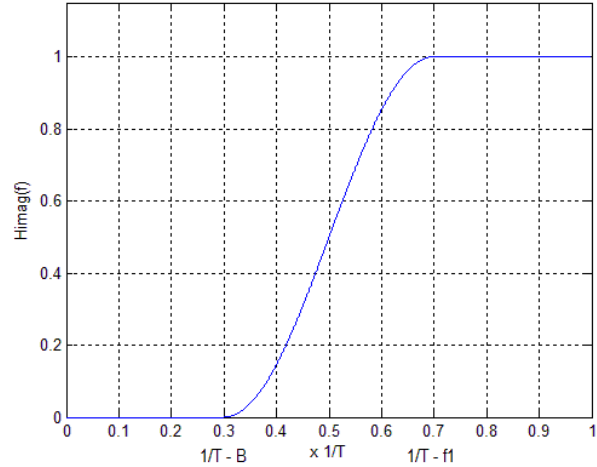
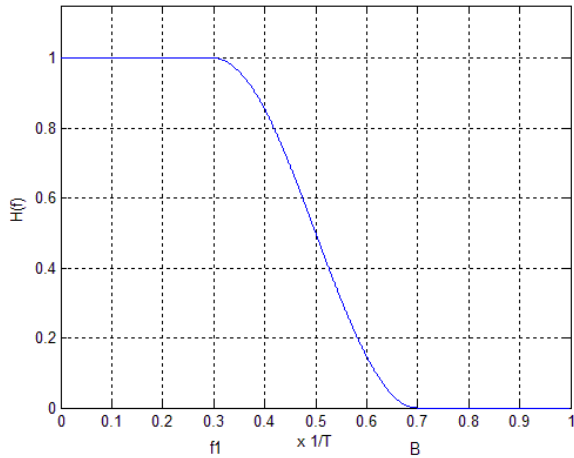


## EE3101 Tutorial 5 Solutions

- (1) (a) symbol rate = 100 MSymbol/s, i.e. BW=100M/2=50MHz  
 (b) symbol rate =  $140/\log_2 16 = 35$  MSymbol/s, i.e. BW=35M/2=17.5MHz
- (2) (a) 6dB bandwidth is the minimum Nyquist bandwidth =  $45/2=22.5$ MHz  
 (b) 100% power bandwidth =  $22.5 \times (1+0.3)$ MHz = 29.25MHz.
- (3) (a) Replace  $f$  with  $\frac{1}{T} - f$ , the mirror image of  $H(f)$  is:

$$\begin{aligned}
 H_{imag}(f) &= 1 & \text{for } \frac{1}{T} - f_1 \leq f \leq \frac{1}{T} \\
 H_{imag}(f) &= \frac{1}{2} \left\{ 1 + \cos \left[ \frac{\pi \left( \frac{1}{T} - f - f_1 \right)}{2f_\Delta} \right] \right\} & \text{for } \frac{1}{T} - B \leq f \leq \frac{1}{T} - f_1 \\
 H_{imag}(f) &= 0 & \text{for } 0 \leq f \leq \frac{1}{T} - B
 \end{aligned}$$



$$f_\Delta = B - \frac{1}{2T} = \frac{1}{2T} - f_1 \quad \Rightarrow \quad \frac{1}{T} - B = f_1 \quad \text{OR} \quad \frac{1}{T} - f_1 = B$$

$$\begin{aligned}
 H_{eq}(f) &= 1 & \text{for } 0 \leq f \leq f_1 \\
 H_{eq}(f) &= \frac{1}{2} \left\{ 1 + \cos \left[ \frac{\pi(f-f_1)}{2f_\Delta} \right] \right\} + \frac{1}{2} \left\{ 1 + \cos \left[ \frac{\pi \left( \frac{1}{T} - f - f_1 \right)}{2f_\Delta} \right] \right\} & \text{for } f_1 \leq f \leq B \\
 H_{eq}(f) &= 1 & \text{for } B \leq f \leq \frac{1}{T}
 \end{aligned}$$

