

Assignment 1 Solution

The assignment consists of 6 questions.

1. (10%) Simplify $\frac{2\sqrt{7}-\sqrt{5}}{\sqrt{7}+\sqrt{5}}$ such that the denominator consists of an integer only.

Solution:

$$\begin{aligned} & \frac{2\sqrt{7}-\sqrt{5}}{\sqrt{7}+\sqrt{5}} \\ &= \frac{(2\sqrt{7}-\sqrt{5})(\sqrt{7}-\sqrt{5})}{(\sqrt{7}+\sqrt{5})(\sqrt{7}-\sqrt{5})} \\ &= \frac{14-3\sqrt{35}+5}{7-5} \\ &= \frac{19-3\sqrt{35}}{2} \end{aligned}$$

2. (20%) Calculate

$$\left(\frac{1-\sqrt{3}i}{6}\right)^2 \left(\frac{\sqrt{3}-i}{2}\right)^3 \left(\overline{1-\sqrt{3}i}\right)^2.$$

Solution:

Step 1: Find the conjugate of $1 - \sqrt{3}i$.

$$\left(\frac{1-\sqrt{3}i}{6}\right)^2 \left(\frac{\sqrt{3}-i}{2}\right)^3 \left(\overline{1-\sqrt{3}i}\right)^2 = \left(\frac{1-\sqrt{3}i}{6}\right)^2 \left(\frac{\sqrt{3}-i}{2}\right)^3 (1+\sqrt{3}i)^2$$

Step 2: Convert to polar form.

$$\begin{aligned} & \left(\frac{1-\sqrt{3}i}{6}\right)^2 \left(\frac{\sqrt{3}-i}{2}\right)^3 (1+\sqrt{3}i)^2 \\ &= \left(\frac{1}{3} \left(\cos\left(-\frac{\pi}{3}\right) + i \sin\left(-\frac{\pi}{3}\right)\right)\right)^2 \left(\cos\left(-\frac{\pi}{6}\right) + i \sin\left(-\frac{\pi}{6}\right)\right)^3 \left(2 \left(\cos\left(\frac{\pi}{3}\right) + i \sin\left(\frac{\pi}{3}\right)\right)\right)^2 \\ &= \left(\frac{1}{3} \operatorname{cis}\left(-\frac{\pi}{3}\right)\right)^2 \operatorname{cis}\left(-\frac{\pi}{6}\right)^3 \left(2 \operatorname{cis}\left(\frac{\pi}{3}\right)\right)^2 \end{aligned}$$

Step 3: Use **de Moivre's Theorem** to calculate the result.

$$\begin{aligned}\left(\frac{1}{3} \operatorname{cis}\left(-\frac{\pi}{3}\right)\right)^2 \operatorname{cis}\left(-\frac{\pi}{6}\right) \left(2 \operatorname{cis}\left(\frac{\pi}{3}\right)\right)^2 &= \frac{2^2}{3^2} \operatorname{cis}\left(\frac{2\pi}{3} - \frac{2\pi}{3} - \frac{3\pi}{6}\right) \\ &= \frac{4}{9} \operatorname{cis}\left(-\frac{\pi}{2}\right) = \frac{4}{9}(0 - i) = -i \frac{4}{9}\end{aligned}$$

3. (20%) the 3rd roots of

$$-4\sqrt{3} + 4i.$$

Solution:

Step 1: Convert to polar form:

$$\begin{aligned}r &= \sqrt{(-4\sqrt{3})^2 + (4)^2} = 8 \\ \theta &= \pi + \arctan^{-1} \frac{4}{-4\sqrt{3}} = \frac{5\pi}{6}\end{aligned}$$

(When real part is negative, π should be added to derive the true argument.)

$$-4\sqrt{3} + 4i = 8 \left(\cos\left(\frac{5\pi}{6}\right) + i \sin\left(\frac{5\pi}{6}\right) \right) = 8 \operatorname{cis}\left(\frac{5\pi}{6}\right)$$

