

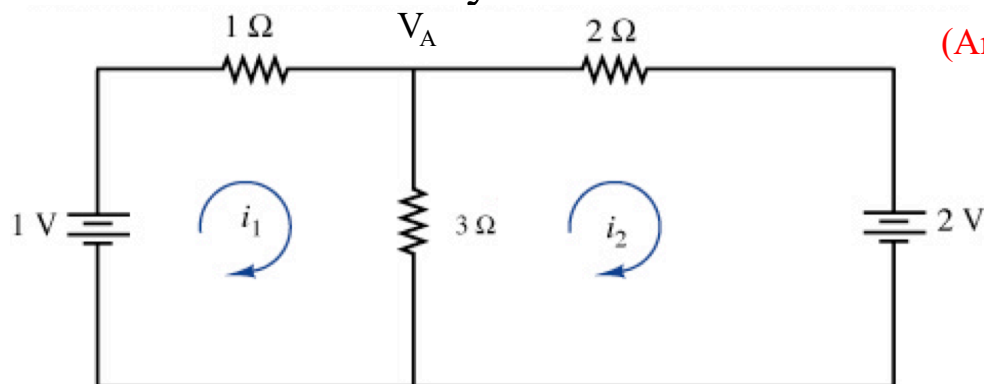
II. Mesh Current Analysis (MCA)

KVL & Ohm's law in action

- The method of mesh current analysis is an application of KVL and Ohm's law together
- The unknown variables that you will solve for are **mesh currents**
- From KVL, the sum of voltage drops and rise must equal zero
- The voltage differences are expressed as the current going through each branch in the mesh

Example on how to do MCA

Use mesh current analysis to find the current through the 3Ω resistor.



(Answer: $i_{3\Omega} = 4/11\text{A}$)

-There are 2 meshes and hence 2 mesh currents

-Apply KVL to each of these 2 meshes

Apply KVL to mesh 1:

$$1 = i_1(1 + 3) - 3i_2$$
$$4i_1 - 3i_2 = 1 \quad (1)$$

Useful tip:

- keep voltages of sources on one side of the equation, and keep voltages of resistors on the other side

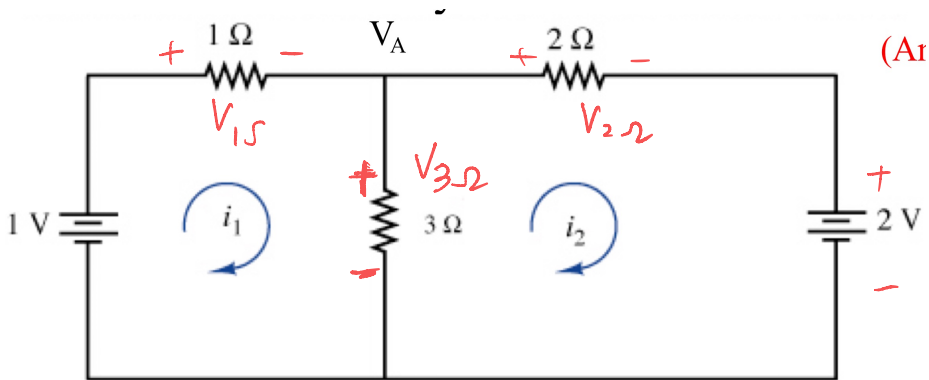
Apply KVL to mesh 2:

Follow the defined direction of the mesh current

$$-2 = i_2(2 + 3) - 3i_1$$
$$5i_2 - 3i_1 = -2 \quad (2)$$

Now, solve equation (1) and (2):

$$i_{3\Omega} = i_1 - i_2$$



mesh analysis $\Rightarrow$ KVL

$$\text{loop } i_1 \quad 1 = V_{1\Omega} + V_{3\Omega} \leftarrow 3(i_1 - i_2)$$

$\uparrow (1)(i_1)$

$$\Rightarrow 1 = i_1 + 3i_1 - 3i_2$$

$$\Rightarrow 1 = 4i_1 - 3i_2 \quad - (1)$$

$$\text{loop } i_2 \quad 2 + V_{2\Omega} - V_{3\Omega} = 0$$

$$2 + \downarrow 2i_2 - 3(i_1 - i_2) = 0$$

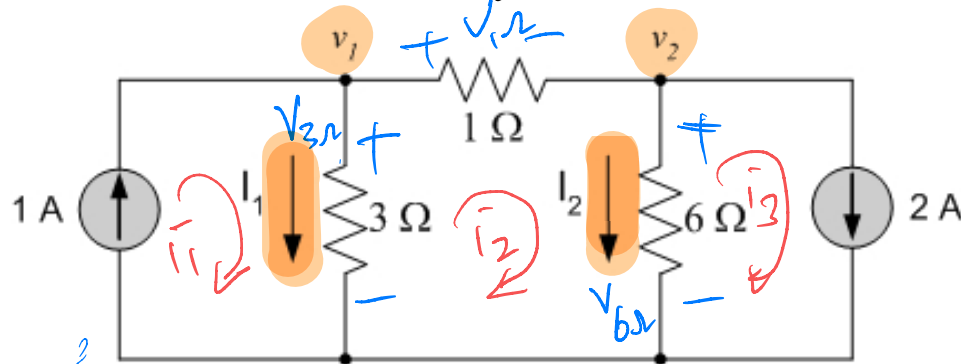
$$\Rightarrow 2 + 2i_2 - 3i_1 + 3i_2 = 0$$

$$\Rightarrow 2 = +3i_1 - 5i_2$$

$$\Rightarrow -2 = -3i_1 + 5i_2 \quad - (2)$$

Worked Example on MCA 1

Use mesh current analysis to find currents I_1 and I_2 . (Answer: $I_1 = I_2 = -0.5A$)



-You see 3 meshes, but only the middle mesh is unknown, the other 2 are defined by the current sources.

-For simplicity, defined the direction of known mesh currents in the same direction as the sources.

$i_1 = 1A$ $i_3 = 2A$

By KVL $V_{3\Omega} - V_{1\Omega} - V_{6\Omega} = 0$

$$3(i_1 - i_2) - 1(i_2) - 6(i_2 - i_3) = 0$$

$$3i_1 - 3i_2 - i_2 - 6i_2 + 6i_3 = 0$$

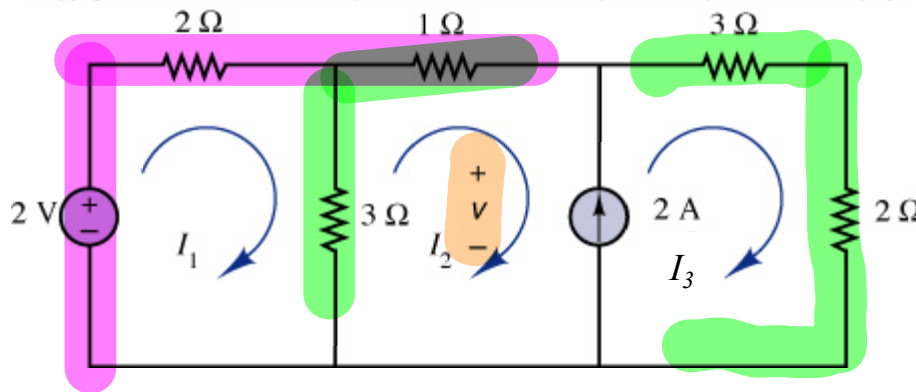
$$3i_1 - 10i_2 + 6i_3 = 0$$

Worked Example on MCA 2

Use mesh current analysis to find the voltage across the current source.

(Answer: $V = 3.89 \text{ V}$)

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-In the process of deriving equations (2) and (3), a new variable V is introduced.