In the overlapped region, i.e.  $f_1 \leq f \leq B$

$$\begin{aligned}
 H_{eq}(f) &= 1 + \frac{1}{2} \left\{ \cos \left[ \frac{\pi(f-f_1)}{2f_\Delta} \right] \right\} + \cos \left[ \frac{\pi \left( \frac{1}{T} - f - f_1 \right)}{2f_\Delta} \right] = 1 + \left\{ \cos \left[ \frac{\pi(f-f_1) + \pi \left( \frac{1}{T} - f - f_1 \right)}{4f_\Delta} \right] \cos \left[ \frac{\pi(f-f_1) - \pi \left( \frac{1}{T} - f - f_1 \right)}{4f_\Delta} \right] \right\} \\
 H_{eq}(f) &= 1 + \left\{ \cos \left[ \frac{\pi \left( \frac{1}{T} - 2f_1 \right)}{4f_\Delta} \right] \cos \left[ \frac{\pi(f-f_1) - \pi \left( \frac{1}{T} - f - f_1 \right)}{4f_\Delta} \right] \right\} \\
 H_{eq}(f) &= 1 + \left\{ \cos \left[ \frac{\pi 2f_\Delta}{4f_\Delta} \right] \cos \left[ \frac{\pi(f-f_1) - \pi \left( \frac{1}{T} - f - f_1 \right)}{4f_\Delta} \right] \right\} = 1
 \end{aligned}$$

$$\text{i.e. } H_{eq}(f) = 1 \quad \text{for } 0 \leq f \leq \frac{1}{T}$$

(b) If there is a sampling phase offset  $\epsilon$ , where  $0 < \epsilon < T$

$$H_{eq}^\epsilon(f) = \exp(-j2\pi f\epsilon) \quad \text{for } 0 \leq f \leq f_1$$

$$H_{eq}^\epsilon(f) = \frac{1}{2} \left\{ 1 + \cos \left[ \frac{\pi(f-f_1)}{2f_\Delta} \right] \right\} \exp(-j2\pi f\epsilon) + \frac{1}{2} \left\{ 1 + \cos \left[ \frac{\pi(\frac{1}{T}-f-f_1)}{2f_\Delta} \right] \right\} \exp[j2\pi \left( \frac{1}{T} - f \right) \epsilon]$$

$$H_{eq}^\epsilon(f) = \exp[j2\pi \left( \frac{1}{T} - f \right) \epsilon] \quad \text{for } f_1 \leq f \leq B$$

$$H_{eq}^\epsilon(f) = \exp[j2\pi \left( \frac{1}{T} - f \right) \epsilon] \quad \text{for } B \leq f \leq \frac{1}{T}$$

$$|H_{eq}^\epsilon|^2 = 1 \quad \text{for } 0 \leq f \leq f_1 \quad \text{and} \quad B \leq f \leq \frac{1}{T}$$

