

L01: Introduction and Review

EE3109 Applied Electromagnetics

Lecture Notes

Dr. Alex M. H. Wong

Jan – Apr 2023

Outline

- **Course Introduction**
- Mathematics Review

Course Information

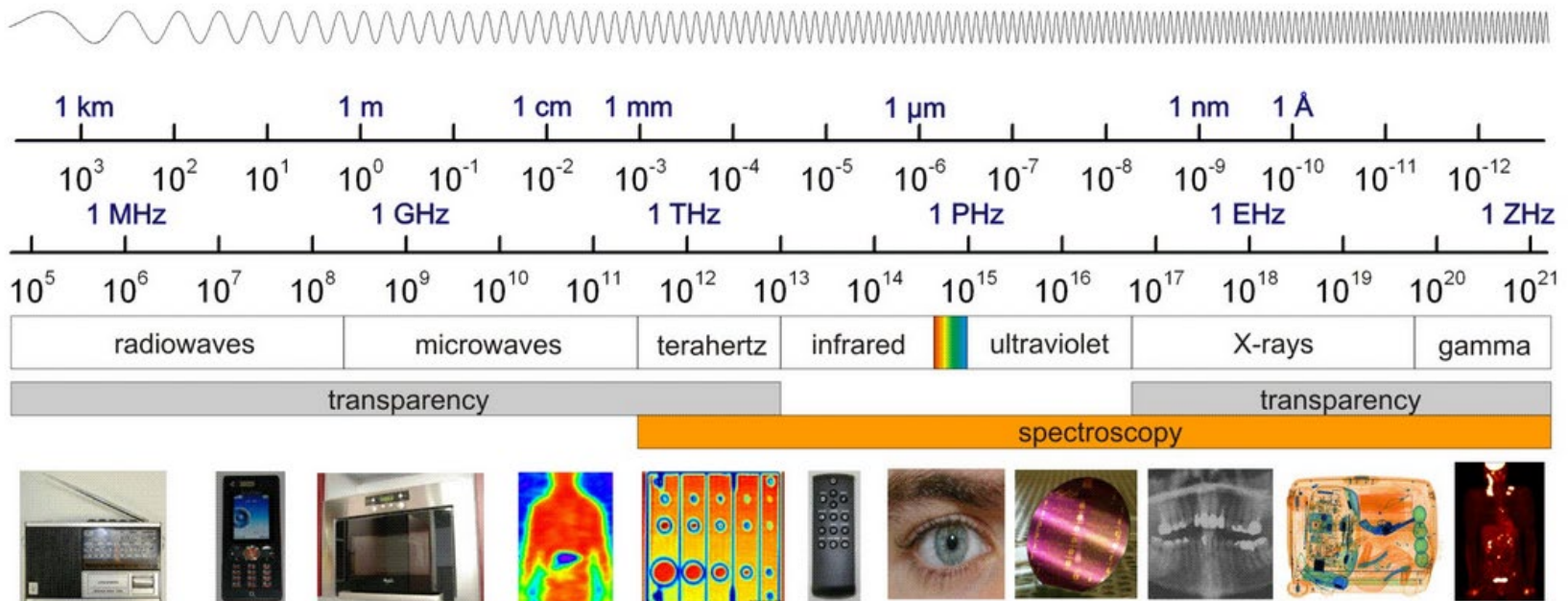
- EE3109 Applied Electromagnetics
- Lecture + Tutorial: Thurs 3-6pm (Yeung B4302)
- Lab Sessions: (Yeung P1615)
 - Session 1: Weeks 10 & 12, Wed 9am - 12nn
 - Session 2: Weeks 9 & 11, Wed 9am - 12nn
 - Session 3: Weeks 10 & 12, Fri 7pm - 10pm (in arrangement)
- Lecturer:
 - Dr. Alex M. H. Wong (Assistant Professor, Yeung G6516)
- Teaching Assistants:
 - Mr. Tayyab Ali KHAN (tkhan6-c AT my.cityu.edu.hk)
 - Mr. Bo XUE (boxue3-c AT my.cityu.edu.hk)
- Office Hours: TBA

Lecture Slide Credits

- Dr. Alex M. H. Wong
- Prof. Ben K. W. Leung, predecessors
- Electronic Teaching Aid for:
Ulaby and Ravaioli, “Fundamentals of Applied Electromagnetics,” (Pearson, 7th ed., Boston, 2015)
- Other Useful Texts: (check syllabus document)

What is Electromagnetics?

The study of **electric** and **magnetic fields**
in time and space



<http://www.physik.uni-kl.de/en/beigang/forschungsprojekte/>

Dimensions and Units

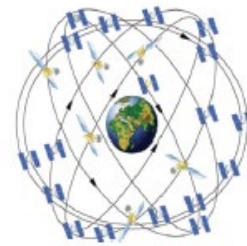
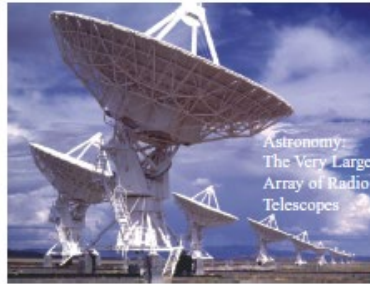
Table 1-1: Fundamental SI units.

Dimension	Unit	Symbol
Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric Current	ampere	A
Temperature	kelvin	K
Amount of substance	mole	mol

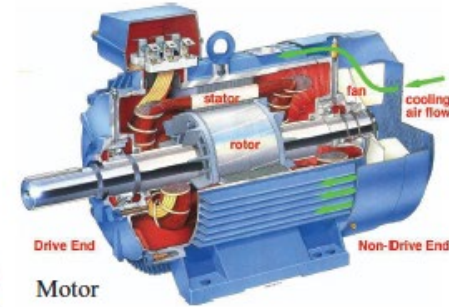
Table 1-2: Multiple and submultiple prefixes.