-Hence, we need one more equation, (4)

Apply KVL at mesh 1:

$$2 = I_1(2+3) - I_2(3)$$
$$\Rightarrow 5I_1 - 3I_2 = 2 \quad (1)$$

Apply KVL at mesh 3:

$$V = I_3(3+2)$$
$$\Rightarrow 5I_3 = V \quad (3)$$

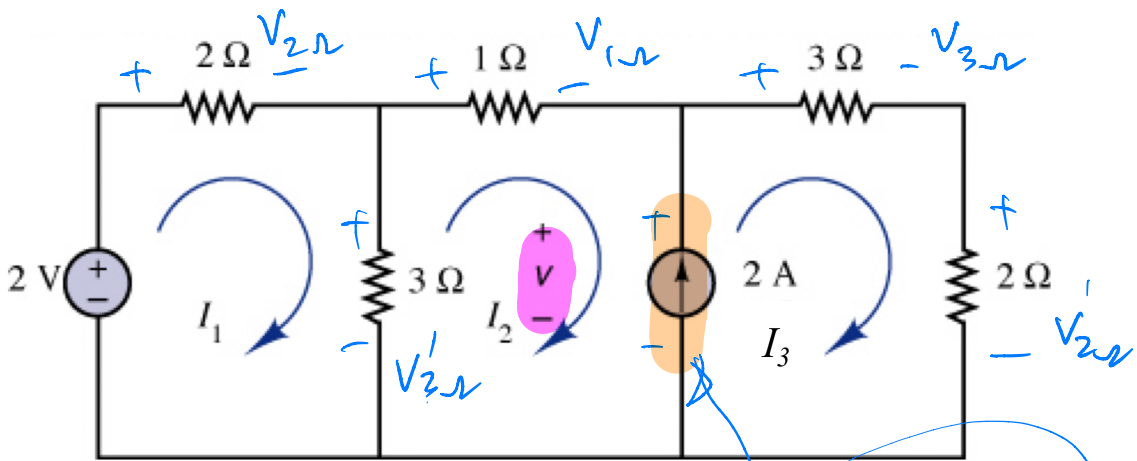
Apply KVL at mesh 2:

$$-V = I_2(3+1) - I_1(3)$$
$$\Rightarrow 4I_2 - 3I_1 = -V \quad (2)$$

One more equation:

$$I_3 - I_2 = 2A \quad (4)$$

Solve for V , verify by $V = (I_3)(3+2)$



KVL loop I₁

$$2 - V_2 - V'_2 = 0$$

$$2 - 2I_1 - 3(I_1 - I_2) = 0$$

$$\Rightarrow 2 - 2I_1 - 3I_1 + 3I_2 = 0$$

$$\Rightarrow 2 - 5I_1 + 3I_2 = 0$$

$$\Rightarrow 2 = 5I_1 - 3I_2 \quad \text{--- (1)}$$

KVL loop I₃

$$V - V_3 - V_2 = 0$$

$$V - 2I_3 - 3I_3 = 0$$

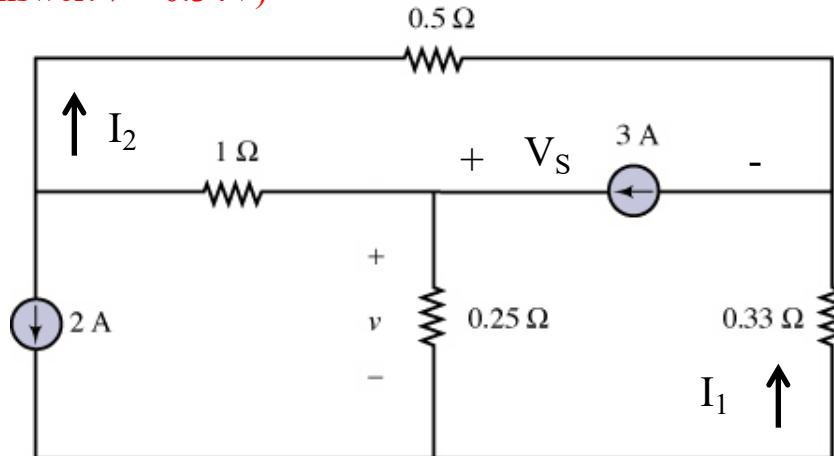
$$V = 5I_3$$

$$I_3 - I_2 = 2A$$

Worked Example on MCA 3

Use mesh current analysis, find the voltage across the $0.25\ \Omega$ resistor.

(Answer: $v = 0.34\text{V}$)



Method:

-Find the current through 0.25Ω resistor first, then use this to find the voltage.

-Although there are 3 meshes, the bottom left mesh is already known (2A going anti-clockwise).

Apply KVL at mesh 1:

$$V_s = I_2(0.5 + 1) + (2)(1)$$
$$\Rightarrow V_s = 1.5I_2 + 2 \quad (1)$$

Apply KVL at mesh 2:

$$V_s = I_1(0.25 + 0.33) - (2)(0.25)$$
$$\Rightarrow V_s = 0.58I_1 - 0.5 \quad (2)$$

Consider the current source:

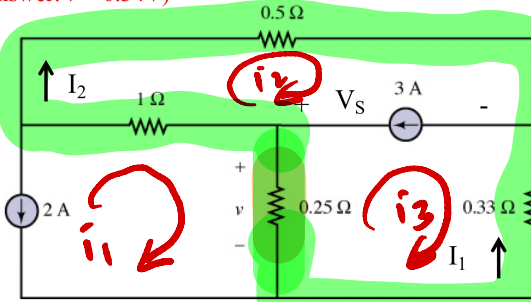
$$I_1 + I_2 = 3 \quad (3)$$

Solve for I_1 and I_2

Current through $0.25\ \Omega$ resistor: $I_1 - 2$

$$V = (I_1 - 2)(0.25)$$

(ANSWER: $v = 0.54 \text{ V}$)



supermesh

$$\begin{aligned} i_1 &= -2 \text{ A} & \text{--- (1)} \\ \text{loop 2} \rightarrow 3 &= i_2 - i_3 & \text{--- (2)} \\ v &= (i_1 - i_2)(0.25) & \text{--- (3)} \end{aligned}$$

Quick note on choosing NVA and MCA

How do you choose between NVA and MCA?

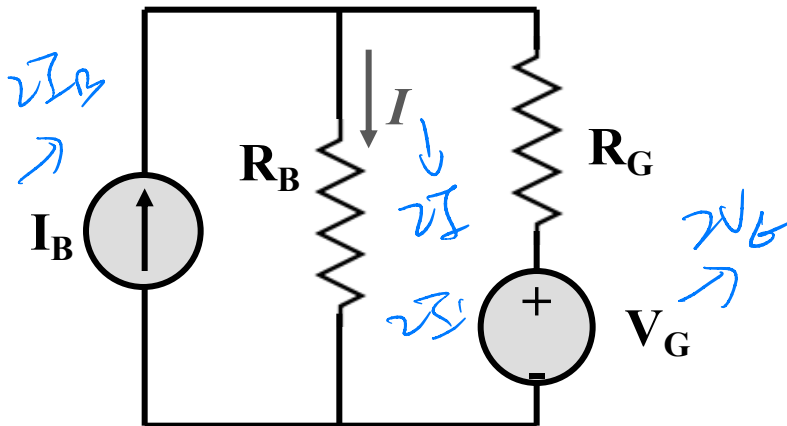
- Your choice should not be due to level of familiarity between the methods.
- One consideration is whichever is simpler to use for a given circuit.
- How then do you decide what is simpler?
- For a start, having fewer equations certainly makes solving easier.

III. Superposition

Law

The method

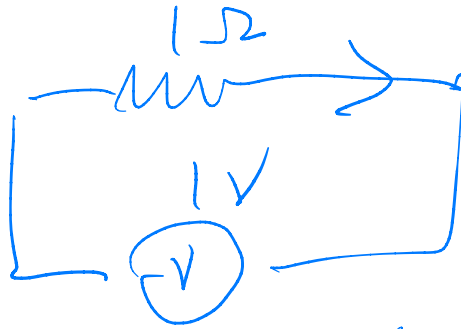
- This method applies only to circuits that have **multiple sources**
- In such case, it can come in handy (or not)
- In a circuit with multiple sources, superposition considers the current or voltage associated with a given branch for one of the sources, while turning the rest off



Aim: Find the current through R_B .

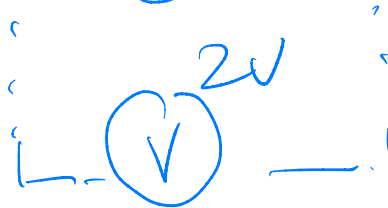
1. Find the current (I_1) through R_B when only I_B is present (V_G is removed)
2. Find the current (I_2) through R_B when only V_G is present (I_B is removed)
3. The net current $I = I_1 + I_2$

Fig 1: Basic circuit with 2 sources



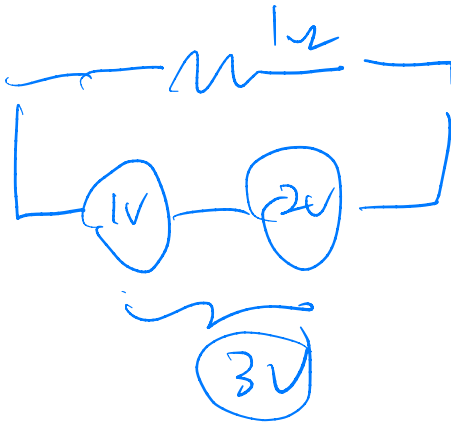
$$I_1 = 1A$$

+



$$I_2 = 2A$$

||



$$I = 3A$$

mathematically
linear

$$\begin{matrix} v_1 & \longrightarrow & y_1 \\ + & & + \\ v_2 & \longrightarrow & y_2 \end{matrix}$$

$y_1 = k v_1$

$$y = \underset{\substack{\phi \\ 1}}{a} x^2$$

$$y = x^2$$

$$x = 1$$

$$x = 2$$

$$y = 1 \quad \text{X} \quad 5$$

$$y = 4$$

$$x = 1 + 2$$

$$= 3$$

$$y = 9$$

How do we “remove” a current source or voltage source?

