

1. Let $\vec{a} = 2\vec{i} - 2\vec{j} + \vec{k}$, $\vec{b} = \vec{i} + 8\vec{j} - 4\vec{k}$, $\vec{c} = 12\vec{i} - 4\vec{j} - 3\vec{k}$. Find
 - (a) $|\vec{a}|$, $\frac{\vec{a}}{|\vec{a}|}$, $|\vec{a} + \vec{b}|$, $|\vec{a}| + |\vec{b}|$, $|\vec{a} - \vec{b}|$, $|\vec{a}| - |\vec{b}|$;
 - (b) $\vec{a} \cdot \vec{b}$ and the angle between $\vec{a}$ and $\vec{b}$;
 - (c) the coefficient of the projection of $\vec{b}$ on $\vec{a}$;
 - (d) $\vec{a} \times \vec{b}$, $\vec{c} \times \vec{a}$, $\vec{a} \times (\vec{b} \times \vec{c})$, $(\vec{a} \times \vec{b}) \times \vec{c}$;
 - (e) the area of the triangle with sides $\vec{a}$ and $\vec{c}$;
 - (f) $\vec{a} \cdot \vec{b} \times \vec{c}$;
 - (g) the volume of the tetrahedron with $\vec{a} + \vec{c}$, $\vec{a} - \vec{c}$ and $\vec{b}$ as adjacent edges.
2. Use vector method to find the volume of the tetrahedron of which the vertices are the points $A(1, 0, 1)$, $B(1, 2, 3)$, $C(-1, -2, -3)$, $D(3, 2, 1)$.
3. Using vector method, show that the midpoints of the sides of an arbitrary quadrilateral form a parallelogram. Further, show that the perimeter of this parallelogram is equal to the sum of the lengths of the diagonals of the original quadrilateral.
4. Let L be the line through $P_1(1, 2, 2)$ and $P_2(0, 2, 5)$. Find the trisection points of the line segment P_1P_2 .
5. Show that for any vectors $\vec{a}$ and $\vec{b}$
 - (a) $|\vec{a} + \vec{b}|^2 + |\vec{a} - \vec{b}|^2 = 2(|\vec{a}|^2 + |\vec{b}|^2)$;
 - (b) $|\vec{a}|^2 |\vec{b}|^2 = (\vec{a} \cdot \vec{b})^2 + |\vec{a} \times \vec{b}|^2$;
 - (c) $|\vec{a} + \vec{b}| \leq |\vec{a}| + |\vec{b}|$.
6. Show that the vectors $\vec{a} = -\vec{i} + \vec{j} + 2\vec{k}$, $\vec{b} = \vec{j} + \vec{k}$ and $\vec{c} = \vec{i} + 2\vec{j}$ are linearly independent. Express the general vector $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ as a linear combination of $\vec{a}$, $\vec{b}$ and $\vec{c}$.
Confirm that $-2\vec{i} + \vec{j} - \vec{k} = 6\vec{a} - 13\vec{b} + 4\vec{c}$.
7. Let l_1 and l_2 be two nonparallel lines with directions $\vec{v}_1 = \vec{i} + \vec{k}$, and $\vec{v}_2 = \vec{i} + \vec{j}$ respectively. Let $P(1, 1, 1)$ be a point on l_1 and $Q(2, 1, 0)$ on l_2 .
 - (a) Find the unit vector $\vec{u}$ in the direction of $\overrightarrow{PQ}$.
 - (b) Show that $\vec{v}_1$, $\vec{v}_2$ and $\vec{u}$ are linearly independent.
 - (c) Using vector methods, find the shortest distance between l_1 and l_2 .

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