Prefix	Symbol	Magnitude
exa	E	10^{18}
peta	P	10^{15}
tera	T	10^{12}
giga	G	10^9
mega	M	10^6
kilo	k	10^3
milli	m	10^{-3}
micro	μ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}
femto	f	10^{-15}
atto	a	10^{-18}

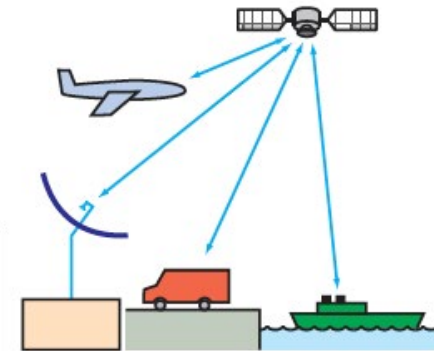
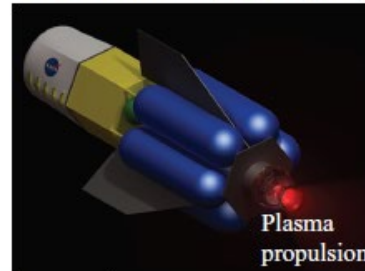
Examples of EM Applications



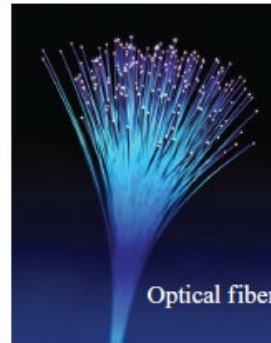
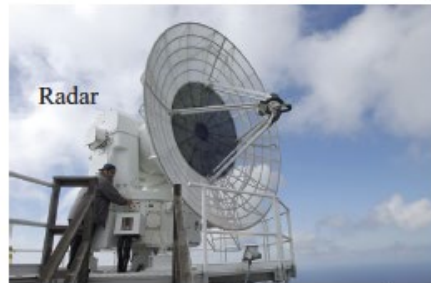
Global Positioning System (GPS)



LCD
Screen

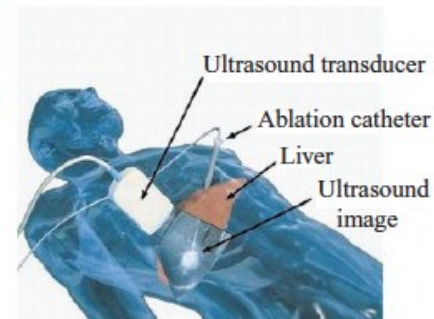


Telecommunication



Electromagnetic sensors

Cell
phone



Microwave ablation for
liver cancer treatment

Electromagnetics in the Classical Era

1827 **Georg Simon Ohm** (German) formulates Ohm's law relating electric potential to current and resistance.

1827 **Joseph Henry** (American) introduces the concept of **inductance**, and builds one of the earliest electric motors. He also assisted Samuel Morse in the development of the **telegraph**.

1831 **Michael Faraday** (English) discovers that a changing magnetic flux can induce an **electromotive force**.

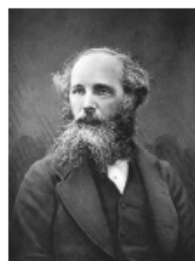


1835 **Carl Friedrich Gauss** (German) formulates **Gauss's law** relating the electric flux flowing through an enclosed surface to the enclosed electric charge.

Gauss's Law for Electricity

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{inside}}}{\epsilon_0}$$

1873 **James Clerk Maxwell** (Scottish) publishes his Treatise on Electricity and Magnetism in which he unites the discoveries of Coulomb, Oersted, Ampère, Faraday, and others into four elegantly constructed mathematical equations, now known as **Maxwell's Equations**.



1887 **Heinrich Hertz** (German) builds a system that can generate **electromagnetic waves** (at radio frequencies) and detect them.



1888 **Nikola Tesla** (Croatian-American) invents the **ac** (alternating current) electric motor.



1895 **Wilhelm Röntgen** (German) discovers **X-rays**. One of his first X-ray images was of the bones in his wife's hands. [1901 Nobel prize in physics.]



1897 **Joseph John Thomson** (English) discovers the **electron** and measures its charge-to-mass ratio. [1906 Nobel prize in physics.]

1905 **Albert Einstein** (German-American) explains the **photoelectric effect** discovered earlier by Hertz in 1887. [1921 Nobel prize in physics.]

Telecommunications

1923



Vladimir Zworykin (Russian-American) invents **television**. In 1926, John Baird (Scottish) transmits TV images over telephone wires from London to Glasgow. Regular **TV broadcasting** began in Germany (1935), England (1936), and the United States (1939).

1958

Jack Kilby (American) builds first **integrated circuit (IC)** on germanium and, independently, **Robert Noyce** (American) builds first IC on silicon.



1926

Transatlantic telephone service between London and New York.

1932

First **microwave telephone link**, installed (by Marconi) between Vatican City and the Pope's summer residence.

1933

Edwin Armstrong (American) invents **frequency modulation (FM)** for radio transmission.

1935

Robert Watson-Watt (Scottish) invents **radar**.



1938

H. A. Reeves (American) invents **pulse code modulation (PCM)**.

1947

William Shockley, Walter Brattain, and John Bardeen (all Americans) invent the junction **transistor** at Bell Labs. [1956 Nobel prize in physics.]