Disabling sources

Voltage source: If the voltage source does not exist, the voltage across the terminals would be zero. It looks like a short circuit. (see Fig 2a)

Current source: If the current source does not exist, the current through it would be zero. It looks like an open circuit. (see Fig 2b)

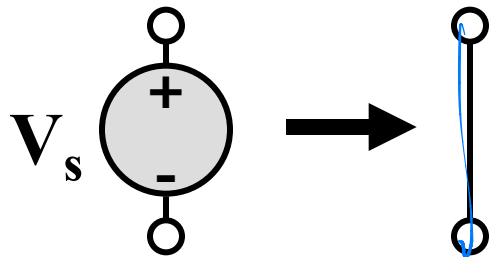


Fig 2a: Disable a voltage source by Replacing with a short circuit.

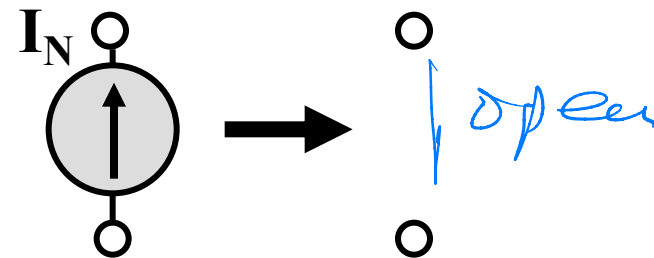


Fig 2b: Disable a current source by Replacing with an open circuit.

Apply the tips to Fig 1

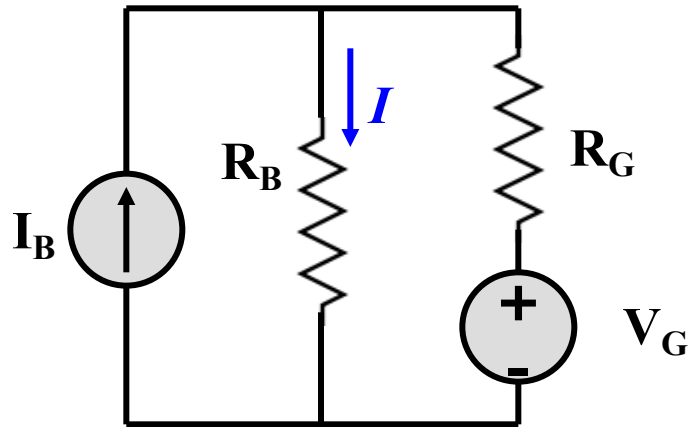
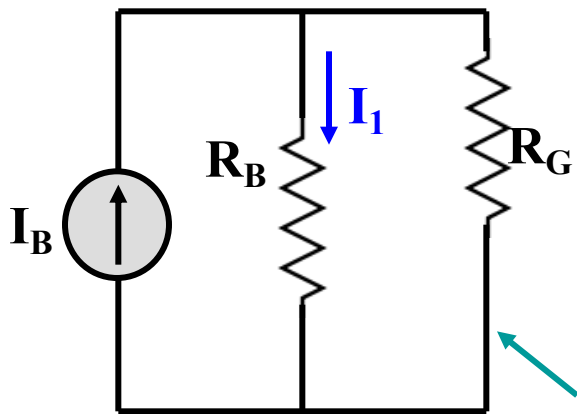
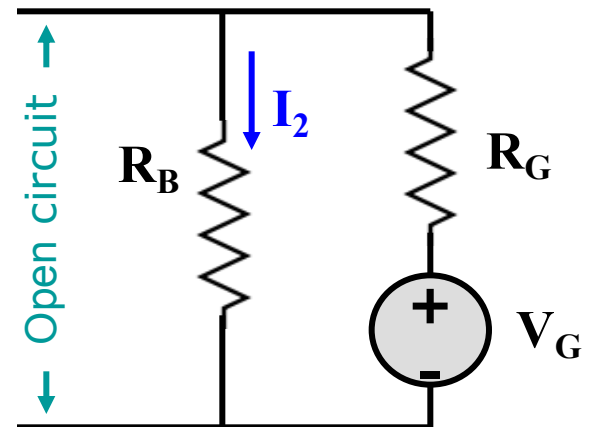


Fig 1: Basic circuit with 2 sources

Disabling V_G , $V_G = 0$



Disabling I_B , $I_B = 0$



$$I = I_1 + I_2$$

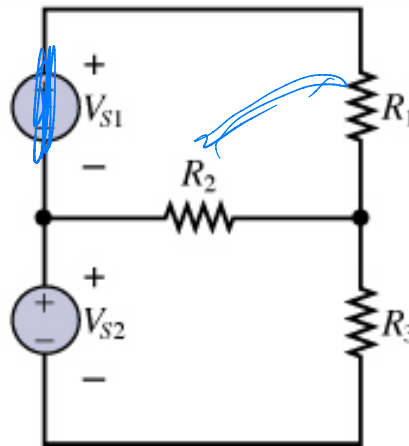
Short circuit

Worked Example on Superposition

Determine the current through R_1 using superposition.

Given $R_1 = 560\Omega$, $R_2 = 3.5k\Omega$, $R_3 = 810\Omega$, $V_{S2} = V_{S1} = 90V$

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a. First, short V_{S1}
 Voltage across R_1 :

$$V_{R1a} = \frac{R_1 \parallel R_2}{(R_1 \parallel R_2) + R_3} V_{S2}$$

$$= \left(\frac{560 \parallel 3500}{(560 \parallel 3500) + 810} \right) 90$$

$$= 33.61V$$

Current through R_1 :

$$I_{R1a} = V_{R1a} / R_{1a}$$

$$= 33.61 / 560$$

$$= 0.06A$$

Worked Example on Superposition

b. Next, short V_{S2}

Current through R_1 :

$$I_{R1b} = \frac{V_{S1}}{R_1 + R_2 \parallel R_3} = \frac{90}{560 + 3500 \parallel 810} = 0.074A$$

Take careful note of the directions defined for each of these currents.

Finally,

$$I_{R1} = I_{R1a} + I_{R1b} = 0.134A$$

1.3 Equivalent Circuit Transformation and Dependent Sources

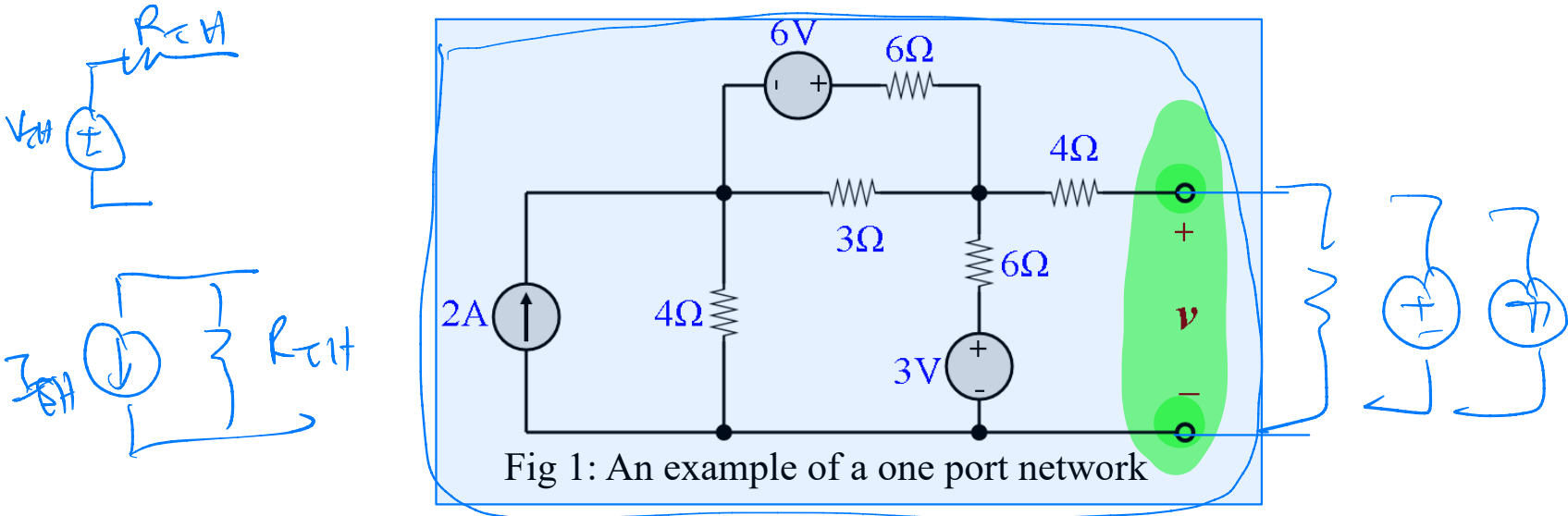
- One port network
 - > Thevenin equivalent circuit
 - > Norton equivalent circuit
 - > Source transformation
- Dependent sources
 - > Dependent voltage source
 - > Dependent current source
- Maximum power transfer

