

EE3124 Tutorial 5 (Solution)

Synchronous Motor

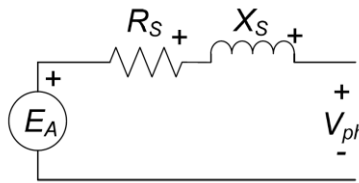
Name:

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Q1 - A 1000 kVA, 11,000 V, three-phase star-connected synchronous motor has an armature resistance and reactance per phase of 3.5Ω and 40Ω respectively.

Draw the vector diagram and determine (a) the voltage drop across armature impedance; (b) the induced e.m.f. and the torque angle of the rotor when it is fully loaded at 0.8 pf lagging.

Solution



Phase voltage: $V_{ph} = \frac{11e3}{\sqrt{3}} = 6350 V$

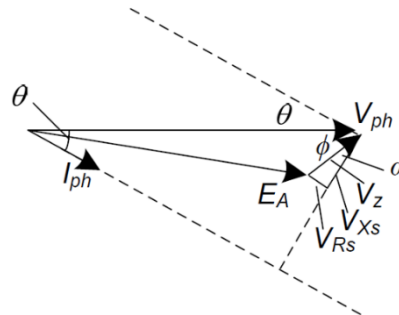
Full load armature current at 0.8 p.f: $|I_{ph}| = \frac{1000e3}{\sqrt{3}11e3} = 52.49 A$

Calculate the induced e.m.f. and torque angle using KVL and complex number

$$Z_s = \sqrt{3.5^2 + 40^2} \angle \tan^{-1} \frac{40}{3.5} = 40.153 \angle 85^\circ$$

$$\begin{aligned} V_{zs} &= 40.153 \angle 85^\circ \times 52.49 \angle -\cos^{-1} 0.8 \\ &= 40.153 \angle 85^\circ \times 52.49 \angle -36.87^\circ \\ &= 2107.63 \angle 48.13^\circ \end{aligned}$$

$$\begin{aligned} E_A &= V_{ph} - V_{zs} \\ &= 6350 - 2107.63 \angle 48.13^\circ \\ &= 6350 - 1406.72 - j1569.47 \\ &= 4943.28 - j1569.47 \\ &= 5186.45 \angle -17.614^\circ \end{aligned}$$



Q2 - A 2000 V 3-phase, 4-pole, Y-connected synchronous motor runs at 1500 r.p.m.. The excitation is constant and corresponds to an open-circuit terminal voltage of 2000 V. The resistance is negligible as compared with synchronous impedance of 3Ω per phase. Determine the power input, power factor and torque developed for a lagging armature current of 200 A.

Solution

Phase voltage: $V_{ph} = \frac{2000}{\sqrt{3}} = 1154.70 \text{ V}$

Induced e.m.f.: 1154.70V

Voltage drop across impedance: $V_Z = 200 \times 3 = 600 \text{ V}$

$$V_{ph}^2 = E_A^2 + V_Z^2 - 2E_A V_Z \cos \phi$$

$$1154.7^2 = 1154.7^2 + 600^2 - 2 \times 1154.7 \times 600 \times \cos \phi$$

$$0 = 360000 - 1385640 \cos \phi$$

$$\phi = 74.941^\circ$$

$$\theta = 90 - \phi$$

$$= 15.06^\circ$$

$$p.f. = \cos 15.06$$

$$= 0.966$$

$$P_{in} = 3 \times V_{ph} \times I_{ph} \times \cos \theta$$

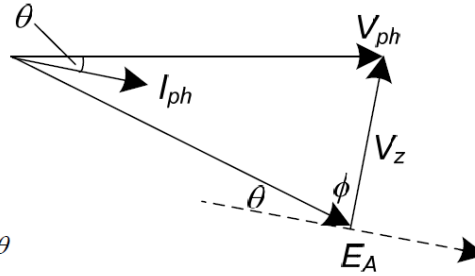
$$\begin{aligned} \text{Input power: } &= 3 \times 1154.7 \times 200 \times 0.966 \\ &= 669264.12 \text{ W} \end{aligned}$$

$$N_s = \frac{r.p.m.}{60 \text{ sec}} = \frac{1500}{60} = 25 \text{ r.p.s.}$$

$$\tau \times \omega = \text{Power}$$

$$\tau = 669264.12 / (2\pi 25)$$

$$= 4260.67 \text{ N} \cdot \text{m}$$



Q3 - A 480-V, 60 Hz, 400-hp 0.8-PF-leading eight-pole Δ -connected synchronous motor has a synchronous reactance of 0.6Ω and negligible armature resistance. Ignore its friction, windage, and core losses for the purposes of this problem. Assume that $|E_A|$ is directly proportional to the field current I_F (in other words, assume that the motor operates in the linear part of the magnetization curve), and that $E_A = 480 \text{ V}$ when $I_F = 4 \text{ A}$.

