

1. Evaluate

$$(a) \det \begin{pmatrix} -3 & 7 & 2 \\ 2 & 4 & -3 \\ 1 & -3 & 7 \end{pmatrix} \quad (b) \det \begin{pmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{pmatrix} \quad (c) \det \begin{pmatrix} 1+a & b & c \\ a & 1+b & c \\ a & b & 1+c \end{pmatrix}$$

2. Consider the following sets of linear equations. In each case use Gaussian elimination to reduce the augmented matrix to echelon form. Determine whether the equations are consistent or inconsistent. If they are consistent, find all possible solutions.

$$(a) \begin{cases} x_1 + 4x_2 - 3x_3 = 3 \\ 2x_1 - 2x_2 + x_3 = 1 \\ 3x_1 + x_2 + 2x_3 = 11 \end{cases} \quad (b) \begin{cases} x_1 - 3x_2 - x_3 + 3x_4 = 0 \\ -x_1 + 2x_2 + x_3 + 4x_4 = 11 \\ 3x_1 - 2x_2 - 2x_3 + x_4 = 6 \end{cases}$$

$$(c) \begin{cases} x_1 + x_2 - x_3 = 1 \\ x_1 + 2x_2 - x_3 = 2 \\ x_1 + x_2 + x_3 = 6 \\ 2x_1 - 2x_2 + x_3 = 1 \end{cases} \quad (d) \begin{cases} x_1 + x_2 = 0 \\ 2x_1 - x_3 = 0 \\ 3x_1 + x_3 = 0 \end{cases}$$

3. (a) Find the matrix in reduced row-echelon form which is row equivalent to

$$\begin{pmatrix} 1 & 1 & 1 & 2 & 1 \\ 2 & 1 & 2 & 1 & 2 \\ 4 & 3 & 4 & 5 & 4 \end{pmatrix}.$$

*(b) Using the result of (a), find a 3×2 matrix A such that $\begin{pmatrix} 1 & 1 & 1 \\ 2 & 1 & 2 \\ 4 & 3 & 4 \end{pmatrix} A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \\ 5 & 4 \end{pmatrix}.$

4. Use the Gauss-Jordan method to find the inverses of the following matrices:

$$(a) \begin{pmatrix} 2 & 1 & 1 \\ 4 & -6 & 0 \\ -2 & 7 & 2 \end{pmatrix} \quad (b) \begin{pmatrix} 1 & 0 & 0 & 0 \\ -\frac{1}{2} & 1 & 0 & 0 \\ 0 & -\frac{2}{3} & 1 & 0 \\ 0 & 0 & -\frac{3}{4} & 1 \end{pmatrix}$$

5. Solve $A\vec{x} = \vec{b}$ with $A = LU = \begin{pmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & 2 & 4 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ by solving the triangular systems

$$L\vec{c} = \vec{b} \text{ and } U\vec{x} = \vec{c}.$$

6. Solve the following systems of equations (A) and (B) by Gaussian elimination and reduce the augmented matrices to echelon forms.

$$(A) \begin{cases} x_1 + 2x_2 - 2x_3 + x_4 + 3x_5 = 1 \\ 2x_1 + 5x_2 - 3x_3 - x_4 + 2x_5 = 2 \\ -3x_1 - 8x_2 + 6x_3 - x_4 - 5x_5 = 1 \end{cases}$$

$$(B) \begin{cases} x_1 + 2x_2 - 2x_3 + x_4 + 3x_5 = 1 \\ 2x_1 + 5x_2 - 3x_3 - x_4 + 2x_5 = 2 \\ -3x_1 - 8x_2 + 6x_3 - x_4 - 5x_5 = -3 \end{cases}$$

Write the solutions in vector form.

*7. For what values of a and b does $\begin{pmatrix} a & 0 & b \\ 0 & a & 1 \\ a & a & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ b \\ 2 \end{pmatrix}$ have

- (a) only one solution,
- (b) infinitely many solutions containing one parameter,
- (c) infinitely many solutions containing two parameters,
- (d) no solution?

Base your arguments upon “rank”.

8. Let $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 0 & 1 & 1 \end{pmatrix}$.

Using elementary row operations,

- (a) find the rank of A ;
- (b) find the inverse of A ;

(c) using (b), find the solution of the system of linear equations $\begin{cases} x + y + z = 2 \\ x + 2y + 3z = 0 \\ y + z = 0 \end{cases}$.

(d) solve the system of linear equations $\begin{cases} x + y + z = 1 \\ x + 2y + 3z = 2 \\ y + z = 3 \end{cases}$.

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