Alexander & Sadiku, “Fundamentals of Electric Circuits” 5th Edition Ch 4.4 – 4.6, 4.8

One-Port Network

Introduction

- A **one port network** is simply a **two terminal device** (See Fig below)



- Given a number of different resistor loads, if you were asked to find the current and voltage at the terminals for each resistor, you would have to recalculate the whole circuit
- A different load will give you different output current & voltage

One-Port Network

Simpler analysis

- Transform any 2 terminal circuit into a circuit that is as simple as having 1 resistor and 1 source (see Fig 2)

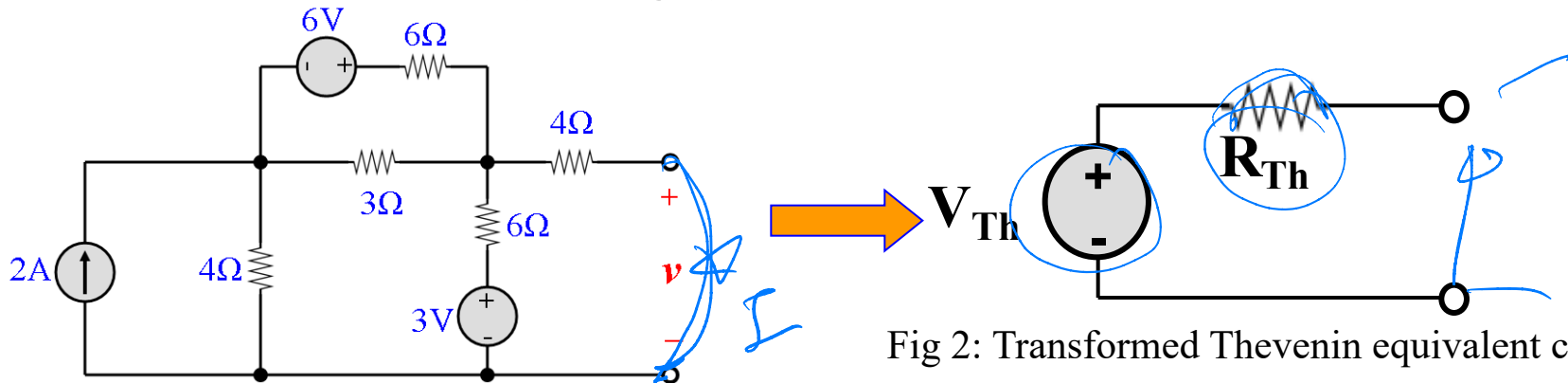
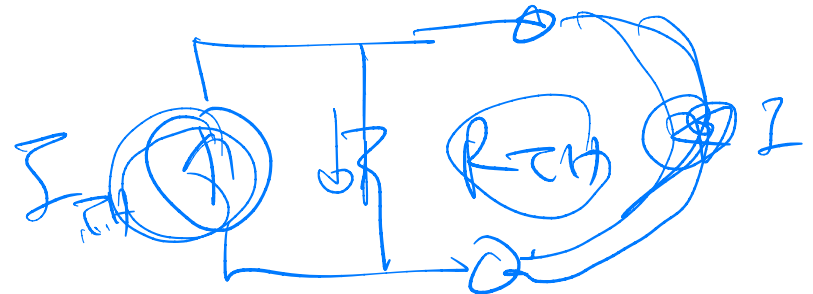


Fig 2: Transformed Thevenin equivalent circuit

The first part of this unit is organized into the following 3 sections:

- Thevenin equivalent
- Norton equivalent
- Source transformation



I. Thevenin

Definition

- Any one-port network composed of **ideal** voltage and current sources, and **linear** resistors, can be represented by an equivalent circuit consisting of an ideal **voltage source** V_{Th} in **series** with an equivalent **resistance** R_{Th}

Deriving the Thevenin equivalent circuit

Step 1: Remove the load from the rest of the one port network. This is the first most fundamental step.

Step 2: Find the equivalent resistance R_T . Disable all ideal sources then find the resistance across the terminals.

For voltage source – replace with short circuit

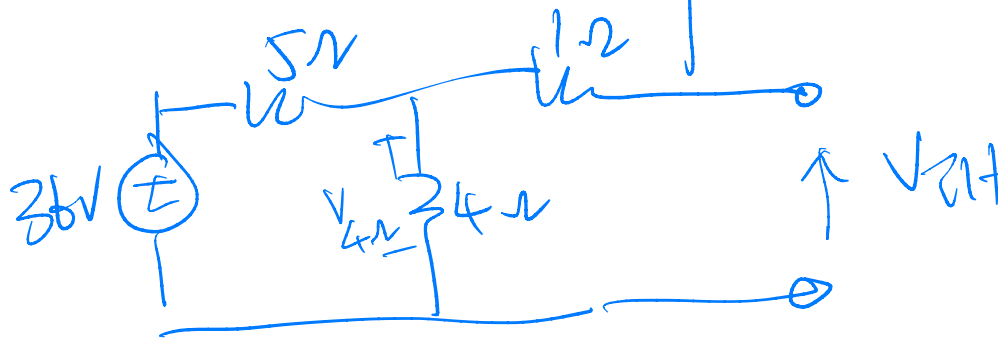
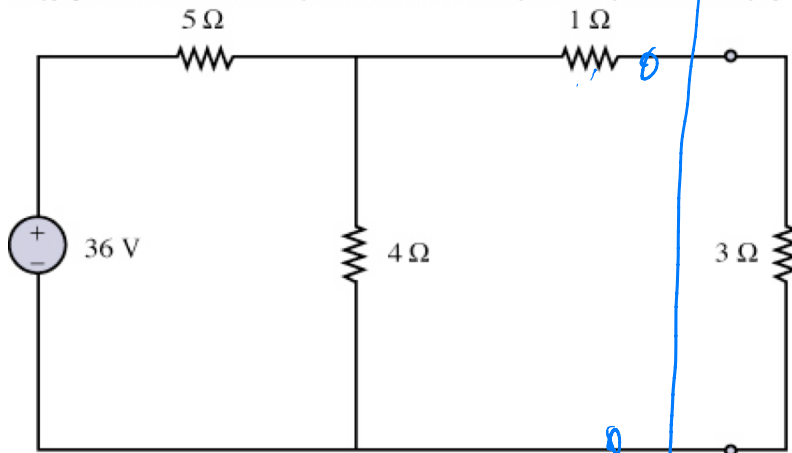
For current source – replace with open circuit

Step 3: Find the Thevenin voltage source. The Thevenin voltage source is equal to the open circuit voltage seen across the terminals (with no load). Solve for this voltage using any preferred method (NVA, MCA, superposition).

Worked Example on Thevenin 1

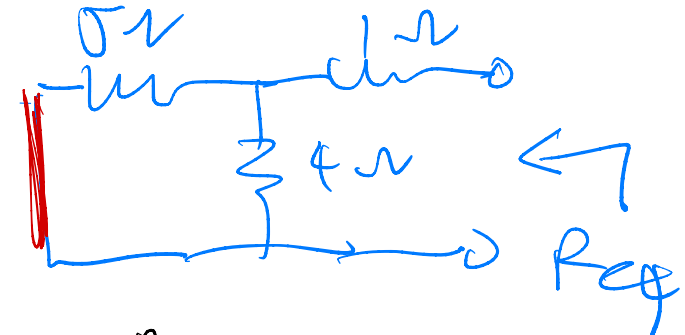
Derive the Thevenin equivalent circuit seen by the 3Ω load.

