0]x[-1]$, the current system output depends on previous input, so the system is not memoryless.

Stable. For $|x[n]| < A$, we have $|y[n]| = |x[n]x[n-1]| = |x[n]| |x[n-1]| \leq A^2$. Therefore, the system is stable.

Casual. $y[0] = x[0]x[-1]$, the current system output does not depend on future input, so it is a causal system.

Nonlinear. Suppose $y_1[n] = x_1[n]x_1[n-1]$, $y_2[n] = x_2[n]x_2[n-1]$,

Assigning $x_3[n] = ax_1[n] + bx_2[n]$, we have

$$y_3[n] = x_3[n]x_3[n-1] = (ax_1[n] + bx_2[n])(ax_1[n-1] + bx_2[n-1])$$

$$ay_1[n] + by_2[n] = ax_1[n]x_1[n-1] + bx_2[n]x_2[n-1]$$

$$y_3[n] \neq ay_1[n] + by_2[n]$$

Therefore, the system is nonlinear.

Time-invariant. Assigning $x_1[n] = x[n - n_0]$, we have

$$y_1[n] = x_1[n]x_1[n-1] = x[n - n_0]x[n - n_0 - 1]$$

$$y[n - n_0] = x[n - n_0]x[n - n_0 - 1]. \text{ Therefore, } y_1[n] = y[n - n_0].$$

The system is time-invariant.

Relevant contents: LT3, P5-P19

Similar questions: Example 3.1 to 3.8; Exercise2 Q2, Q3

Question 6 (20 marks)

Let $x(t) = \begin{cases} t, & 0 \leq t \leq 1 \\ 2-t, & 1 \leq t \leq 2 \end{cases}$ be a periodic signal with fundamental period $T=2$ and Fourier coefficients a_k .

- (a) Determine the value of a_0 .
- (b) Determine the Fourier series for the signal $dx(t)/dt$.
- (c) Prove the differentiation property of the continuous-time Fourier series:
If $x(t) \leftrightarrow a_k$, then $dx(t)/dt \leftrightarrow jk\Omega_0 a_k$

Solution:

$$(a) a_0 = \frac{1}{T} \int_0^T x(t) dt = \frac{1}{2} \int_0^1 t dt + \frac{1}{2} \int_1^2 (2-t) dt = \frac{1}{2}$$

$$(b) g(t) = \frac{dx(t)}{dt} = \begin{cases} 1, & 0 \leq t \leq 1 \\ -1, & 1 < t \leq 2 \end{cases}$$

$$\text{Let } g(t) \leftrightarrow b_k, b_0 = \frac{1}{T} \int_0^T x(t) dt = \frac{1}{2} \int_0^1 dt - \frac{1}{2} \int_1^2 dt = 0$$

$$\begin{aligned} b_k &= \frac{1}{2} \int_0^1 e^{-jk(2\pi/2)t} dt - \frac{1}{2} \int_1^2 e^{-jk(2\pi/2)t} dt \\ &= \frac{2 \sin(k\pi/2)}{k\pi} e^{-jk\pi/2} = \frac{1-e^{-jk\pi}}{jk\pi}, k \neq 0 \end{aligned}$$

$$(c) x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\Omega_0 t}$$

By differentiating both side of the Fourier Series, we obtain

$$g(t) = \frac{dx(t)}{dt} = \sum_{k=-\infty}^{\infty} jk\Omega_0 a_k e^{jk\Omega_0 t}$$

Therefore, the Fourier series of signal $g(t)$ is $b_k = jk\Omega_0 a_k$, i.e.,

$$dx(t)/dt \leftrightarrow jk\Omega_0 a_k$$

Relevant contents: LT4, P4

Similar questions: Example 4.3; Example 4.4; Exercise3 Q3, Q4