1960



Echo, the first passive communication satellite is launched, and successfully reflects radio signals back to Earth. In 1963, the first **communication satellite** is placed in geosynchronous orbit.

1969

ARPANET is established by the U.S. Department of Defense, to evolve later into the Internet.

1979

Japan builds the first cellular telephone network:

- 1983 cellular phone networks start in the United States.
- 1990 **electronic beepers** become common.
- 1995 **cell phones** become widely available.
- 2002 cell phone supports **video and Internet**.

1984

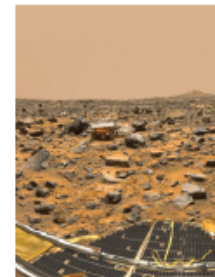
Worldwide **Internet** becomes operational.

1988

First **transatlantic optical fiber cable** between the U.S. and Europe.

1997

Mars Pathfinder sends images to Earth.



1955

Pager is introduced as a radio communication product in hospitals and factories.

1955

Narinder Kapany (Indian-American) demonstrates the **optical fiber** as a low-loss, light-transmission medium.

2004

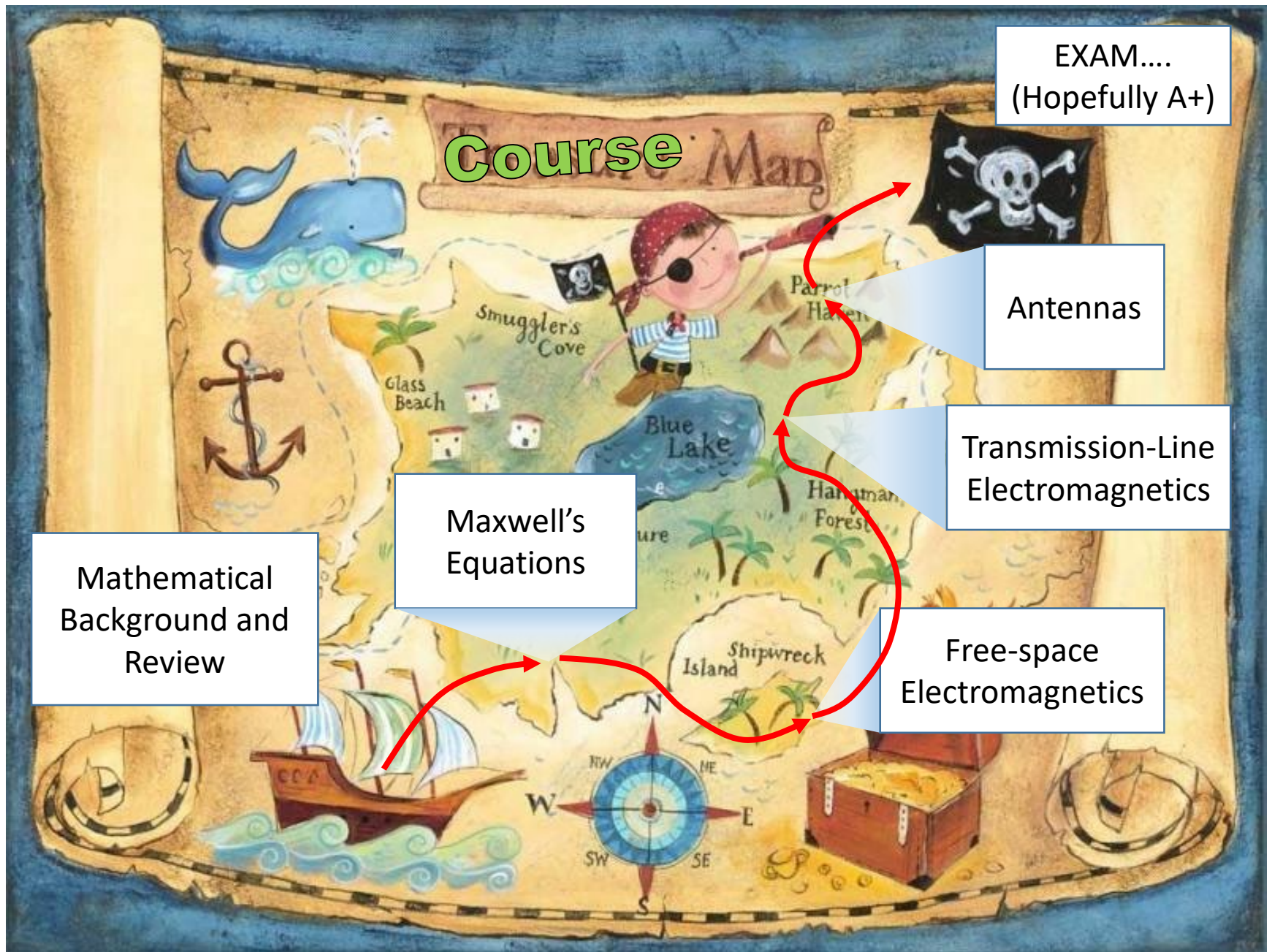
Wireless communication supported by many airports, university campuses, and other facilities.

2012

Smartphones worldwide exceed 1 billion.

How can we study Electromagnetics?





