

EE 2005



week 13

DC Review

Kirchhoff's law:

KCL -> Nodal Analysis (current source)

KVL -> Mesh Analysis (voltage source)

superposition -> multiple sources

tera	T	10^{12}
giga	G	10^9
mega	M	10^6
kilo	k	10^3
milli	m	10^{-3}
micro	μ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}
femto	f	10^{-15}

Thevenin

Definition

- Any one-port network composed of **ideal** voltage and current sources, and **linear** resistors, can be represented by an equivalent circuit consisting of an ideal **voltage source** V_{Th} in **series** with an equivalent **resistance** R_{Th}

Deriving the Thevenin equivalent circuit

Step 1: Remove the load from the rest of the one port network. This is the first most fundamental step.

Step 2: Find the **equivalent resistance R_T** . Disable **all ideal sources** then find the resistance across the terminals.

For **voltage source** – replace with **short circuit**

For **current source** – replace with **open circuit**

Step 3: Find the **Thevenin voltage source**. The Thevenin voltage source is equal to the **open circuit voltage seen across the terminals** (with no load). Solve for this voltage using any preferred method (NVA, MCA, superposition).

Norton

Definition

- Any one-port network composed of **ideal** voltage and current sources, and **linear** resistors, can be represented by an equivalent circuit consisting of an ideal **current source** I_N in **parallel** with an equivalent **resistance** R_N

Deriving the Norton equivalent circuit

Steps 1 and 2 for deriving the Norton equivalent circuit are exactly the same as that for Thevenin.

Step 3: Find the Norton current source. The Norton current source is equal to the short circuit current seen across the terminals (with no load). Solve for this voltage using any preferred method (NVA, MCA, superposition).

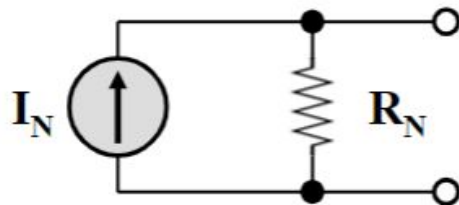
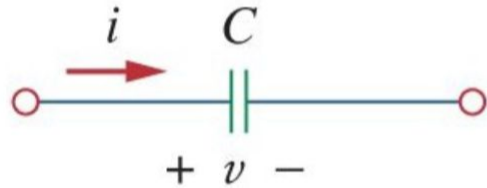


Fig 3: Transformed Norton equivalent circuit

Capacitors



$$Q(t) = CV(t)$$

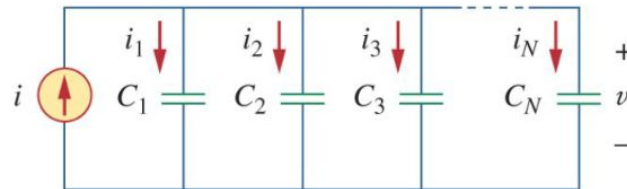
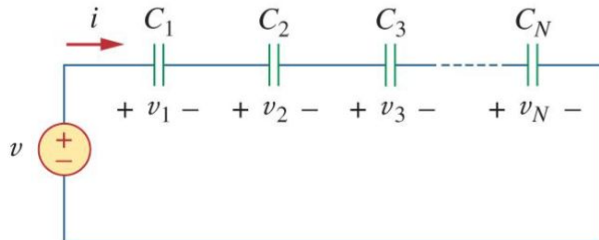
$$i(t) = C \frac{dV(t)}{dt}$$

Differentiation and Integration

$$Q = \int i(t) dt$$

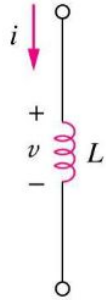
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots + \frac{1}{C_N}$$

$$C_{eq} = C_1 + C_2 + C_3 + \dots + C_N$$



Inductors

If the current through an inductor is time-varying, then the generated voltage must also be time-varying:



$$\text{Flux: } \phi = Li$$

$$\text{Faraday's Law: } v(t) = \frac{d\phi(t)}{dt}$$

$$\text{Differentiating with respect to time (t): } v(t) = L \frac{di(t)}{dt}$$

Voltage depends on the **rate of change** of current

The equivalent inductance in series and in parallel can be calculated the same way as the resistors.