(a) What is the speed of this motor?

(b) If this motor is initially supplying 400 hp at 0.8 PF lagging, what are the magnitudes and angles of $\mathbf{E}_A$ and $\mathbf{I}_A$?

(c) According to (b), how much torque is this motor producing? What is the torque angle δ ? How near is this value to the maximum possible induced torque of the motor for this field current setting?

(d) If $\mathbf{E}_A$ is increased by 30 percent, what is the new magnitude of the armature current? What is the motor's new power factor?

Solution

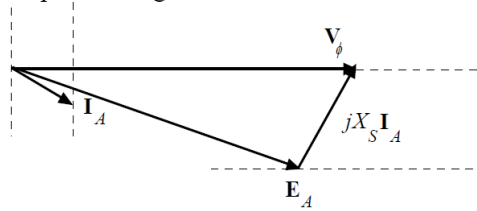
(a) The speed of this motor is given by

$$n_m = \frac{120 f_{se}}{P} = \frac{120(60 \text{ Hz})}{8} = 900 \text{ r/min}$$

(b) If losses are being ignored, the output power is equal to the input power, so the input power will be

$$P_{IN} = (400 \text{ hp})(746 \text{ W/hp}) = 298.4 \text{ kW}$$

This situation is shown in the phasor diagram below:



The line current flow under these circumstances is

$$I_L = \frac{P}{\sqrt{3} V_T \text{ PF}} = \frac{298.4 \text{ kW}}{\sqrt{3} (480 \text{ V})(0.8)} = 449 \text{ A}$$

Because the motor is Δ -connected, the corresponding phase current is $I_A = 449 / \sqrt{3} = 259 \text{ A}$. The angle of the current is $-\cos^{-1}(0.80) = -36.87^\circ$, so $\mathbf{I}_A = 259 \angle -36.87^\circ \text{ A}$. The internal generated voltage $\mathbf{E}_A$ is

$$\mathbf{E}_A = \mathbf{V}_\phi - jX_S \mathbf{I}_A$$

$$\mathbf{E}_A = (480 \angle 0^\circ \text{ V}) - j(0.6 \Omega)(259 \angle -36.87^\circ \text{ A}) = 406 \angle -17.8^\circ \text{ V}$$

(c) This motor has 6 poles and an electrical frequency of 60 Hz, so its rotation speed is $n_m = 1200$ r/min. The induced torque is

$$\tau_{\text{ind}} = \frac{P_{\text{OUT}}}{\omega_m} = \frac{298.4 \text{ kW}}{(900 \text{ r/min}) \left(\frac{1 \text{ min}}{60 \text{ s}} \right) \left(\frac{2\pi \text{ rad}}{1 \text{ r}} \right)} = 3166 \text{ N} \cdot \text{m}$$

The maximum possible induced torque for the motor at this field setting is the maximum possible power divided by ω_m

$$\tau_{\text{ind,max}} = \frac{3V_\phi E_A}{\omega_m X_s} = \frac{3(480 \text{ V})(406 \text{ V})}{(900 \text{ r/min}) \left(\frac{1 \text{ min}}{60 \text{ s}} \right) \left(\frac{2\pi \text{ rad}}{1 \text{ r}} \right) (0.6 \Omega)} = 10,340 \text{ N} \cdot \text{m}$$

The current operating torque is about 1/3 of the maximum possible torque.

(d) If the magnitude of the internal generated voltage E_A is increased by 30%, the new torque angle can be found from the fact that $E_A \sin \delta \propto P = \text{constant}$.

$$E_{A2} = 1.30 E_{A1} = 1.30(406 \text{ V}) = 487.2 \text{ V}$$

$$\delta_2 = \sin^{-1} \left(\frac{E_{A1} \sin \delta_1}{E_{A2}} \right) = \sin^{-1} \left(\frac{406 \text{ V}}{487.2 \text{ V}} \sin(-17.8^\circ) \right) = -14.8^\circ$$

The new armature current is

$$\mathbf{I}_{A2} = \frac{\mathbf{V}_\phi - \mathbf{E}_{A2}}{jX_s} = \frac{480 \angle 0^\circ \text{ V} - 487.2 \angle -14.8^\circ \text{ V}}{j0.6 \Omega} = 208 \angle -4.1^\circ \text{ A}$$

The magnitude of the armature current is 208 A, and the power factor is $\cos(-4.1^\circ) = 0.913$ lagging.