Tentative Course Schedule

Week #	Week of	Course Coverage
1	Jan 9	Course Introduction, Math Review
2	Jan 16	Introduction to Maxwell Equations, Materials and Boundaries
	Jan 23	<i>Happy Chinese New Year!</i>
3	Jan 30	Wave Equations, Plane Waves, Propagation in Lossy Media
4	Feb 6	Polarizations, EM Potential <i>Asst. 1a Due</i>
5	Feb 13	Reflection and Transmission <i>Asst. 1b Due</i>
6	Feb 20	Test 1 , Electromagnetics R&D
7	Feb 27	Poynting vector, Power and Velocity
8	Mar 6	Waveguides and Transmission Lines (TL)
9	Mar 13	VIP in a TL, Terminated and Lossy TLs, <i>Asst. 2 Due, Lab 1 Begins</i>
10	Mar 20	Impedance in a TL, Smith Charts,
11	Mar 27	Smith Charts Cont'd, Impedance Matching, Quarter-wave Transformers, <i>Lab 2 Begins</i>
12	Apr 3	Test 2 , Introduction to Antennas
13	Apr 10	Dipole Antennas, <i>Asst. 3 Due</i>

Evaluations

- 2 Tests (15% ea.)
 - 1 Hour time length
 - 1 A4-sized aid sheet for Test 1; 2 sheets for Test 2 and Exam
- 2 Lab Reports (5% ea.)
 - Due one week after your lab session, end of the day
- 3 Regular Assignments (3% ea.)
 - Due end of LECTURE day (Thursday)
- Conceptual Questions (1%)
 - Conceptual questions throughout the lecture, posted online
 - Given in class (& on Canvas), answer on Canvas in time
 - Scoring all correct answers will lead to bonus points (1.5 / 1.0 %)

Course Policies

- Late assignments and lab reports
 - **-20% from total** for submissions that are 1 work day late
 - **-40% from total** for submissions that are 2 work days late
 - Late submissions **not accepted after two work days**
- Hybrid lecture and course recordings
 - As per University Policy, we have returned to face-to-face teaching mode (**no hybrid lectures**)
 - **Lecture recordings** will be accessible for those who **attend lecture** and **requests for access** with the TA
 - Requests are made to the **TA** at the **beginning of class**
 - **Beginning of course**: Recordings for **Lectures 1 and 2** will be made available to everyone

Discussion Items

- Class Demographics
- Tutorial Usage
- Office hours

Outline

- Course Introduction
- **Mathematics Review**

Mathematics Review

This lecture reviews mathematical background which is key understanding the contents for this course.

Students are strongly suggested to work through the tutorial questions and perform necessary background readings to understand the mathematics covered here.

Main topics of coverage:

- Vector calculus
- Phasors

Revision of Vector Analysis

$$\mathbf{B} \times \mathbf{A} = -\mathbf{A} \times \mathbf{B}$$

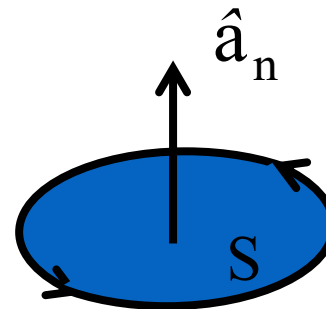
$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) \neq (\mathbf{A} \times \mathbf{B}) \times \mathbf{C}$$

$$\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = \mathbf{B} \cdot (\mathbf{C} \times \mathbf{A}) = \mathbf{C} \cdot (\mathbf{A} \times \mathbf{B})$$

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B})$$

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$\oint_S \vec{\mathbf{A}} \cdot d\vec{\mathbf{s}} = \oint_S \vec{\mathbf{A}} \cdot \hat{\mathbf{a}}_n ds$$



The “Dell operator”

- Consider ∇ as the “Dell operator”:

$$\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right) = \frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j} + \frac{\partial}{\partial z} \hat{k}$$

(in Cartesian co-ordinates)

Vector Derivative Operations

$$\nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right) = \frac{\partial f}{\partial x} \hat{i} + \frac{\partial f}{\partial y} \hat{j} + \frac{\partial f}{\partial z} \hat{k}$$

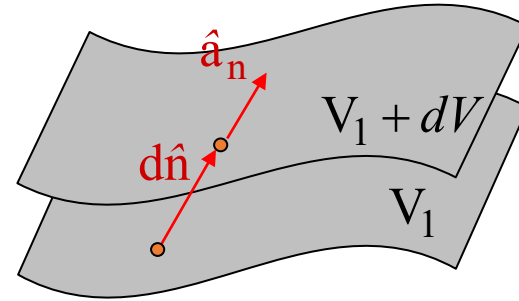
$$\nabla \cdot \mathbf{A} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right) \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right) \times \mathbf{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}, \frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}, \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right)$$

Gradient:

$$\text{grad}V \triangleq \hat{a}_n \frac{dV}{dn}$$

$$\nabla V \triangleq \hat{a}_n \frac{dV}{dn}$$



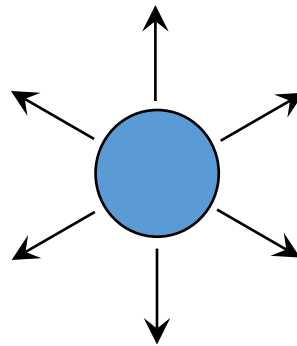
It represents both the magnitude and the direction of the maximum space rate of increase of a scalar

Divergence

$$\nabla \cdot \bar{A} \triangleq \lim_{\Delta v \rightarrow 0} \frac{\oint_s \bar{A} \cdot d\bar{s}}{\Delta v}$$

(u=x, y or z)

Net outward flux of A per unit volume as the volume about the point tends to zero.