If current is constant with time $\rightarrow di/dt = 0 \rightarrow V = 0$ (No voltage)

No current change $\rightarrow$ No voltage difference

If current changes with time $\rightarrow di/dt \neq 0 \rightarrow V \neq 0$ (There is voltage)

Current change $\rightarrow$ Voltage difference

C & L

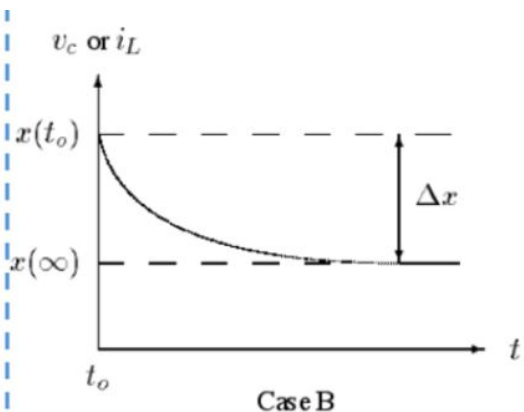
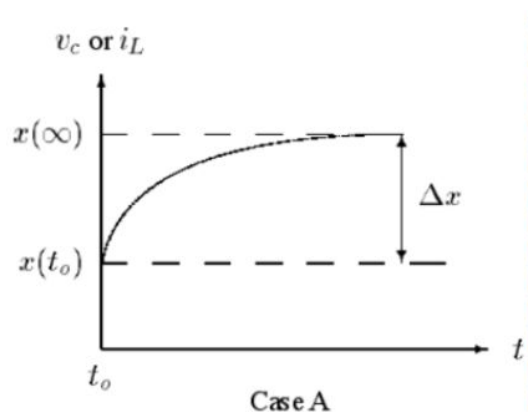
Relation	Resistor (R)	Capacitor (C)	Inductor (L)
v-i	$v = iR$	$v = \frac{1}{C} \int_{t_0}^t i(\tau) d\tau + v(t_0)$	$v = L \frac{di}{dt}$
i-v	$i = v/R$	$i = C \frac{dv}{dt}$	$i = \frac{1}{L} \int_{t_0}^t v(\tau) d\tau + i(t_0)$
Two in series	$R_{eq} = R_1 + R_2$	$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$	$L_{eq} = L_1 + L_2$
Parallel	$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$	$C_{eq} = C_1 + C_2$	$L_{eq} = \frac{L_1 L_2}{L_1 + L_2}$
At DC	Same	Open circuit	Short circuit
Circuit variable that cannot change abruptly	Not applicable	v	i

Resistance:

C: $1/j\omega C = -j/(\omega C)$

L: $j\omega L$

Transient



- Simple 1st order RC circuit

$$\tau = CR$$

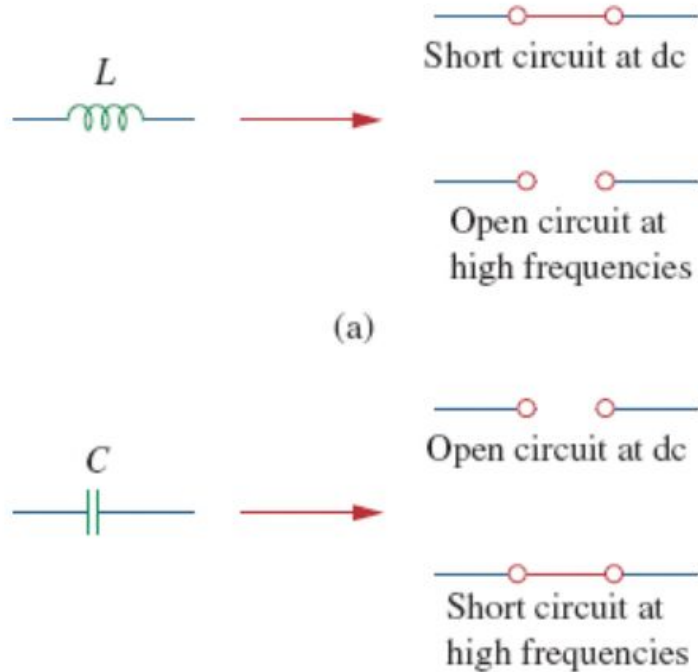
- Simple 1st order RL circuit

$$\tau = \frac{L}{R}$$

$$x(t) = x(t_0) + \Delta x(1 - e^{-(t-t_0)/\tau}) \quad x(t) = x(\infty) + \Delta x e^{-(t-t_0)/\tau}$$

- Find the time constant τ
- Find initial value $x(t_0)$ and final value $x(\infty)$
- Determine whether Case A or Case B expression shall be used

AC circuits



Impedances and admittances of passive elements.

