

MA1502/GE1359 Algebra
Semester A, 2021-22
Test

Date: Wednesday, 27 October 2021

Time allowed: 90 minutes (9:15 am – 10:45 am)

Submission time: 15 minutes (10:45 am – 11:00 am)

Instructions to candidates:

This paper has FOUR pages (including this cover page).

The formula sheet of trigonometric identities is provided on page 4.

This paper has SEVEN questions.

The questions will begin on page 2.

Answer ALL questions. Show ALL steps to get full mark.

Please submit your work as a single pdf file (with the signed Academic Honesty Pledge as the front page) to Canvas before 11:00 am HK time on 27 October 2021.

Question 1

Find the largest possible domain and largest possible range of the following function. Give your answers in set/interval notations.

$$F(x) = \frac{2}{x^2 - 2x - 3}. \quad [13 \text{ marks}]$$

Question 2

Let f and g be real-valued functions defined by

$$f(x) = \sqrt{7-x} \quad \text{and} \quad g(x) = \frac{x}{x-2}.$$

For each of the following functions, find its expression and largest possible domain.

(a) $\left(\frac{f}{g}\right)(x)$ [6 marks]

(b) $(f \circ f)(x)$ [7 marks]

(c) $(g \circ f)(x)$ [7 marks]

Question 3

Consider the real-valued function

$$G(x) = \sqrt{x^2 - 6x + 11} \quad \text{for } x \leq 1.$$

(a) Determine whether $G^{-1}(x)$, the inverse function of $G(x)$, exists. Give reasons. [4 marks]

(b) Find $G^{-1}(x)$ if it exists. [4 marks]

(c) Find the largest possible domain and largest possible range of $G^{-1}(x)$ if it exists. [4 marks]

Question 4

Given that $\sin A = \frac{2}{\sqrt{5}}$ where $90^\circ < A < 180^\circ$, and $\cos B = -\frac{5}{13}$ where $-180^\circ < B < -90^\circ$.

Without using calculator, find the values of $\sin(2A + B)$ and $\cos(2A + B)$. Then determine the quadrant containing $2A + B$. Give reasons. [16 marks]

Question 5

Find the general solution (in radians) of each of the following equations:

(a) $\cot x = \sqrt{3}$ [5 marks]

(b) $\cos(6x) - 3 \sin(3x) + 1 = 0$ [11 marks]

Question 6

Starting with $\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$, without using calculator, find the exact value of $\sin \frac{\pi}{24}$.

(You do not need to simplify your answer.) [7 marks]

Question 7

Express $\frac{x^5 - x^4 + 2x^3 + 15x^2 - 16x + 36}{x^4 - 16}$ into partial fractions. [16 marks]

- End -

Trigonometric identities - Formula sheet

➤ Compound Angle Formulae

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

➤ Double Angle Formulae

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

➤ Half Angle Formulae

$$\cos^2 A = \frac{1}{2}(1 + \cos 2A)$$

$$\sin^2 A = \frac{1}{2}(1 - \cos 2A)$$

➤ Product-to-Sum Formulae

$$\sin A \cos B = \frac{1}{2}[\sin(A + B) + \sin(A - B)]$$

$$\cos A \sin B = \frac{1}{2}[\sin(A + B) - \sin(A - B)]$$

$$\cos A \cos B = \frac{1}{2}[\cos(A + B) + \cos(A - B)]$$

$$\sin A \sin B = -\frac{1}{2}[\cos(A + B) - \cos(A - B)]$$

➤ Sum-to-Product Formulae

$$\sin x + \sin y = 2 \sin\left(\frac{x + y}{2}\right) \cos\left(\frac{x - y}{2}\right)$$

$$\sin x - \sin y = 2 \cos\left(\frac{x + y}{2}\right) \sin\left(\frac{x - y}{2}\right)$$

$$\cos x + \cos y = 2 \cos\left(\frac{x + y}{2}\right) \cos\left(\frac{x - y}{2}\right)$$

$$\cos x - \cos y = -2 \sin\left(\frac{x + y}{2}\right) \sin\left(\frac{x - y}{2}\right)$$