Divergence theorem:

$$\int_v \nabla \cdot \bar{A} dV = \oint_s \bar{A} \cdot d\bar{s}$$

Volume integral of the divergence of a vector equals total outward flux of vector through the surface that bounds the volume

Curl

$$(\nabla \times \mathbf{A})_u \triangleq \lim_{\Delta S_u \rightarrow 0} \frac{1}{\Delta S_u} \oint_{C_u} \vec{A} \cdot d\vec{l}$$

(u=x, y or z)

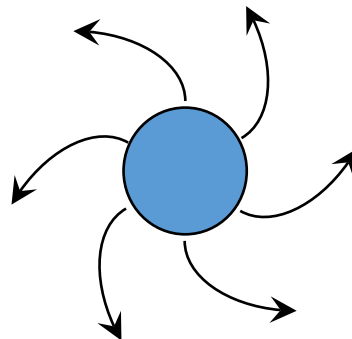
Net circulation flux of A per unit area as the area about the point tends to zero

Direction is the normal direction of the area

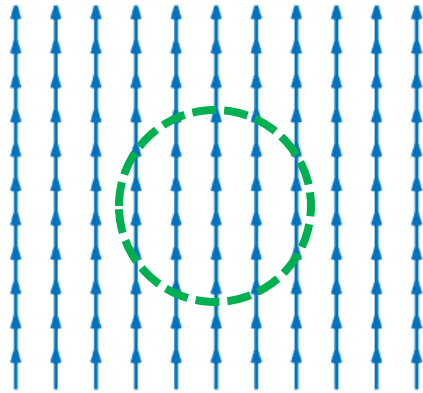
Stokes' theorem:

$$\int_s (\nabla \times \vec{A}) \cdot d\vec{s} = \oint_c \vec{A} \cdot d\vec{l}$$

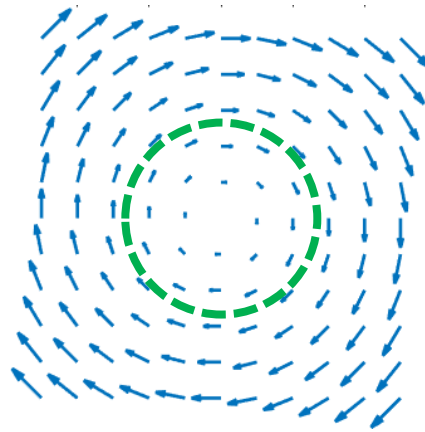
Surface integral of the curl of a vector field over an open surface is equal to the closed line integral of the vector along the contour bounding the surface



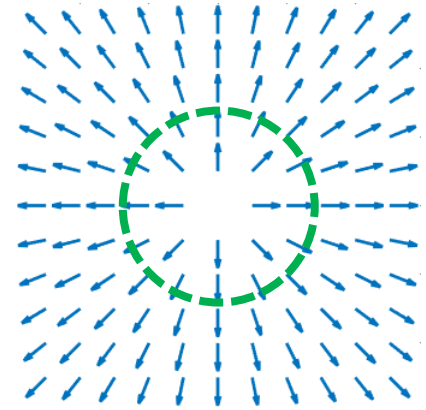
What is the divergence / curl of the following vector functions?



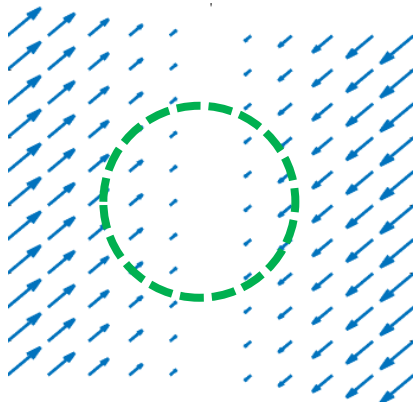
(a)



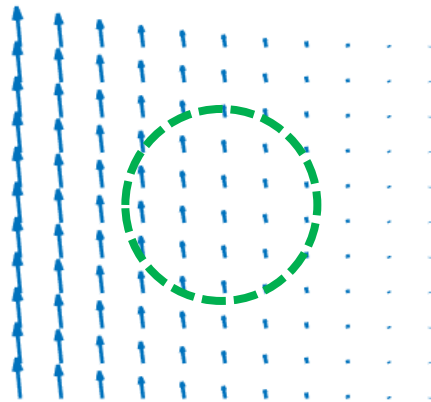
(b)



(c)



(d)



(e)

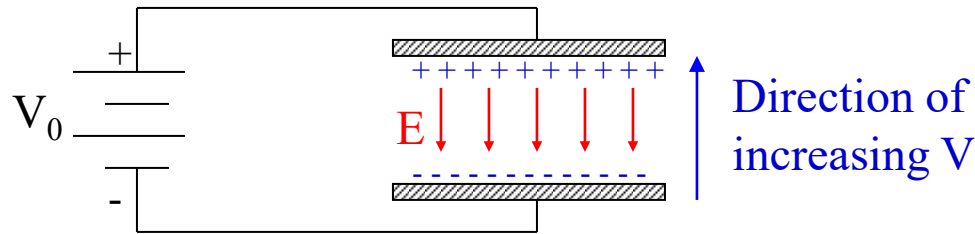
- (a) $\text{Div } A = \text{Curl } A = 0$
- (b) $\text{Div } A = 0; \text{Curl } A < 0$
- (c) $\text{Div } A > 0; \text{Curl } A = 0$
- (d) $\text{Div } A < 0; \text{Curl } A < 0$
- (e) $\text{Div } A = 0; \text{Curl } A < 0$

Two Null identities: $\nabla \times \nabla V = 0$; $\nabla \cdot \nabla \times \vec{A} = 0$

If $\nabla \times \vec{E} = 0$, we can define

$$\vec{E} = -\nabla V \quad \text{Definition in electrostatics}$$

Vector can be expressed as the gradient of a scalar field



If $\nabla \cdot \vec{B} = 0$, we can define

$$\vec{B} = \nabla \times \vec{A}$$

Vector can be expressed as the curl of another vector field.