Element	Impedance	Admittance
R	$Z = R$	$Y = \frac{1}{R}$
L	$Z = j\omega L$	$Y = \frac{1}{j\omega L}$
C	$Z = \frac{1}{j\omega C}$	$Y = j\omega C$

Phasors:

- Euler's Form
- Rectangular Form
- Polar Form

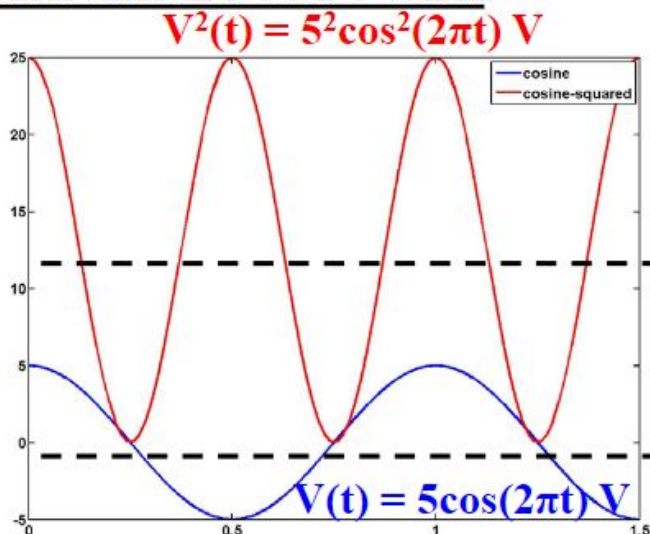
which is the easiest to calculate?

De Moivre's Law

AC Power: Instantaneous vs. Average

Although the average voltage across a resistor with a sinusoidal AC voltage across it is zero, note that the average power dissipated is not zero. Since $P = V^2/R$, we should consider the square of the voltage.

The following graph shows the voltage across a 1Ω resistor in blue and the corresponding square of this voltage in red. The red curve therefore shows the INSTANTANEOUS POWER.



The instantaneous power is clearly sinusoidal with a DC offset:

$$\begin{aligned} V^2(t) &= 5^2 * 0.5 * (1 + \cos(4\pi t)) \\ &= V_{\text{peak}}^2 * 0.5 * \cos(4\pi t) + \\ &\quad V_{\text{peak}}^2 * 0.5 \\ &= 12.5\cos(4\pi t) + 12.5 \end{aligned}$$

Mean of $V^2 = 12.5$ (DC offset)

Average power of $V^2(t)$ is 12.5 W
Average power of $V^2(t) = 0.5 * V_{\text{peak}}^2$

Mean of $V = 0$

Frequency Response

Low pass filter

Let us consider the response of the output V_o in relation to the input V_i . We keep the amplitude of V_i constant but vary its frequency ω .

By voltage divider rule: $\frac{V_o}{V_i}(j\omega) = \frac{1/j\omega C}{R + 1/j\omega C}$

$$= \frac{1}{1 + j\omega CR}$$

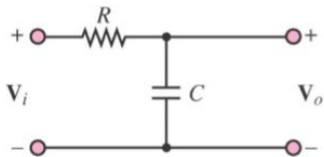
Note that CR is a constant based on the circuit values unlike ω , which is a variable.

Re-write CR as a constant with the same unit as angular frequency:

$$\omega_c = \frac{1}{RC} \quad \text{This frequency is called the cutoff radian frequency (ω_c) and is a CONSTANT}$$

Sub back into above equation:

$$\frac{V_o}{V_i}(j\omega) = \frac{1}{1 + j(\omega/\omega_c)}$$



High pass filter

Let us consider the response of the output V_o in relation to the input V_i . We keep the amplitude of V_i constant but vary its frequency ω .