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Steps: $V_{TH} = 36 \frac{4}{5+4} = 16 \text{ V}$

1. Remove the 3Ω load
2. Find R_{TH} (disable the voltage source)
3. Find V_{TH} (with no load)
4. Draw the Thevenin circuit

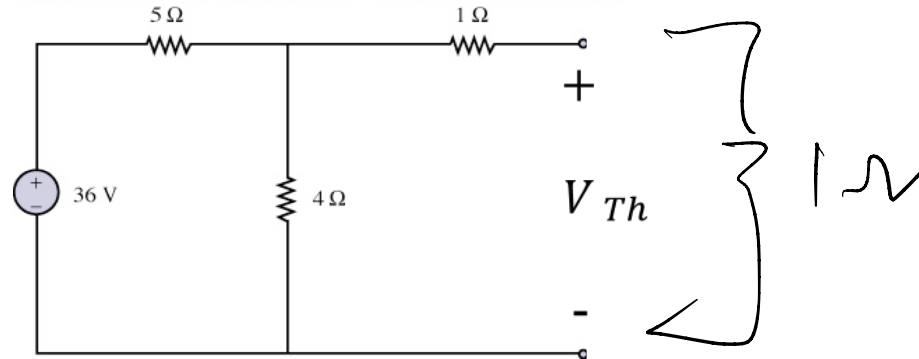


$$R_{TH} = 1 + 4 \parallel 5$$

Worked Example on Thevenin 1

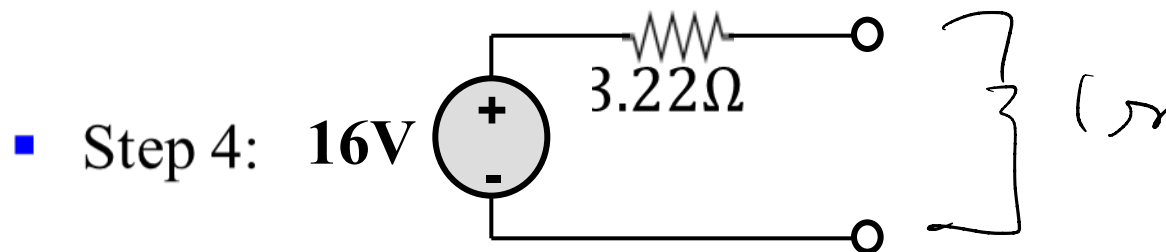
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■ Step 1:



■ Step 2: $R_{Th} = 1 + 5 || 4 = 3.22 \Omega$

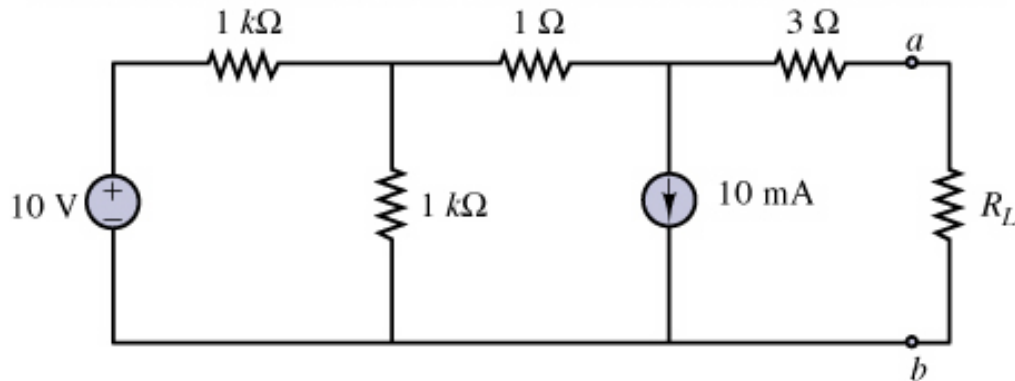
■ Step 3: $V_{Th} = 36 \times \frac{4}{5+4} = 16V$



Worked Example on Thevenin 2

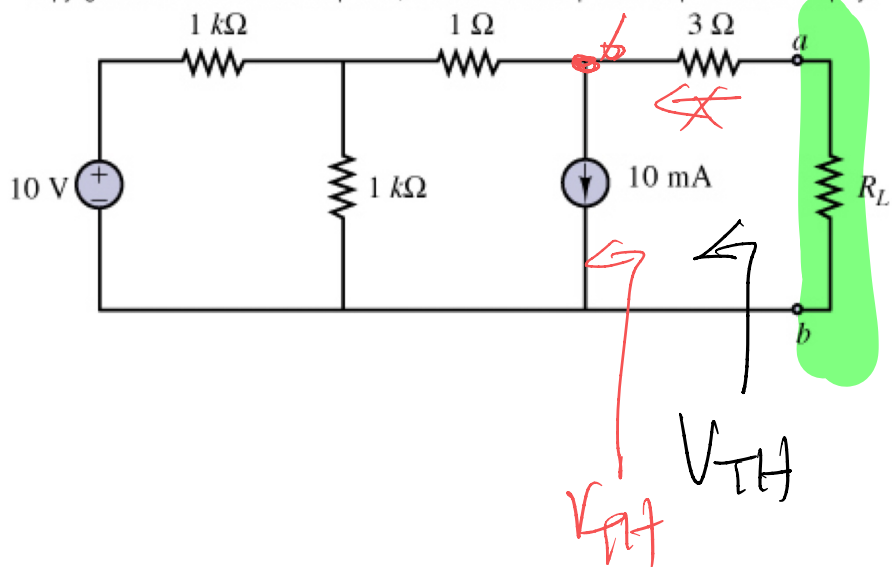
Derive the Thevenin equivalent circuit as seen by the resistor R_L .

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Steps:

1. Remove the load
2. Find R_{Th} (disable the voltage and current sources)
3. Find V_{Th} (with no load)
4. Draw the Thevenin circuit



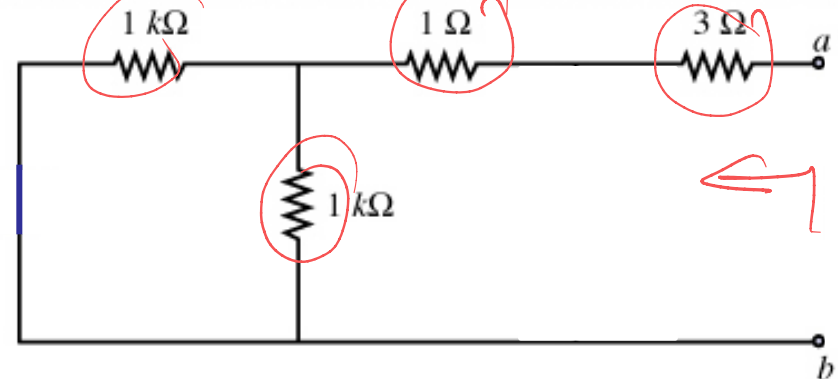
Worked Example on Thevenin 2

■ Step 1 & 2:

$$R_{Th} = 1 + 3 + 1000 || 1000$$

$$= 504 \Omega$$

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■ Step 3:

$$(i) \frac{V_1 - V_{Th}}{1} = 10 \times 10^{-3}$$

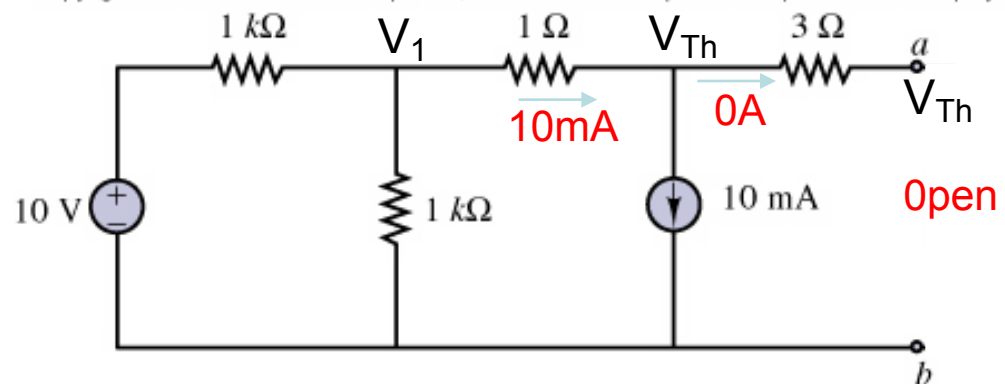
$$(ii) \frac{10 - V_1}{1000} = \frac{V_1}{1000} + 10 \times 10^{-3}$$

$$\text{From (i), } V_{Th} = V_1 - 10 \times 10^{-3}$$

$$\text{From (ii), } 10 - V_1 = V_1 + 10 \Rightarrow 2V_1 = 0 \Rightarrow V_1 = 0V$$

$$\text{Therefore, } V_{Th} = -10 \text{ mV}$$

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II. Norton

Definition

- Any one-port network composed of **ideal** voltage and current sources, and **linear** resistors, can be represented by an equivalent circuit consisting of an ideal **current source** I_N in **parallel** with an equivalent **resistance** R_N

Deriving the Norton equivalent circuit

Steps 1 and 2 for deriving the Norton equivalent circuit are exactly the same as that for Thevenin.