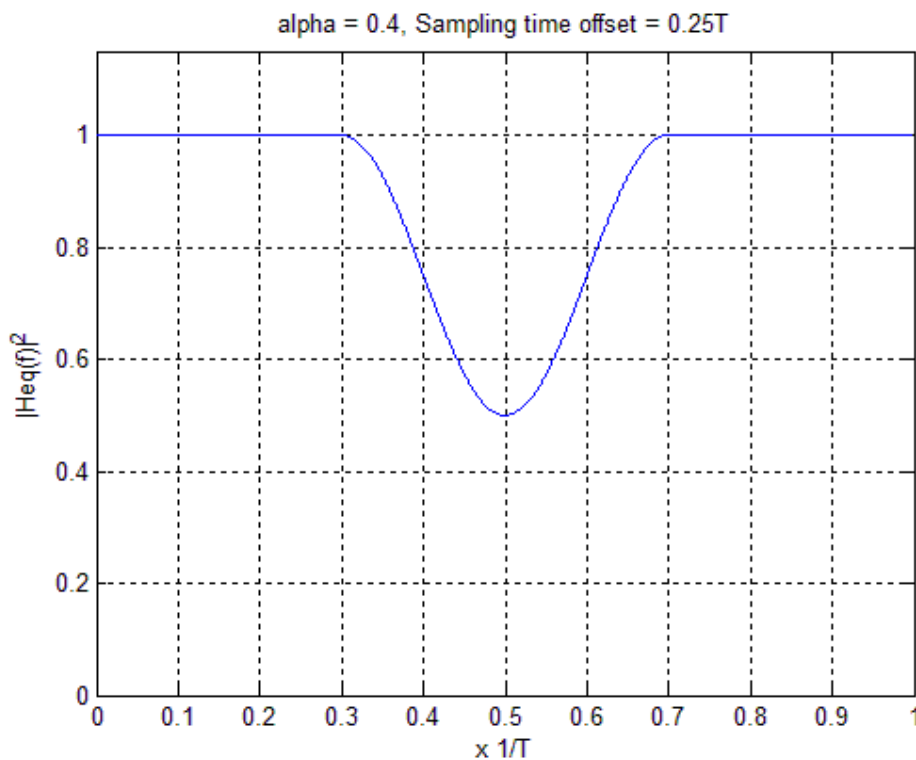
**In the overlapped region,**  $H_{eq}^\epsilon(f) = a \exp(-j\phi) + b \exp(j\theta)$  where  $\phi = 2\pi f\epsilon$ ,  $\theta = 2\pi \left( \frac{1}{T} - f \right) \epsilon$

$$|H_{eq}^\epsilon(f)|^2 = a^2 + b^2 + ab \exp[j(\phi + \theta)] + ab \exp[-j(\phi + \theta)]$$

$$|H_{eq}^\epsilon(f)|^2 = a^2 + b^2 + 2ab \cos(\phi + \theta) = a^2 + b^2 + 2ab \cos\left(\frac{2\pi\epsilon}{T}\right)$$

$$\text{i.e. } |H_{eq}^{0 < \epsilon < T}(f)|^2 < |H_{eq}^{\epsilon=0}(f)|^2$$

$$\rightarrow P_{av}^{0 < \epsilon < T} < P_{av}^{\epsilon=0}$$



Since there is no overlapped region for raised cosine filter with  $\alpha = 0$ ,  $P_{av}$  doesn't depend on the sampling time offset  $\epsilon$ .

(c) No, since there is no overlapping region in  $H_{eq}(f)$ ,  $|H_{eq}(f)|^2$  becomes independent of  $\epsilon$ , i.e. the average sampling power  $P_{av}$  will be independent of  $\epsilon$ .

\*\* (4) Take “1” and “0” to represent  $+\sin(\frac{\pi t}{T})$  and  $-\sin(\frac{\pi t}{T})$  {both from  $t = 0$  to  $T$ } respectively. Use discrete impulses with amplitudes +1 and -1 to represent “1” and “0”. The PSD of this discrete impulse sequence is:

$$P(\omega) = \frac{1}{T}$$

$$H(\omega) = \int_0^T \sin\left(\frac{\pi t}{T}\right) \exp(-j2\pi f t) dt$$

$$\int u(x)v'(x)dx = u(x)v(x) - \int u'(x)v(x)dx$$

$$H(\omega) = \left(\frac{1}{-j2\pi f}\right) \left\{ \sin\left(\frac{\pi t}{T}\right) \exp(-j2\pi f T) \Big|_0^T - \int_0^T \frac{\pi}{T} \cos\left(\frac{\pi t}{T}\right) \exp(-j2\pi f t) dt \right\}$$

$$H(\omega) = \left(\frac{1}{-j2\pi f}\right) \left(-\frac{\pi}{T}\right) \left\{ \int_0^T \cos\left(\frac{\pi t}{T}\right) \exp(-j2\pi f t) dt \right\}$$