Working with Gradients:

$$V_1 = V_1(x, y, z), V_2 = V_2(x, y, z)$$

$$\nabla(V_1 + V_2) = \nabla V_1 + \nabla V_2$$

$$\nabla(V_1 V_2) = V_1 \nabla V_2 + V_2 \nabla V_1$$

Example 1: Gradient

If $\phi(x, y, z) = 3x^2y - y^3z^2$, find $\nabla\phi$ (or $\text{grad } \phi$) at point $(1, -2, -1)$.

$$\begin{aligned}\nabla\phi &= (\hat{i} \frac{\partial}{\partial x} + \hat{j} \frac{\partial}{\partial y} + \hat{k} \frac{\partial}{\partial z})(3x^2y - y^3z^2) \\ &= 6xy\hat{i} + (3x^2 - 3y^2z^2)\hat{j} - 2y^3z\hat{k}\end{aligned}$$

At point $(1, -2, -1)$

$$\nabla\phi = -12\hat{i} - 9\hat{j} - 16\hat{k}$$

Example 2: Divergence

If $\vec{A} = x^2 z \hat{i} - 2y^3 z^2 \hat{j} + xy^2 z \hat{k}$, find $\nabla \cdot \vec{A}$ (or $\text{div } \vec{A}$) at the point $(1, -1, 1)$.

$$\begin{aligned}\nabla \cdot A &= \left(\frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j} + \frac{\partial}{\partial z} \hat{k} \right) \cdot (x^2 z \hat{i} - 2y^3 z^2 \hat{j} + xy^2 z \hat{k}) \\ &= 2xz - 6y^2 z^2 + xy^2\end{aligned}$$

At point $(1, -1, 1)$

$$\nabla \cdot A = -3$$

Example 3: Curl

If $\vec{A} = xz^3\hat{i} - 2x^2yz\hat{j} + 2yz^4\hat{k}$, find $\nabla \times \vec{A}$ (or curl $\vec{A}$) at the point $(1, -1, 1)$.

$$\begin{aligned}\nabla \times \vec{A} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xz^3 & -2x^2yz & 2yz^4 \end{vmatrix} \\ &= \hat{i}(2z^4 + 2x^2y) + \hat{j}(3xz^2) + \hat{k}(-4xyz)\end{aligned}$$

$$\text{At point } (1, -1, 1) \quad \nabla \times A = 3\hat{j} + 4\hat{k}$$

Helmholtz's Theorem :

A vector field is determined to within an additive constant if **both** its divergence and its curl are specified everywhere.

Optional Homework: Prove it!

Complex Numbers

We will find it is useful to represent sinusoids as complex numbers $j = \sqrt{-1}$

$$z = x + jy$$

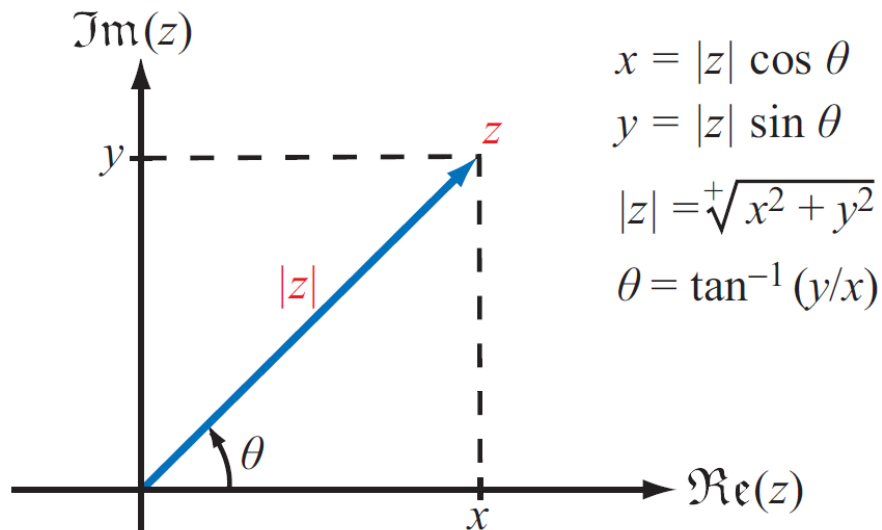
Rectangular coordinates

$$\operatorname{Re}(z) = x$$

$$z = |z| \angle \theta = |z| e^{j\theta}$$

Polar coordinates

$$\operatorname{Im}(z) = y$$



**Relations based
on Euler's Identity**

$$e^{\pm j\theta} = \cos \theta \pm j \sin \theta$$

Relations for Complex Numbers

Euler's Identity: $e^{j\theta} = \cos \theta + j \sin \theta$

$$\sin \theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

$$\cos \theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

$$\mathbf{z} = x + jy = |\mathbf{z}|e^{j\theta}$$

$$\mathbf{z}^* = x - jy = |\mathbf{z}|e^{-j\theta}$$

$$x = \Re(\mathbf{z}) = |\mathbf{z}| \cos \theta$$

$$|\mathbf{z}| = \sqrt[4]{\mathbf{z}\mathbf{z}^*} = \sqrt{x^2 + y^2}$$

$$y = \Im(\mathbf{z}) = |\mathbf{z}| \sin \theta$$

$$\theta = \tan^{-1}(y/x)$$

$$\mathbf{z}^n = |\mathbf{z}|^n e^{jn\theta}$$

$$\mathbf{z}^{1/2} = \pm |\mathbf{z}|^{1/2} e^{j\theta/2}$$

$$\mathbf{z}_1 = x_1 + jy_1$$

$$\mathbf{z}_2 = x_2 + jy_2$$

$$\mathbf{z}_1 = \mathbf{z}_2 \text{ iff } x_1 = x_2 \text{ and } y_1 = y_2$$

$$\mathbf{z}_1 + \mathbf{z}_2 = (x_1 + x_2) + j(y_1 + y_2)$$

$$\mathbf{z}_1 \mathbf{z}_2 = |\mathbf{z}_1| |\mathbf{z}_2| e^{j(\theta_1 + \theta_2)}$$