Step 3: Find the Norton current source. The Norton current source is equal to the short circuit current seen across the terminals (with no load). Solve for this voltage using any preferred method (NVA, MCA, superposition).

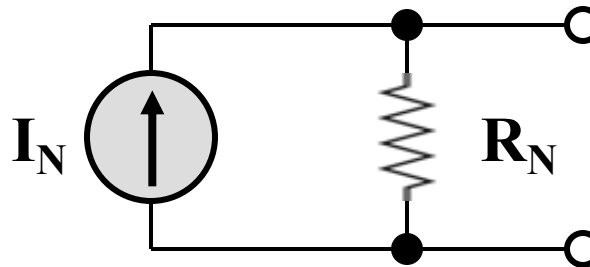
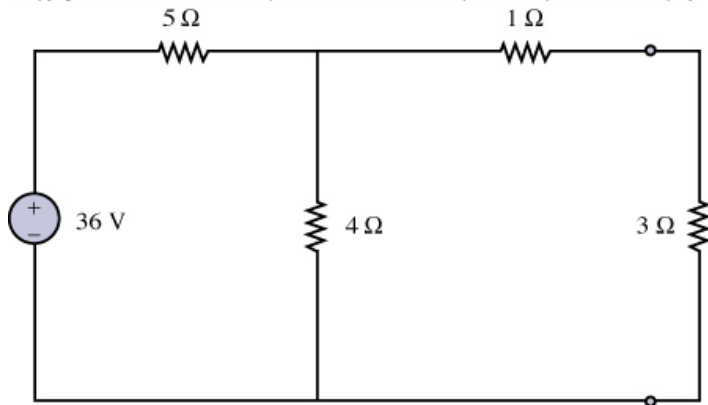


Fig 3: Transformed Norton equivalent circuit

Worked Example on Norton 1

Derive the Norton equivalent circuit seen by the 3Ω load.

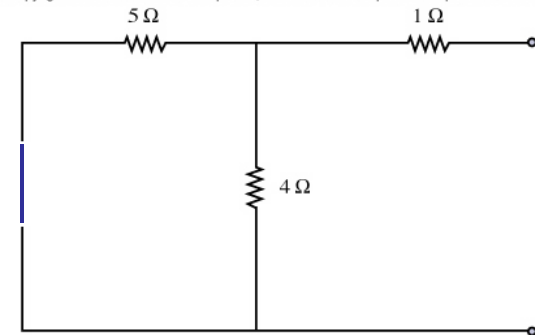
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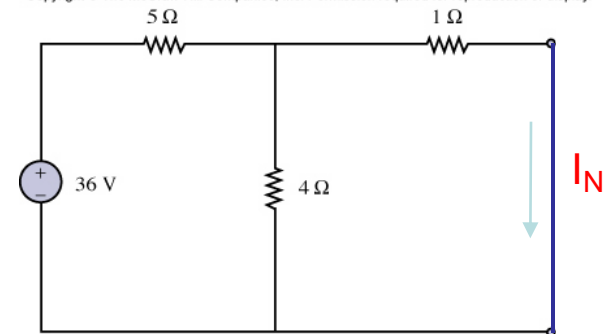
$$\text{Step 1\&2: } R_N = 1 + 4 \parallel 5 = 1 + \frac{4(5)}{4+5} = 3.22\Omega$$

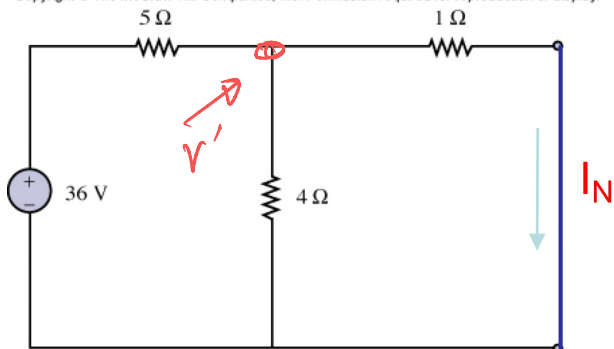
$$\text{Step 3: } I_N = \frac{36 \left(\frac{4(1)}{4+1} \right)}{5 + \frac{4}{5}} = 4.966\text{ A}$$

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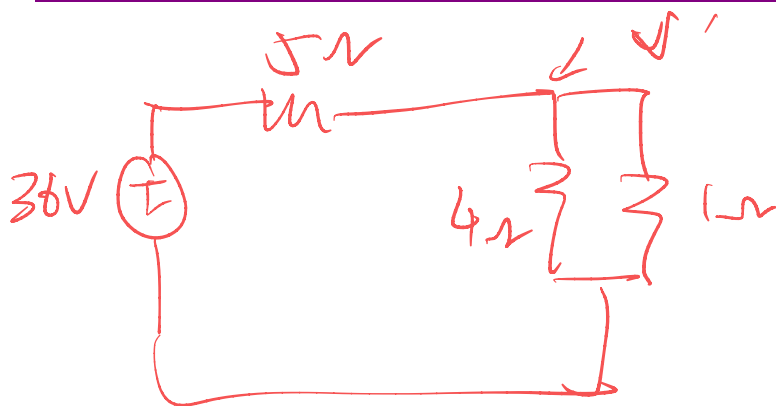


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$$I_N = \frac{V'}{1\Omega}$$

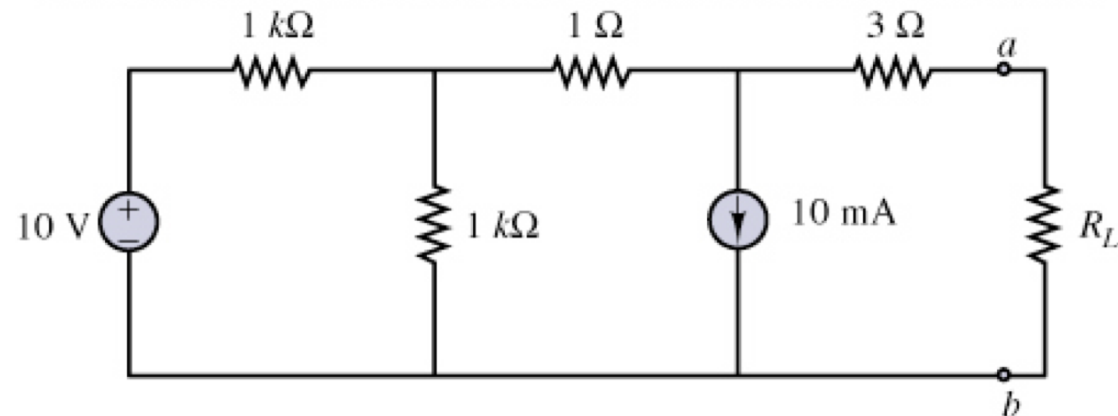


$$V' = 36 \frac{4 || 1}{5 + 4 || 1}$$

Worked Example on Norton 2

Derive the Norton equivalent circuit seen by the load R_L .