$$H(\omega) = \left(\frac{1}{-j2\pi f}\right)^2 \left(-\frac{\pi}{T}\right) \left\{ \cos\left(\frac{\pi t}{T}\right) \exp(-j2\pi f t) \Big|_0^T - \int_0^T -\frac{\pi}{T} \sin\left(\frac{\pi t}{T}\right) \exp(-j2\pi f t) dt \right\}$$

$$H(\omega) = \left(\frac{1}{-j2\pi f}\right)^2 \left(-\frac{\pi}{T}\right) \left\{ -\exp(-j2\pi f T) - 1 + \int_0^T \frac{\pi}{T} \sin\left(\frac{\pi t}{T}\right) \exp(-j2\pi f t) dt \right\}$$

$$H(\omega) = \int_0^T \sin\left(\frac{\pi t}{T}\right) \exp(-j2\pi f t) dt = \left(\frac{1}{-j2\pi f}\right)^2 \left(-\frac{\pi}{T}\right) \left\{ -\exp(-j2\pi f T) - 1 + \int_0^T \frac{\pi}{T} \sin\left(\frac{\pi t}{T}\right) \exp(-j2\pi f t) dt \right\}$$

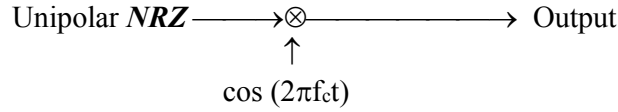
$$\left(4\pi f^2 T - \frac{\pi}{T}\right) \int_0^T \sin\left(\frac{\pi t}{T}\right) \exp(-j2\pi f t) dt = -\exp(-j2\pi f T) - 1$$

$$H(\omega) = \int_0^T \sin\left(\frac{\pi t}{T}\right) \exp(-j2\pi f t) dt = -\frac{-T[\exp(-j2\pi f T)+1]}{\pi(4f^2 T^2-1)} = -\frac{2T\exp(-j\pi f T)\cos(\pi f T)}{\pi(4f^2 T^2-1)}$$

$$S(\omega) = P(\omega)|H(\omega)|^2 = \frac{1}{T} \left| -\frac{2T\exp(-j\pi f T)\cos(\pi f T)}{\pi(4f^2 T^2-1)} \right|^2 = \frac{4T\cos^2(\pi f T)}{\pi^2(4f^2 T^2-1)^2}$$

(5)

(a) **On-Off Keying**

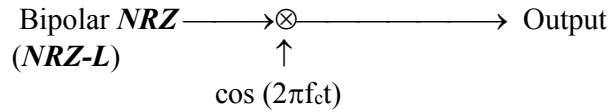


$P_g$  of unipolar **NRZ** is (Section 1.2.1):  $\frac{A^2 T_b}{4} Sa^2\left(\frac{2\pi f T_b}{2}\right) + a \delta(f)$

where “a” is a constant, i.e.

$$P_v(f) = \frac{1}{4} \left\{ \frac{A^2 T_b}{4} Sa^2[\pi(f - f_c)T_b] + \frac{A^2 T_b}{4} Sa^2[\pi(f + f_c)T_b] + a\delta[(f - f_c)] + a\delta[(f + f_c)] \right\}$$

(b) **BPSK**



$P_g$  of Bipolar **NRZ** is (Section 1.2.2):  $A^2 T_b [Sa^2(\frac{2\pi f T_b}{2})]$ , i.e.

$$P_v(f) = \frac{A^2 T_b}{4} \{ Sa^2[\pi(f - f_c)T_b] + Sa^2[\pi(f + f_c)T_b] \}$$

(c) **QPSK**

The required  $P_v(f)$  is equivalent to adding 2 **BPSK** spectrum together. Since the spectrum of the 2 **BPSK** are identical and the 2 **BPSK** signals are uncorrelated and orthogonal, the final  $P_v(f)$  is equivalent to the summation of the two BPSK spectrum with  $T = 2T_b$  { $T_b$  is the bit duration}, i.e.

$$P_v(f) = A^2 T_b \{ Sa^2[2\pi(f - f_c)T_b] + Sa^2[2\pi(f + f_c)T_b] \}$$