

EE3121 Diff Equations for EE

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Week 1. Introduction

First-Order Differential Equation

- Separable Equations
- Exact Equations
- Integrating Factors
- Bernoulli Equations.

Separable Equations

- Type 1) $f(x)dx + g(y)dy = 0 \Rightarrow \int f(x)dx + \int g(y)dy = C$

General Solution: $F(x) + G(y) = C$

- Type 2) $\frac{dy}{dx} = f(x)g(y) \Rightarrow \int f(x)dx = \int \frac{dy}{g(y)} \Rightarrow$ General Solution: $F(x) = \int \frac{dy}{g(y)} + C$

- Type 3) $\frac{dy}{dx} = h\left(\frac{y}{x}\right) \Rightarrow$ Denote $u = \frac{y}{x}$, then $\frac{dy}{dx} = \frac{du}{dx}x + u \Rightarrow \frac{du}{dx}x = h(u) - u$
 $\Rightarrow \frac{du}{h(u)-u} = \frac{dx}{x} \Rightarrow$ General Solution: $\ln x + C = \int \frac{du}{h(u)-u}$

Separable Equations

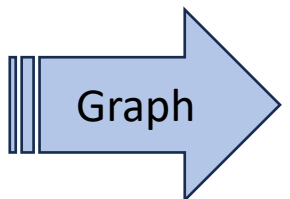
- Example) Exponential Growth DE

$$\frac{dy}{dt} = k \cdot y \implies y(t) = y_0 \cdot e^{kt}, \text{ where } y(0) = y_0$$

- Example) Logistic Growth

$$\frac{dy}{dt} = k \cdot y \cdot \left(1 - \frac{y}{M}\right) \implies y(t) = \frac{C \cdot k \cdot e^{kt}}{1 + C \cdot e^{kt}}$$

Given $y(0) = y_0$, we can prove $C = \frac{y_0}{M - y_0}$ and general solution $y(t) = \frac{y_0 \cdot M \cdot e^{kt}}{(M - y_0) + y_0 \cdot e^{kt}}$



Separable Equations

- Example) Logistic Growth DE

$$\frac{dy}{dt} = k \cdot y \cdot \left(1 - \frac{y}{M}\right) \Rightarrow y(t) = \frac{y_0 \cdot M \cdot e^{kt}}{(M - y_0) + y_0 \cdot e^{kt}}$$

$y(t)$: Population at time t ,	y_0 : Initial Population
M : Carrying capacity,	k : Growth rate

- Example) A population of rabbits in a meadow is observed to be 200 rabbits at time $t=0$. After a month, the rabbit population is observed to have increased by 4. Using an initial population of 200 and a growth rate of 0.04, predict the population with a carrying capacity of 750 rabbits.

Exact Differential Equation

Definition

If a continuous DE $F(x, y)$ exist, such that

$$I(x, y)dx + J(x, y)dy = 0 \Rightarrow \frac{\partial F}{\partial x} = I(x, y), \frac{\partial F}{\partial y} = J(x, y),$$

the differential equation is exact if and only if $\frac{\partial I(x, y)}{\partial y} = \frac{\partial J(x, y)}{\partial x}$.

$$I_y(x, y) = J_x(x, y)$$