

EE3109 Applied Electromagnetics
2024/25 Semester B
Week 8: Tutorial Solution

Q1. Sol'n.

2.5 For a parallel-plate transmission line, the line parameters are given by:

$$\begin{aligned}R' &= 2 \quad (\Omega/\text{m}), \\L' &= 335 \quad (\text{nH}/\text{m}), \\G' &= 0, \\C' &= 344 \quad (\text{pF}/\text{m}).\end{aligned}$$

Find α , β , u_p , and Z_0 at 1 GHz.

Solution: At 1 GHz, $\omega = 2\pi f = 2\pi \times 10^9$ rad/s. Application of (2.22) gives:

$$\begin{aligned}\gamma &= \sqrt{(R' + j\omega L')(G' + j\omega C')} \\&= [(2 + j2\pi \times 10^9 \times 335 \times 10^{-9})(0 + j2\pi \times 10^9 \times 344 \times 10^{-12})]^{1/2} \\&= [(2 + j2100)(j2.2)]^{1/2} \\&= 48e^{j89.97^\circ} = 0.023 + j48.\end{aligned}$$

Hence,

$$\begin{aligned}\alpha &= 0.023 \text{ Np/m}, \\ \beta &= 48 \text{ rad/m}.\end{aligned}$$

$$\begin{aligned}u_p &= \frac{\omega}{\beta} = \frac{2\pi f}{\beta} = \frac{2\pi \times 10^9}{48} = 1.31 \times 10^8 \text{ m/s.} \\Z_0 &= \left[\frac{R' + j\omega L'}{G' + j\omega C'} \right]^{1/2} \\&= \left[\frac{2100e^{j89.95^\circ}}{2.2e^{j90^\circ}} \right]^{1/2} \\&= \left[954e^{-j0.05^\circ} \right]^{1/2} \\&= 31e^{-j0.025^\circ} \approx (31 - j0.01) \Omega.\end{aligned}$$

Q2. Sol'n.

2.13 In addition to not dissipating power, a lossless line has two important features: (1) it is dispersionless (μ_p is independent of frequency) and (2) its characteristic impedance Z_0 is purely real. Sometimes, it is not possible to design a transmission line such that $R' \ll \omega L'$ and $G' \ll \omega C'$, but it is possible to choose the dimensions of the line and its material properties so as to satisfy the condition

$$R'C' = L'G' \quad (\text{distortionless line}).$$

Such a line is called a *distortionless* line because despite the fact that it is not lossless, it does nonetheless possess the previously mentioned features of the loss line. Show that for a distortionless line,

$$\alpha = R' \sqrt{\frac{C'}{L'}} = \sqrt{R'G'}, \quad \beta = \omega \sqrt{L'C'}, \quad Z_0 = \sqrt{\frac{L'}{C'}}.$$

Solution: Using the distortionless condition in Eq. (2.22) gives

$$\begin{aligned} \gamma &= \alpha + j\beta = \sqrt{(R' + j\omega L')(G' + j\omega C')} \\ &= \sqrt{L'C'} \sqrt{\left(\frac{R'}{L'} + j\omega\right) \left(\frac{G'}{C'} + j\omega\right)} \\ &= \sqrt{L'C'} \sqrt{\left(\frac{R'}{L'} + j\omega\right) \left(\frac{R'}{L'} + j\omega\right)} \\ &= \sqrt{L'C'} \left(\frac{R'}{L'} + j\omega\right) = R' \sqrt{\frac{C'}{L'}} + j\omega \sqrt{L'C'}. \end{aligned}$$

Hence,

$$\alpha = \Re(\gamma) = R' \sqrt{\frac{C'}{L'}}, \quad \beta = \Im(\gamma) = \omega \sqrt{L'C'}, \quad u_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{L'C'}}.$$

Similarly, using the distortionless condition in Eq. (2.29) gives

$$Z_0 = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}} = \sqrt{\frac{L'}{C'}} \sqrt{\frac{R'/L' + j\omega}{G'/C' + j\omega}} = \sqrt{\frac{L'}{C'}}.$$

Q3. Sol'n.

2.14 For a distortionless line [see Problem 2.13] with $Z_0 = 50 \Omega$, $\alpha = 10$ (mNp/m), $u_p = 2.5 \times 10^8$ (m/s), find the line parameters and λ at 100 MHz.

