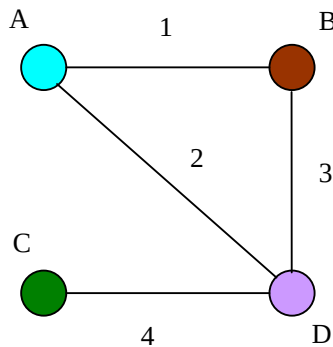


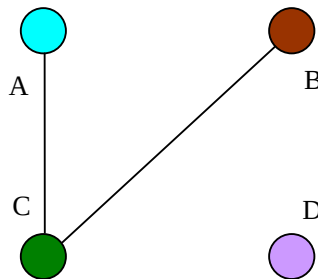
3-1



(i) Matrices

$$A = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}, \quad M = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}, \quad L = \begin{bmatrix} 2 & -1 & 0 & -1 \\ -1 & 2 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ -1 & -1 & -1 & 3 \end{bmatrix}$$

(ii) Complementary graph

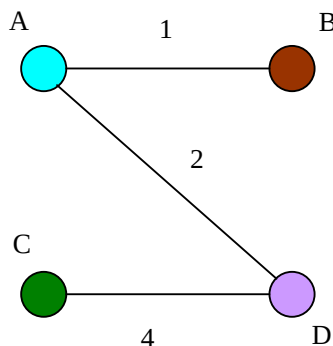


(iii) The two graphs are not isomorphic, because their numbers of edges are not the same.

Disconnected set: {CD}, {AB, AD}, {AB, BD}, {AD, BD}, {AD, CD}, {BD, CD}, {AB, CD},
 {AB, AD, BD}, {AB, BD, CD}, {AD, BD, CD}, {AB, AD, CD}, {AB, AD, BC, CD}
 Cut set: {CD}, {AB, AD}, {AB, BD}, {AD, BD}

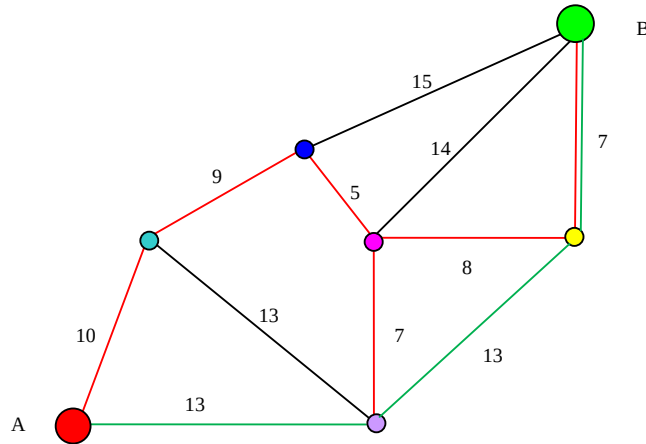
(iv) Edge connectivity = 1 (removing Edge CD) and Node connectivity = 1 (removing Node D)

(v) A spanning tree (not unique) can be



3-2 **Yes.** You only need to draw the graph from a given matrix. Then you can write out the other two matrices from the graph.

3-3 (a) Minimum spanning tree (**red**); (b) Shortest path length (**green**) = 33



3-4

Average node-betweenness = $\frac{(N-1)(N-2)}{2N}$, because only the central node N has non-zero

node-betweenness, so starting from Node 1, one has

$$B_1(N) = \frac{(1, N, 2)}{(1, N, 2)} + \frac{(1, N, 3)}{(1, N, 3)} + \dots + \frac{(1, N, N-1)}{(1, N, N-1)} = N-2$$

which is the same for all other non-central nodes. Therefore, totally, $B = (N-1)(N-2)$, which has been doubly counted, so divided by N nodes, one finally has the above result.

Average edge-betweenness = $N-2$, because for each edge (node i , center), similar to the above, the edge-betweenness is $N-2$. There are $N-1$ of such values, giving totally $(N-1)(N-2)$. After divided by $N-1$ edges, one obtains

$$\frac{(N-1)(N-2)}{N-1} = N-2$$