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$$\begin{aligned} R_N &= 1k || 1k + 1 + 3 \\ &= 504 \Omega \end{aligned}$$

- (1) $I_N + 10 \times 10^{-3} = i_2$
- (2) $i_1(1000) + i_2(1) + I_N(3) = 10$
- (3) $i_1(1000) + (i_1 - i_2)1000 = 10$



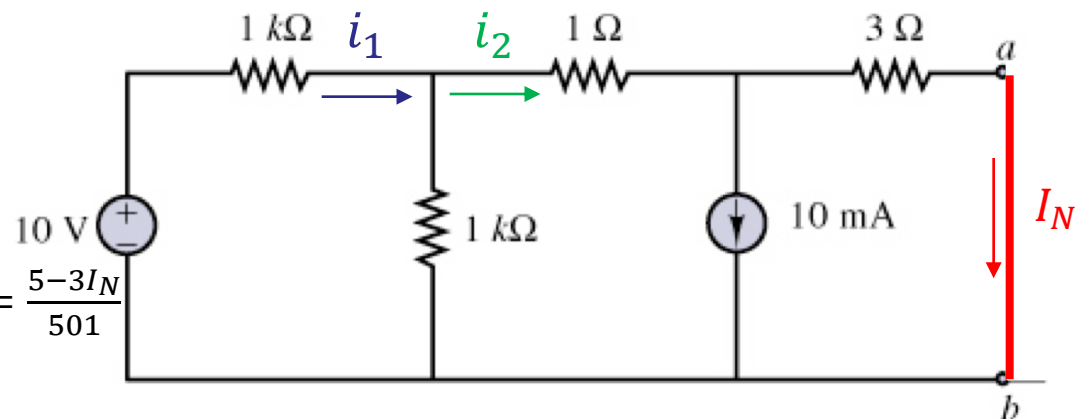
$$(3) i_1 = 0.5i_2 + 0.005$$

$$(2) i_2 = 10 - 3I_N - 1000(0.5i_2 + 0.005) = \frac{5 - 3I_N}{501}$$

$$(1) I_N + 10 \times 10^{-3} = \frac{5 - 3I_N}{501}$$



$$I_N = -19.8 \mu A$$



III. Source Transformation

Definition

- This trick rests on the fact that the Thevenin and Norton forms are equivalent to each other and therefore are also interchangeable

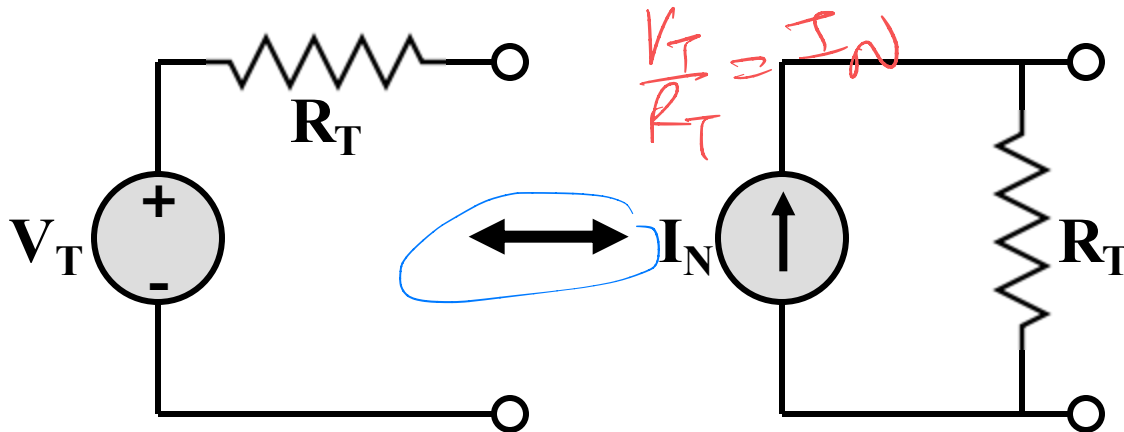


Fig 4a: Thevenin equivalent circuit

Fig 4b: Norton equivalent circuit

We can transform between the two equivalent circuits, observing each time that:

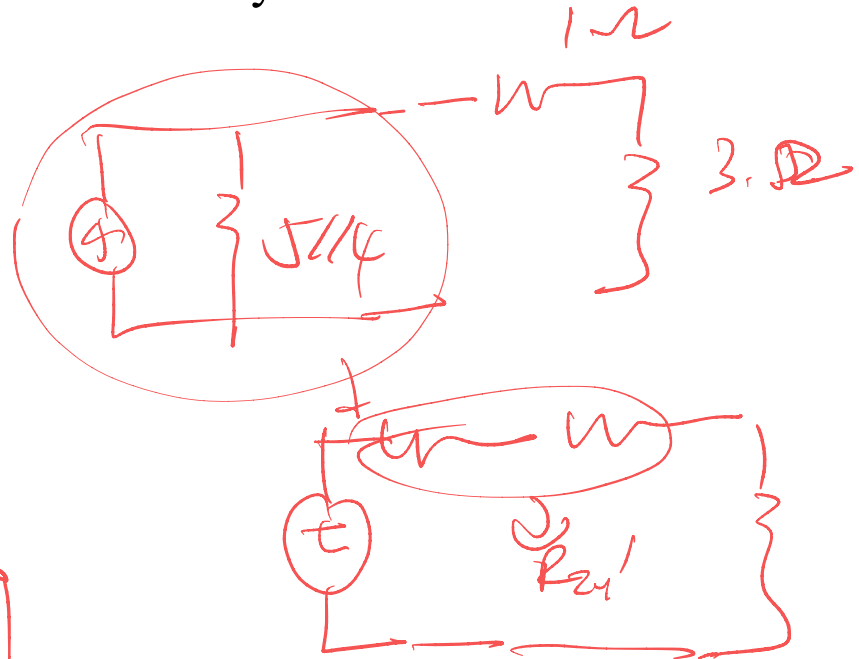
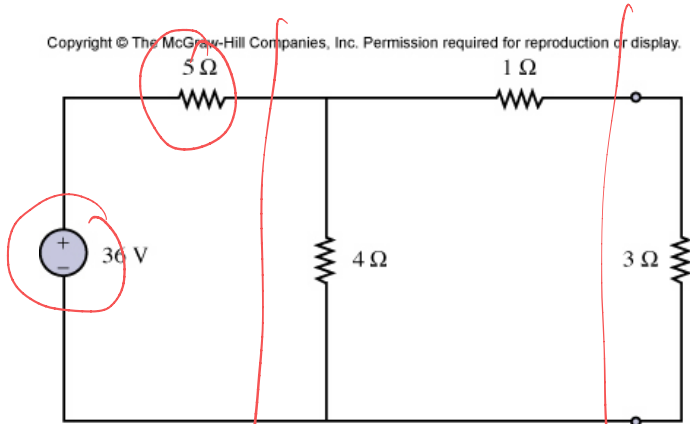
$$V_T = I_N R_T$$

- Rather than transform the whole circuit in one go to Thevenin or Norton equivalent forms, we can instead transform part of the circuit
- The process of merge transform and merge again can be repeated

Worked Example on Source Transformation 1

Derive either of the equivalent circuit forms seen by the 3Ω load.

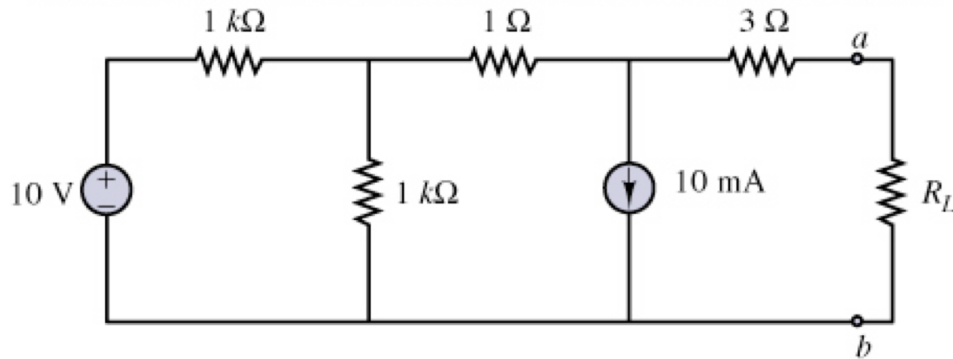
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Worked Example on Source Transformation 2

Derive either of the equivalent circuit forms seen by the load R_L .

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III. Dependent Sources

- All the sources we have come across are independent source
 - > For a voltage source, the voltage maintained across the source is fixed
 - > For a current source, the current through the source is fixed
- There also exist dependent sources in circuit theory
- Unlike independent sources, the value of a dependent source is not predetermined, but is set by the current or voltage through a specific branch

Symbol

- The symbol for an independent source is a circle
- For a dependent source, the symbol is a diamond

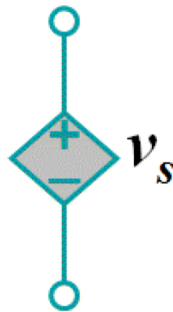


Fig 5a: Dependent voltage source

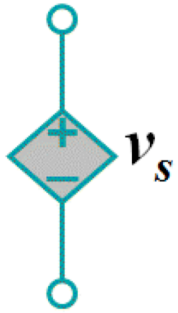


Fig 5b: Dependent current source

Dependent Sources

Dependent Voltage Source

v_S is the voltage source across the terminals.