$$\frac{\mathbf{z}_1}{\mathbf{z}_2} = \frac{|\mathbf{z}_1|}{|\mathbf{z}_2|} e^{j(\theta_1 - \theta_2)}$$

$$-1 = e^{j\pi} = e^{-j\pi} = 1 \angle \pm 180^\circ$$

$$j = e^{j\pi/2} = 1 \angle 90^\circ$$

$$-j = e^{-j\pi/2} = 1 \angle -90^\circ$$

$$\sqrt{j} = \pm e^{j\pi/4} = \pm \frac{(1 + j)}{\sqrt{2}}$$

$$\sqrt{-j} = \pm e^{-j\pi/4} = \pm \frac{(1 - j)}{\sqrt{2}}$$

**Learn how to
perform these
with your
calculator/computer**

Phasors

$$\begin{aligned} V(z, t) &= A \cos(\omega t - \beta z + \phi) \\ &= A \operatorname{Re}\{\cos(\omega t - \beta z + \phi) + j\sin(\omega t - \beta z + \phi)\} \\ &= \operatorname{Re}\{A \exp\{j(\omega t - \beta z + \phi)\}\} \\ &= \operatorname{Re}\{A e^{j\phi} e^{-j\beta z} e^{j\omega t}\} \end{aligned}$$

Define: $\bar{V}(z) = A e^{j\phi} e^{-j\beta z} \stackrel{\text{def}}{=} \text{“The phasor of } V(z, t)\text{”}$
($V_0 = A e^{j\phi}$ is call the “complex amplitude”)

Converting back into time domain:

$$V(z, t) = \operatorname{Re}\{\bar{V}(z) \exp(j\omega t)\}$$

Example 4: Time domain to phasors

If $V(z, t) = 35 \sin(\omega t - 12z + 120^\circ)$, find $\bar{V}(z)$.

$$\begin{aligned} V(z, t) &= 35 \sin(\omega t - 12z + 120^\circ) \\ &= 35 \cos[90^\circ - (\omega t - 12z + 120^\circ)] \\ &= 35 \cos[(\omega t - 12z + 120^\circ) - 90^\circ] \\ &= \boxed{35} \cos(\omega t \boxed{-12z} + \boxed{30^\circ}) \\ &\Rightarrow \bar{V}(z) = \textcolor{blue}{35} e^{j\textcolor{green}{\pi}/6} e^{-j\textcolor{red}{12}z} \end{aligned}$$

Example 5: Phasors to Time domain

$$\bar{V}(x, y) = (3 + j4)e^{j(16x+32y)}, \text{ find } V(x, y, t).$$

$$\begin{aligned} V(x, y, t) &= \text{Re}\{\bar{V}(x, y)\exp(j\omega t)\} \\ &= \text{Re}\{(3 + j4)e^{j(16x+32y)}\exp(j\omega t)\} \\ &= \text{Re}\{\sqrt{3^2 + 4^2}e^{j \tan^{-1}(4/3)}e^{j(16x+32y)}\exp(j\omega t)\} \\ &= \text{Re}\{5 e^{j 0.927}e^{j(16x+32y)}\exp(j\omega t)\} \\ &= \text{Re}\{5 e^{j (\omega t + 16x + 32y + 0.927)}\} \\ &= 5 \cos(\omega t + 16x + 32y + 0.927) \end{aligned}$$

Time and Phasor Domain

$x(t)$		$\mathbf{X}$
$A \cos \omega t$	$\longleftrightarrow$	A
$A \cos(\omega t + \phi)$	$\longleftrightarrow$	$Ae^{j\phi}$
$-A \cos(\omega t + \phi)$	$\longleftrightarrow$	$Ae^{j(\phi \pm \pi)}$
$A \sin \omega t$	$\longleftrightarrow$	$Ae^{-j\pi/2} = -jA$
$A \sin(\omega t + \phi)$	$\longleftrightarrow$	$Ae^{j(\phi - \pi/2)}$
$-A \sin(\omega t + \phi)$	$\longleftrightarrow$	$Ae^{j(\phi + \pi/2)}$
$\frac{d}{dt}(x(t))$	$\longleftrightarrow$	$j\omega \mathbf{X}$
$\frac{d}{dt}[A \cos(\omega t + \phi)]$	$\longleftrightarrow$	$j\omega Ae^{j\phi}$
$\int x(t) dt$	$\longleftrightarrow$	$\frac{1}{j\omega} \mathbf{X}$
$\int A \cos(\omega t + \phi) dt$	$\longleftrightarrow$	$\frac{1}{j\omega} Ae^{j\phi}$

It is much easier to deal with exponentials in the phasor domain than sinusoidal relations in the time domain