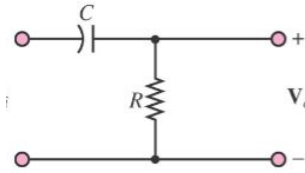
By voltage divider rule: $\frac{V_o}{V_i}(j\omega) = \frac{R}{R + 1/j\omega C}$

$$= \frac{1}{1 + 1/j\omega CR}$$

Once again we re-write RC to define the cut off radian frequency, ω_c whereby $\omega_c = 1/RC$:

Sub back into above equation:

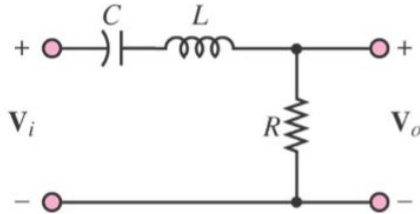
$$\frac{V_o}{V_i}(j\omega) = \frac{1}{1 + (j\omega_c / \omega)}$$



RLC Series Resonator (Bandpass filter)

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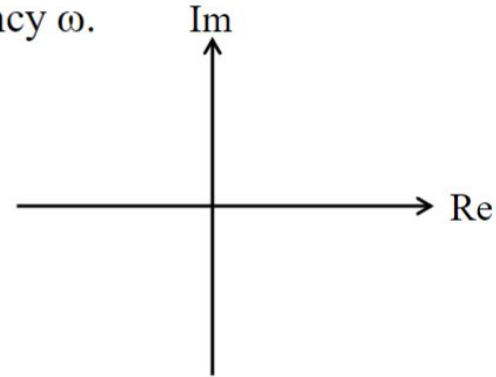
RLC bandpass filter. The circuit preserves frequencies within a band.



Let us consider the response of the output V_o in relation to the input V_i . We keep the amplitude of V_i constant but vary its frequency ω .

By voltage divider rule:

$$\begin{aligned}\frac{V_o}{V_i}(j\omega) &= \frac{R}{R + j\omega L + 1/j\omega C} \\ &= \frac{1}{1 + j\left(\frac{\omega L}{R} - \frac{1}{\omega RC}\right)}\end{aligned}$$



$|V_o/V_i|$ is max when denominator is minimized

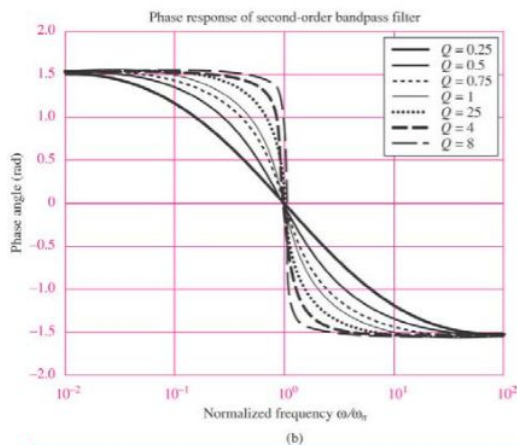
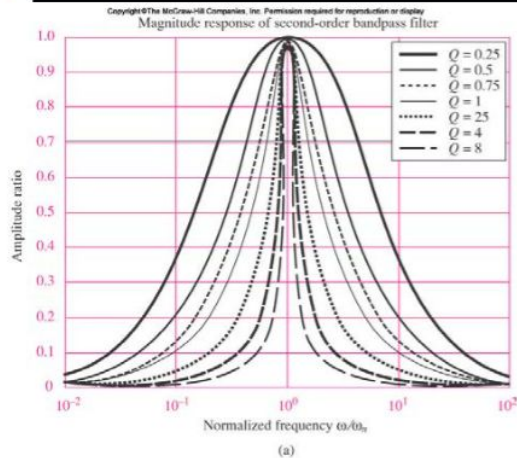
Plot of denominator for V_o/V_i

We can see that the output will be at its maximum when the imaginary part of the denominator is zero:

$$(\omega L/R) - [1/(\omega RC)] = 0 \rightarrow \boxed{\omega^2 = 1/(LC)}$$

When this happens, the impedances of the capacitor and inductor are equal and opposite. This is known as resonance. Max value of V_o/V_i for all frequencies is 1 in this case.