Solution: The product of the expressions for α and Z_0 given in Problem 2.6 gives

$$R' = \alpha Z_0 = 10 \times 10^{-3} \times 50 = 0.5 \quad (\Omega/\text{m}),$$

and taking the ratio of the expression for Z_0 to that for $u_p = \omega/\beta = 1/\sqrt{L'C'}$ gives

$$L' = \frac{Z_0}{u_p} = \frac{50}{2.5 \times 10^8} = 2 \times 10^{-7} \text{ (H/m)} = 200 \quad (\text{nH/m}).$$

With L' known, we use the expression for Z_0 to find C' :

$$C' = \frac{L'}{Z_0^2} = \frac{2 \times 10^{-7}}{(50)^2} = 8 \times 10^{-11} \text{ (F/m)} = 80 \quad (\text{pF/m}).$$

The distortionless condition given in Problem 2.6 is then used to find G' .

$$G' = \frac{R'C'}{L'} = \frac{0.5 \times 80 \times 10^{-12}}{2 \times 10^{-7}} = 2 \times 10^{-4} \text{ (S/m)} = 200 \quad (\mu\text{S/m}),$$

and the wavelength is obtained by applying the relation

$$\lambda = \frac{u_p}{f} = \frac{2.5 \times 10^8}{100 \times 10^6} = 2.5 \text{ m.}$$

Q4. Sol'n.

2.15 Find α and Z_0 of a distortionless line whose $R' = 8 \Omega/\text{m}$ and $G' = 2 \times 10^{-4} \text{ S/m}$.

Solution: From the equations given in Problem 2.13,

$$\alpha = \sqrt{R'G'} = [8 \times 2 \times 10^{-4}]^{1/2} = 4 \times 10^{-2} \quad (\text{Np/m}),$$

$$Z_0 = \sqrt{\frac{L'}{C'}} = \sqrt{\frac{R'}{G'}} = \left(\frac{8}{2 \times 10^{-4}} \right)^{1/2} = 200 \Omega.$$

Q5. Sol'n.

2.16 A transmission line operating at 125 MHz has $Z_0 = 40 \Omega$, $\alpha = 0.01$ (Np/m), and $\beta = 0.75$ rad/m. Find the line parameters R' , L' , G' , and C' .

Solution: Given an arbitrary transmission line, $f = 125$ MHz, $Z_0 = 40 \Omega$, $\alpha = 0.01$ Np/m, and $\beta = 0.75$ rad/m. Since Z_0 is real and $\alpha \neq 0$, the line is distortionless. From Problem 2.13, $\beta = \omega\sqrt{L'C'}$ and $Z_0 = \sqrt{L'/C'}$, therefore,

$$L' = \frac{\beta Z_0}{\omega} = \frac{0.75 \times 40}{2\pi \times 125 \times 10^6} = 38.2 \text{ nH/m}.$$

Then, from $Z_0 = \sqrt{L'/C'}$,

$$C' = \frac{L'}{Z_0^2} = \frac{38.2 \text{ nH/m}}{40^2} = 23.9 \text{ pF/m}.$$

From $\alpha = \sqrt{R'G'}$ and $R'C' = L'G'$,

$$R' = \sqrt{R'G'} \sqrt{\frac{R'}{G'}} = \sqrt{R'G'} \sqrt{\frac{L'}{C'}} = \alpha Z_0 = 0.01 \text{ Np/m} \times 40 \Omega = 0.4 \Omega/\text{m}$$

and

$$G' = \frac{\alpha^2}{R'} = \frac{(0.01 \text{ Np/m})^2}{0.4 \Omega/\text{m}} = 0.25 \text{ mS/m}.$$

Q6. Sol'n.

2.17 Polyethylene with $\epsilon_r = 2.25$ is used as the insulating material in a lossless coaxial line with characteristic impedance of 50Ω . The radius of the inner conductor is 1.2 mm.

- (a) What is the radius of the outer conductor?
- (b) What is the phase velocity of the line?

Solution: Given a lossless coaxial line, $Z_0 = 50 \Omega$, $\epsilon_r = 2.25$, $a = 1.2$ mm:

- (a) From Table 2-2, $Z_0 = (60/\sqrt{\epsilon_r}) \ln(b/a)$ which can be rearranged to give

$$b = a \exp Z_0 \sqrt{\epsilon_r} / 60 = (1.2 \text{ mm}) \exp 50 \sqrt{2.25} / 60 = 4.2 \text{ mm}.$$

- (b) Also from Table 2-2,

$$u_p = \frac{c}{\sqrt{\epsilon_r}} = \frac{3 \times 10^8 \text{ m/s}}{\sqrt{2.25}} = 2.0 \times 10^8 \text{ m/s}.$$