Current Controlled Voltage Source

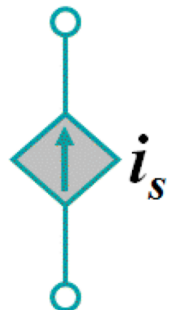
For example, $v_S = 5i_X$. The current i_X is not the current through the source but belongs to another branch.

Voltage Controlled Voltage Source

For example, $v_S = 7v_X$. The voltage v_X is not the source voltage. As v_X changes, so also v_S according to the above relations.

Dependent Current Source

i_S is the current source through the terminals.



Current Controlled Current Source

For example, $i_S = 5i_X$. The current i_X is not the current through the source but belongs to another branch.

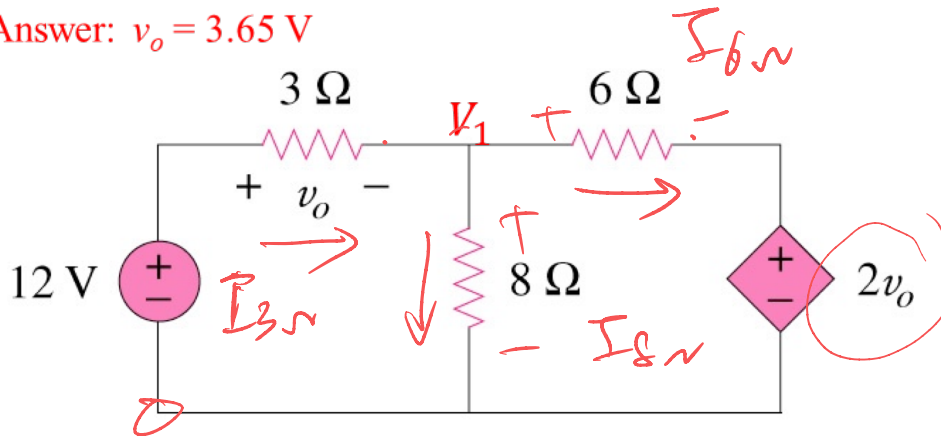
Voltage Controlled Current Source

For example, $i_S = 7v_X$. The voltage v_X is not the source voltage. As v_X changes, so also i_S according to the above relations.

Worked Example on Dependent Source 1

Calculate v_o in the circuit.

Answer: $v_o = 3.65 \text{ V}$



Hints:

1. The dependent source here is a voltage controlled voltage source
2. Apply KCL

(i) $V_1 = 12 - v_o$ $12 - v_o - V_1 = 0$

(ii) $\frac{v_o}{3} + \frac{2v_o - V_1}{6} = \frac{V_1}{8}$

$V_1 = 12 - v_o$

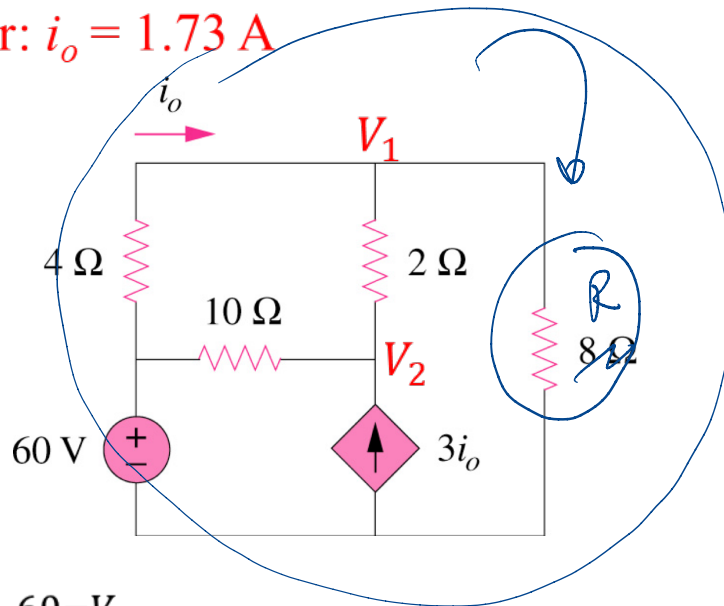
By KCL, $I_{3\Omega} - I_{8\Omega} - I_{6\Omega} = 0$

$$\frac{12 - V_1}{3} - \frac{V_1}{8} - \frac{V_1 - 2v_o}{6} = 0$$

Worked Example on Dependent Source 2

Calculate i_o in the circuit.

Answer: $i_o = 1.73 \text{ A}$



Hints:

1. The dependent source here is a current controlled current source
2. Apply KCL

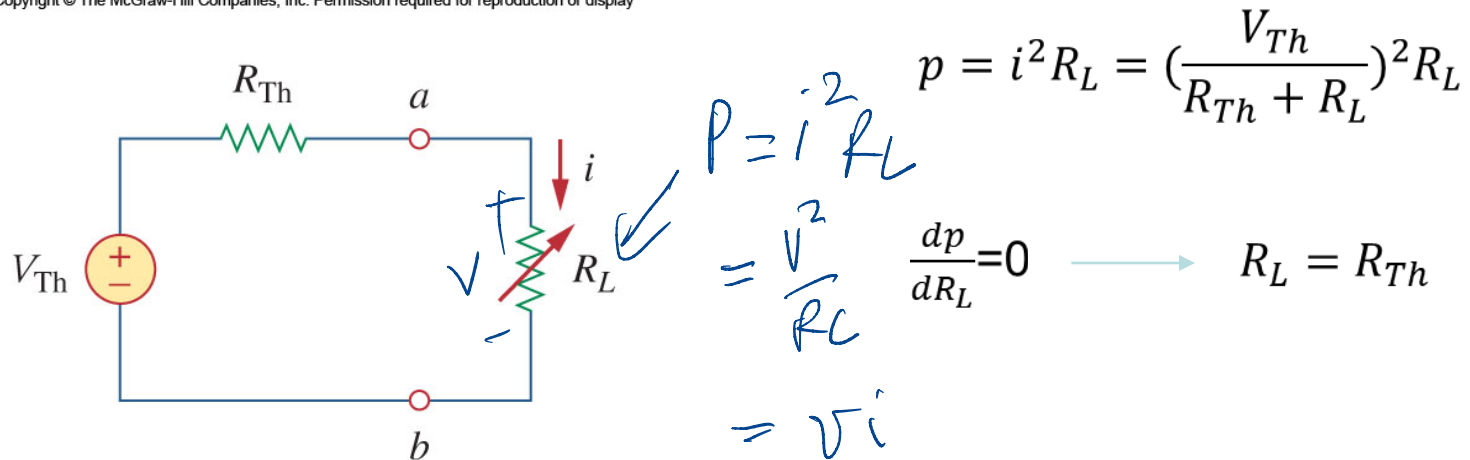
$$(i) i_o = \frac{60 - V_1}{4}$$

$$(ii) i_o = \frac{V_1 - V_2}{2} + \frac{V_1}{8}$$

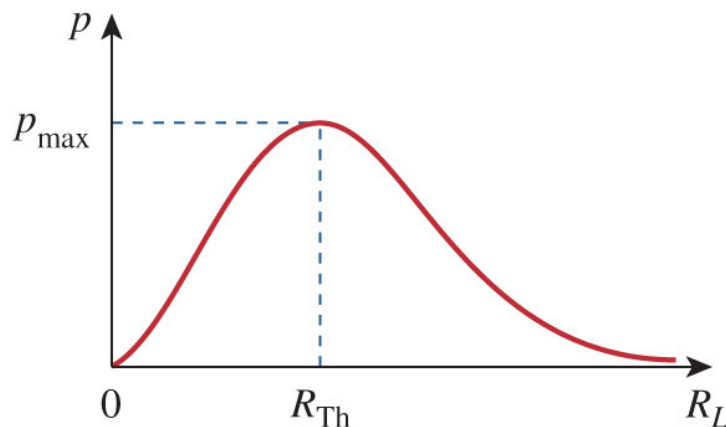
$$(iii) \frac{60 - V_2}{10} + 3i_o = \frac{V_2 - V_1}{2}$$

Maximum Power Transfer

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$$p_{max} = \frac{V_{Th}^2}{4R_{Th}}$$