

## In-Class Activity 4 Solutions

**Question 1:** The matrices  $A$  and  $B$  are defined as follows:

$$A = \begin{pmatrix} 2 & 7 & 6 \\ 3 & 1 & 5 \end{pmatrix}$$

$$B = \begin{pmatrix} 4 & 0 \\ 3 & 7 \\ 1 & 5 \end{pmatrix}$$

Find  $(AB)^T$ .

$$AB = \begin{pmatrix} (2 \times 4) + (7 \times 3) + (6 \times 1) & (2 \times 0) + (7 \times 7) + (6 \times 5) \\ (3 \times 4) + (1 \times 3) + (5 \times 1) & (3 \times 0) + (1 \times 7) + (5 \times 5) \end{pmatrix} = \begin{pmatrix} 35 & 79 \\ 20 & 32 \end{pmatrix}$$

$$(AB)^T = \begin{pmatrix} 35 & 20 \\ 79 & 32 \end{pmatrix}$$

**Question 2:** Use the method of elementary row operations to solve the following system of equations

$$3x_1 - 3x_2 + 5x_3 = 6$$

$$x_1 + 7x_2 + 5x_3 = 4$$

$$5x_1 + 10x_2 + 15x_3 = 9$$

The augmented matrix is

$$\left( \begin{array}{ccc|c} 3 & -3 & 5 & 6 \\ 1 & 7 & 5 & 4 \\ 5 & 10 & 15 & 9 \end{array} \right)$$

We will now use elementary row operations to reduce the augmented matrix to echelon form:

$$\xrightarrow{\frac{1}{3}R_1 \rightarrow R_1} \left( \begin{array}{ccc|c} 1 & -1 & 5/3 & 2 \\ 1 & 7 & 5 & 4 \\ 5 & 10 & 15 & 9 \end{array} \right)$$

$$\begin{aligned}
&\xrightarrow{R_3 - 5R_1 \rightarrow R_3} \left( \begin{array}{ccc|c} 1 & -1 & 5/3 & 2 \\ 1 & 7 & 5 & 4 \\ 0 & 15 & 20/3 & -1 \end{array} \right) \\
&\xrightarrow{R_2 - R_1 \rightarrow R_2} \left( \begin{array}{ccc|c} 1 & -1 & 5/3 & 2 \\ 0 & 8 & 10/3 & 2 \\ 0 & 15 & 20/3 & -1 \end{array} \right) \\
&\xrightarrow{\frac{1}{8}R_2 \rightarrow R_2} \left( \begin{array}{ccc|c} 1 & -1 & 5/3 & 2 \\ 0 & 1 & 5/12 & 1/4 \\ 0 & 15 & 20/3 & -1 \end{array} \right) \\
&\xrightarrow{R_3 - 15R_2 \rightarrow R_3} \left( \begin{array}{ccc|c} 1 & -1 & 5/3 & 2 \\ 0 & 1 & 5/12 & 1/4 \\ 0 & 0 & 5/12 & -19/4 \end{array} \right) \\
&\xrightarrow{R_1 - 4R_2 \rightarrow R_1} \left( \begin{array}{ccc|c} 1 & -5 & 0 & 1 \\ 0 & 1 & 5/12 & 1/4 \\ 0 & 0 & 5/12 & -19/4 \end{array} \right) \\
&\xrightarrow{R_2 - R_3 \rightarrow R_2} \left( \begin{array}{ccc|c} 1 & -5 & 0 & 1 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 5/12 & -19/4 \end{array} \right) \\
&\xrightarrow{\frac{12}{5}R_3 \rightarrow R_3} \left( \begin{array}{ccc|c} 1 & -5 & 0 & 1 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & -57/5 \end{array} \right) \\
&\xrightarrow{R_1 + 5R_2 \rightarrow R_1} \left( \begin{array}{ccc|c} 1 & 0 & 0 & 26 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & -57/5 \end{array} \right)
\end{aligned}$$

Therefore,

$$x_1 = 26, \quad x_2 = 5, \quad x_3 = -57/5$$

**Question 3:** Perform elementary row operations to solve the following system of equations

$$x_1 + 2x_2 + x_3 = 1$$

$$2x_1 + 2x_2 = 2$$

$$3x_1 + 4x_2 + x_3 = 3$$

The augmented matrix is

$$\left( \begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 2 & 2 & 0 & 2 \\ 3 & 4 & 1 & 3 \end{array} \right)$$

We will now use elementary row operations to reduce the augmented matrix to echelon form:

$$\xrightarrow{3R_1 - R_3 \rightarrow R_3} \left( \begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 2 & 2 & 0 & 2 \\ 0 & 2 & 2 & 0 \end{array} \right)$$

$$\xrightarrow{2R_1 - R_2 \rightarrow R_2} \left( \begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & 2 & 2 & 0 \\ 0 & 2 & 2 & 0 \end{array} \right)$$

$$\xrightarrow{R_2 - R_3 \rightarrow R_3} \left( \begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & 2 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

$$\xrightarrow{\frac{1}{2}R_2 \rightarrow R_2} \left( \begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

This last matrix represents the system

$$x_1 + 2x_2 + x_3 = 1$$

$$x_2 + x_3 = 0$$

$$0x_1 + 0x_2 + 0x_3 = 0$$

The third of these equations conveys no information at all about  $x_1, x_2$  and  $x_3$ , for it simply tells us that  $0 = 0$ . Consequently, the original system has the same solutions as the following

system of two equations in three unknowns:

$$x_1 + 2x_2 + x_3 = 1$$

$$x_2 + x_3 = 0$$

These equations tell us first that, given  $x_3$ , we have  $x_2 + x_3 = 0$ , so  $x_2 = -x_3$ .

Then we have  $x_1 = 1 - 2x_2 - x_3 = 1 + 2x_3 - x_3 = 1 + x_3$ , so both  $x_1$  and  $x_2$  are determined in terms of  $x_3$ . If we let  $x_3$  be any real number  $s$ , then

$$x_1 = 1 + s, \quad x_2 = -s, \quad x_3 = s$$

is a solution to the system. So there are infinitely many solutions to the system.