

Assignment 2 Solution

The assignment consists of 7 questions.

1. (15%) Prove using mathematical induction that for all $n \geq 1$,

$$1 + 4 + 7 + \dots + (3n - 2) = \frac{n(3n - 1)}{2}.$$

2. (15%) Use contrapositive proof to prove: If x and $y \in \mathbb{Z}$, $x + y + xy$ is even, then x and y have the same parity (either both are even, or both are odd).

3. (15%) Suppose $A = \{\text{red, green, blue}\}$, $B = \{\text{red, yellow, orange}\}$, $C = \{\text{red, orange, green, purple}\}$, and the universal set $U = A \cup B \cup C$. Find the following:

- 1) $A \cap (B \cup C)$.
- 2) $(A \cap B) \cup C^c$.
- 3) $(A - B) \cap C$.

4. (15%) Suppose $A = \{\text{toast, juice}\}$, $B = \{\text{egg, milk}\}$, $H = \{\text{cat, dog, rabbit, mouse}\}$, $F = \{\text{dog, cow, duck, pig, rabbit}\}$, $W = \{\text{duck, rabbit, deer, frog, mouse}\}$.

- 1) Find cartesian product $A \times B$.
- 2) Use Venn diagram to illustrate $(H \cap F)^c \cap W$.

5. (15%) Using the properties of sets to prove

$$(A - C) - (C - B) = A - C$$

Start the proof from the Left-hand side. Use algebraic rules to prove it. State the name of the algebraic rule you used in each step. Do **not** skip any steps.

6. (15%) Determine if the follow functions are injective, surjective, and bijective.

1) $f: \mathbb{R} \rightarrow \mathbb{R}, f(x) = -x^2 + 4x + 9$.

2) $g: \mathbb{N} \rightarrow \mathbb{N}, g(x) = x + 3$.

3) $h: \mathbb{R} \rightarrow (0, +\infty), h(x) = \log(e^{-x} + 1)$.

7. (10%) Find the composition function of the following functions:

1) $f(x) = 4x + 3, g(x) = -2x^2 + 3x + 5$. Find $(g \circ f)(x)$.

2) $f(x) = \frac{3}{5}x + 6, g(x) = 6x^2 - 15x + 20$. Find $(f \circ g)\left(\frac{5}{3}\right)$.