Step 2: Solve

$$z^3 = 8 \operatorname{cis}\left(\frac{5\pi}{6} + 2k\pi\right).$$

We have

$$z = 2 \operatorname{cis}\left(\frac{5\pi}{18} + \frac{2}{3}k\pi\right)$$

Step 3: Obtain three roots by substituting $k = 0, 1, 2$.

$$\begin{aligned}k = 0, z &= 2 \operatorname{cis}\left(\frac{5\pi}{18}\right) \\ k = 1, z &= 2 \operatorname{cis}\left(-\frac{\pi}{18} + \frac{2}{3}\pi\right) = 2 \operatorname{cis}\left(\frac{17\pi}{18}\right) \\ k = 2, z &= 2 \operatorname{cis}\left(-\frac{\pi}{18} + \frac{4}{3}\pi\right) = 2 \operatorname{cis}\left(\frac{29\pi}{18}\right).\end{aligned}$$

4. (10%) Convert the following hexadecimal number
 $(7A9E)_{16}$
 to 1) decimal number and 2) 16 bits equivalent binary number.

Solution:

1) We use the **power series expansion**:

$$\begin{aligned}(7A9E)_{16} &= 7 \times 16^3 + 10 \times 16^2 + 9 \times 16^1 + 14 \times 16^0 \\ &= 28672 + 2560 + 144 + 14 = (31390)_{10}\end{aligned}$$

2) Convert each hex digit to 4-bit binary number.

Hex	7	A	9	E
	↓	↓	↓	↓
Bin	0111	1010	1001	1110

Therefore, $(7A9E)_{16} = (0111\ 1010\ 1001\ 1110)_2$.

5. (20%) Assume 12-bit 2's complement format with 1 bit for the sign, 8 bits for the integer part, and 3 bits for the fractional part. Find the 12-bit fixed-point **2's complement** representation of the following fractional number:

$$(-88.125)_{10}.$$

Solution:

Step 1: Convert to binary number

$$\begin{aligned}(-88.125)_{10} &= -(64 + 16 + 8 + 0.125) \\ &= -(2^6 + 2^4 + 2^3 + 2^{-3}) \\ &= (-101\ 1000.001)_2\end{aligned}$$

Step 2: Find the 2's complement:

$$\begin{aligned}(-101\ 1000.001)_2 &\rightarrow \quad \mathbf{1\ 0101\ 1000.001} \quad \text{Sign-Magnitude Form} \\ &\quad \mathbf{1\ 1010\ 0111.110} \quad \text{1's Complement} \\ &\quad \mathbf{1\ 1010\ 0111.111} \quad \text{2's Complement}\end{aligned}$$

Therefore, 12-bit fixed-point 2's complement representation of $(-88.125)_{10}$ is **1 1010 0111.111**.

6. (20%) Use IEEE 754 32-bit floating point format to perform the following addition

$$X + Y$$

X : 46th largest negative number that IEEE 754 32-bit floating point format can represent.

Y : [1][0000 0100] [1101 0000 0000 1010 1010 011]

Show your answer in IEEE 754 32-bit floating-point format.

IEEE 754 32-bit floating-point format is defined as:

Sign (1 bit)	Exponent (8 bits)	Mantissa (23 bits)
0	0000 0000	0000 0000 0000 0000 0000 000

Solution:

X is derived as:

Sign: 1 (Negative)

Exponent: 0000 0001 (Smallest exponent)

Mantissa: 0000 0000 0000 0000 0101 101 (46th smallest mantissa)

X : [1][0000 0001] [0000 0000 0000 0000 0101 101]

Exponents of X and Y are different, shift the radix point of smaller number X by 3 places.

Exponent (8 bits)	Mantissa (23 bits with hidden 1)
0000 0001	1.0000 0000 0000 0000 0101 101
↓	↓
0000 0100	0.0010 0000 0000 0000 0000 101

Then perform the addition of X and Y .

X	0.0010 0000 0000 0000 0000 101
$+Y$	1.1101 0000 0000 1010 1010 011
$X + Y$	1.1111 0000 0000 1010 1011 000

The result of addition in IEEE 754 is 32-bit floating point format is:

[1] [0000 0100] [1111 0000 0000 1010 1011 000]