Quality factor



- (1) There is no “flat” part in the frequency response curve
- (2) Response peaks at one frequency: $\omega = 1/\sqrt{LC}$, this is known as the resonance frequency, ω_0
- (3) For frequencies move further away from ω_0 (whether higher or lower), $|V_o/V_i|$ gets increasingly smaller

$$\frac{V_o}{V_i}(j\omega) = \frac{j\omega CR}{1 - \omega^2 LC + j\omega CR}$$

Make the subst. using: $\omega_0 = \frac{1}{\sqrt{LC}}$

$$\frac{V_o}{V_i}(j\omega) = \frac{j\left(\frac{\omega}{\omega_0}\right) \frac{R}{\sqrt{L/C}}}{1 - \left(\frac{\omega}{\omega_0}\right)^2 + j\left(\frac{\omega}{\omega_0}\right) \frac{R}{\sqrt{L/C}}}$$

If we define: $Q = \frac{\sqrt{L/C}}{R}$

$$\frac{V_o}{V_i}(j\omega) = \frac{j(\omega/\omega_0)(1/Q)}{1 - (\omega/\omega_0)^2 + j(\omega/\omega_0)(1/Q)}$$

Q is known as the **Quality Factor** and it describes the sharpness or width of the peak relative to ω_0

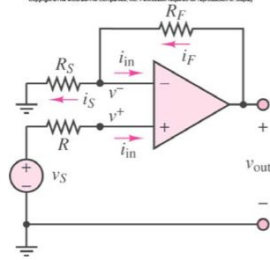


Figure 5 : Non-inverting amplifier

- Closed loop gain (V_{out}/V_{in})

- Gain: $\frac{v_{out}}{v_S} = 1 + \frac{R_F}{R_S}$

- Circuit input resistance : Infinite

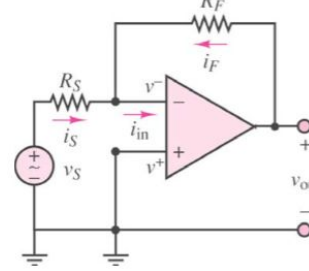


Figure 4 : Inverting amplifier

- Closed loop gain (V_{out}/V_{in})

- Gain: $\frac{v_{out}}{v_S} = -\frac{R_F}{R_S}$

- Circuit input resistance : R_S

Closed loop gain (G)

- Note that $i_{in} = 0$ since it is going into the op amp input
 $\rightarrow v_S = v^+ = v^-$ (1)
- Note there is also no current running into the inverting input of op amp as well ($i_{in} = 0$ for either inputs)
 $\rightarrow R_S$ and R_F are thus in series

- Note that $v^- = v^+ = 0$
- Apply KCL at inverting input (V^-)

$$\frac{v_S - 0}{R_S} + \frac{v_{out} - 0}{R_F} = 0$$

$$\frac{V_{out}}{V_S} = -\frac{R_F}{R_S}$$

OpAmp

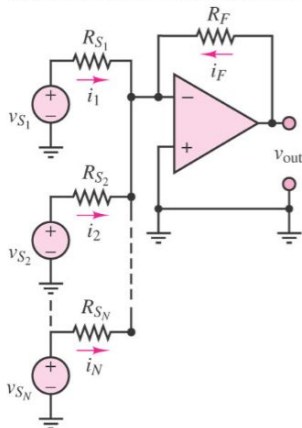


Figure 6 : Summing amplifier

- Note that since $v^+ = 0$, therefore $v^- = 0$
- Therefore:

$$i_1 = \frac{v_{S1}}{R_{S1}}, i_2 = \frac{v_{S2}}{R_{S2}}, \dots, i_N = \frac{v_{SN}}{R_{SN}}$$

- Applying KCL at v^- :

$$i_F + i_1 + i_2 + \dots + i_N = 0$$

- Hence by nodal voltage analysis:

$$\frac{v_{out}}{R_F} + \frac{v_{S1}}{R_{S1}} + \frac{v_{S2}}{R_{S2}} + \dots + \frac{v_{SN}}{R_{SN}} = 0$$

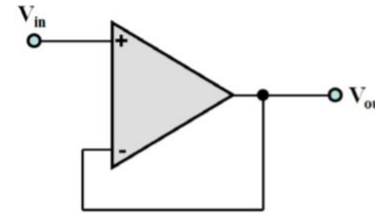


Figure 3: Source follower

- Closed loop gain (V_{out}/V_{in})

- Gain: $\frac{V_{out}}{V_{in}} = 1$

- Circuit input resistance : Infinite

Active Filter

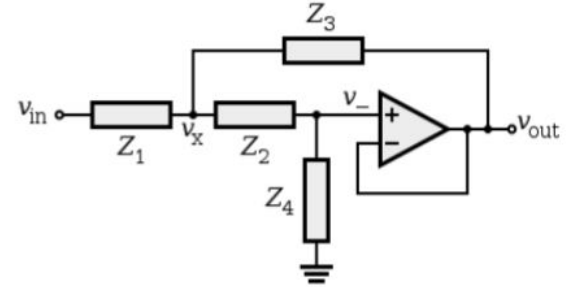
Implementation of:

- Comparator
- Differentiating Amplifier
- Integrating Amplifier
- Decibels(dB)
- 2nd order Filter
- Active Band Pass Filter

Passband gain is R_F/R_S

Sallen-Key Filter Topology

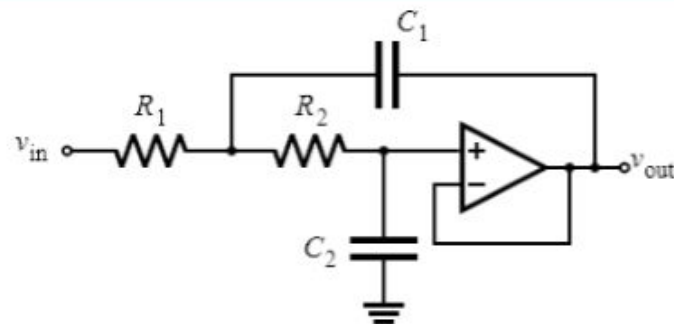
Assuming that the op-amp is ideal, transfer function of this filter is given by



$$H(j\omega) = \frac{v_{out}}{v_{in}} = \frac{Z_3 Z_4}{Z_1 Z_2 + Z_3 (Z_1 + Z_2) + Z_3 Z_4}$$

different types of R, L and C for Z's components
→ different kinds of filter responses.

Sallen-Key Filter Topology: Low Pass Filter



$$H(j\omega) = \frac{v_{out}}{v_{in}} = \frac{Z_3 Z_4}{Z_1 Z_2 + Z_3 (Z_1 + Z_2) + Z_3 Z_4}$$

1. It can be formed by choosing $Z_1=R_1$, $Z_2=R_2$, $Z_3=1/j\omega C_1$, $Z_4=1/j\omega C_2$;

2. Convert the above equation to the following form $H(j\omega) = \frac{\omega_o^2}{(j\omega)^2 + \left(\frac{\omega_o}{Q}\right)(j\omega) + \omega_o^2}$

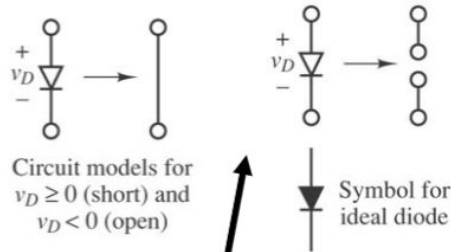
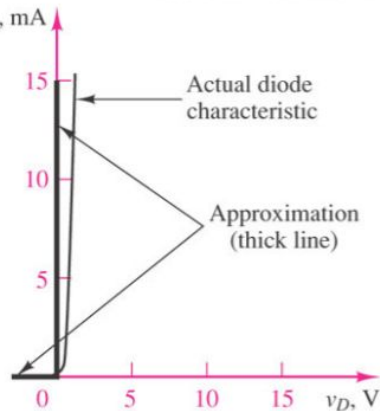
3. The undamped natural frequency ω_o is given by $\omega_o = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}$

4. The Q factor is given by $Q = \frac{\sqrt{R_1 R_2 C_1 C_2}}{C_2 (R_1 + R_2)}$

Diodes

Ideal diode model

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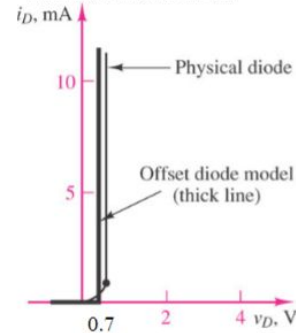
In other words, diode is treated like a switch here

$v_D > 0$: Short circuit

$v_D < 0$: Open circuit

Offset diode model

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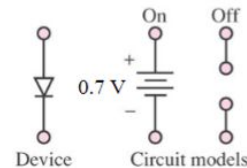
V_γ is typically used to denote the offset voltage

$v_D \geq 0.7 \text{ V}$

Diode $\rightarrow$ 0.7 V battery

$v_D < 0.7 \text{ V}$

Diode $\rightarrow$ open circuit



MOSFET

Summary of operating modes

- **Cut-Off:**

$$V_{GS} < V_T, V_{GD} < V_T$$

- **Linear/Triode/Ohmic:**

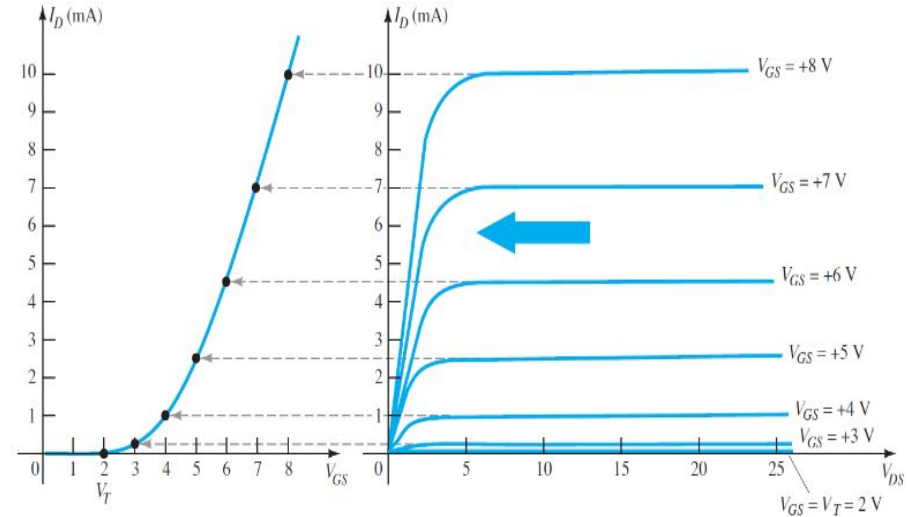
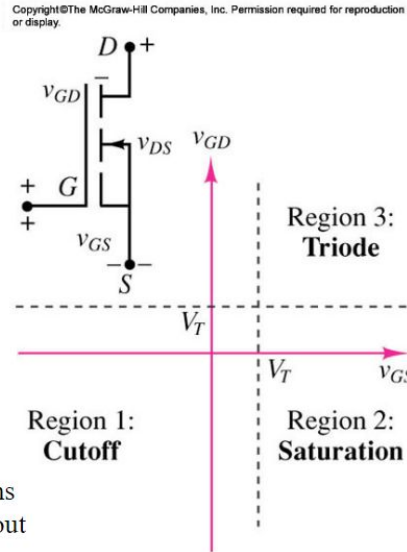
$$V_{GS} > V_T, V_{GD} > V_T$$

- **Saturation:**

$$V_{GS} > V_T, V_{GD} < V_T$$

For n-channel
enhancement MOS

Power consumed by MOSFET = $I_D V_{DS}$
This sets a limit on the operating conditions
Too much power and the transistor burns out



SSEC

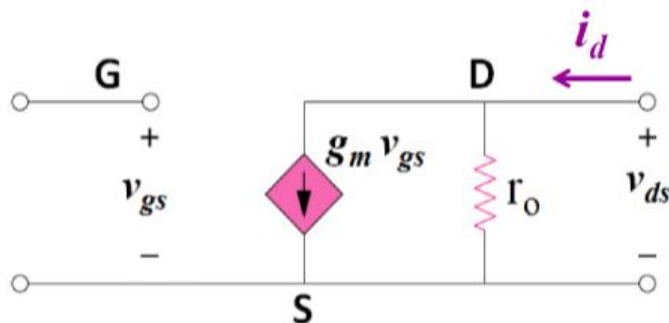
$$i_d = g_m v_{gs} + v_{ds}/r_o$$

$$\Delta I_D = \left. \frac{\partial I_D}{\partial V_{GS}} \right|_{V_{DS}} \Delta V_{GS} + \left. \frac{\partial I_D}{\partial V_{DS}} \right|_{V_{GS}} \Delta V_{DS}$$

The chain rule equation is
now a simple KCL equation:

$$i_d = g_m v_{gs} + v_{ds}/r_o$$

$$r_o = \left. \frac{\partial V_{DS}}{\partial I_D} \right|_{V_{GS}}, \quad g_m = \left. \frac{\partial I_D}{\partial V_{GS}} \right|_{V_{DS}}$$



NOTE: The SSEC only shows variables that are small signals.
DC values have no small signals and thus assigned to be ZERO.