Just need to track magnitude/phase, knowing that everything is at frequency ω

Phasors in Circuits

Phasor Relation for Resistors

Current through resistor

Time domain

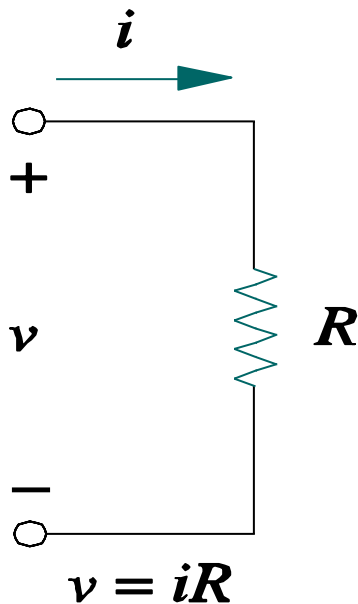
$$i = I_m \cos(\omega t + \phi)$$

$$v = iR = RI_m \cos(\omega t + \phi)$$

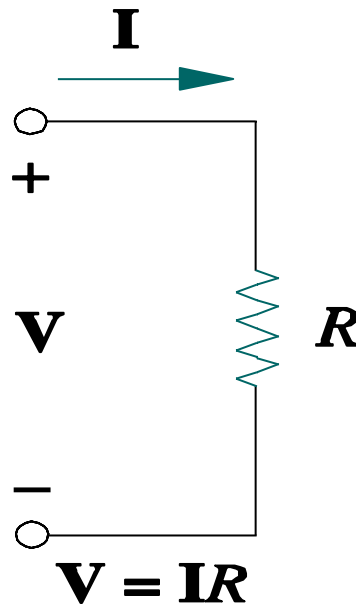
Phasor Domain

$$\begin{aligned} \mathbf{V} &= R\mathbf{I} \\ &= RI_m \angle \phi \end{aligned}$$

Time Domain

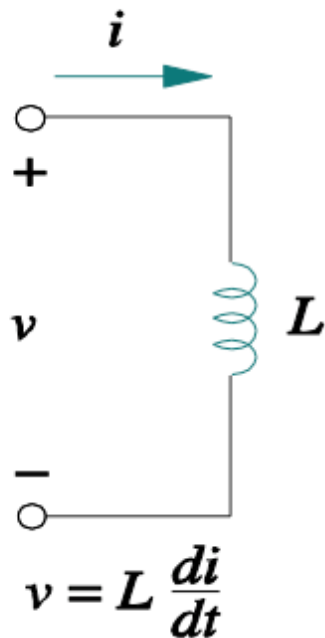


Frequency Domain

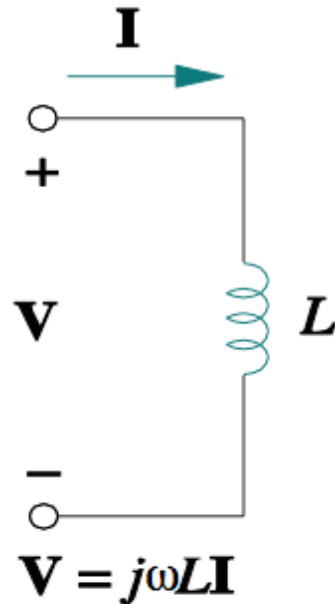


Phasor Relation for Inductors

Time Domain



Frequency Domain



Time domain $v = L \frac{di}{dt}$

Phasor Domain $v_L = \Re[\mathbf{V}_L e^{j\omega t}]$

and

$$i_L = \Re[\mathbf{I}_L e^{j\omega t}].$$

Consequently,

$$\begin{aligned}\Re[\mathbf{V}_L e^{j\omega t}] &= L \frac{d}{dt} [\Re(\mathbf{I}_L e^{j\omega t})] \\ &= \Re[j\omega L \mathbf{I}_L e^{j\omega t}],\end{aligned}$$

which leads to

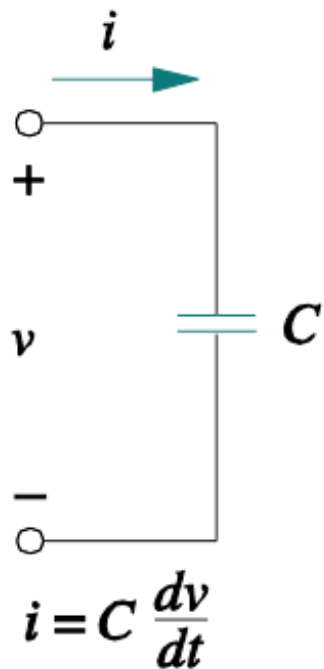
$$\mathbf{V}_L = j\omega L \mathbf{I}_L$$

and

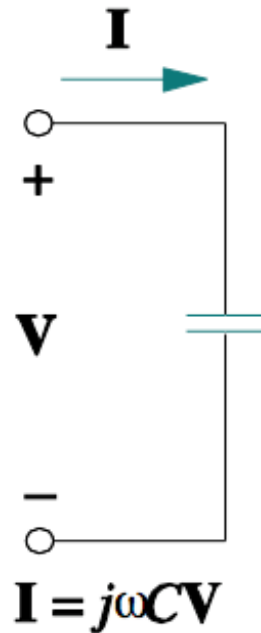
$$\mathbf{Z}_L = \frac{\mathbf{V}_L}{\mathbf{I}_L} = j\omega L.$$

Phasor Relation for Capacitors

Time Domain



Frequency Domain



Time domain

$$i = C \frac{dv}{dt}$$

Phasor Domain

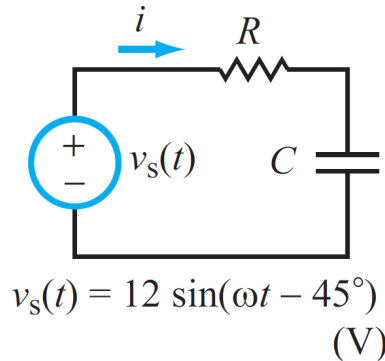
$$\mathbf{I}_C = j\omega C \mathbf{V}_C$$

$$\mathbf{Z}_C = \frac{\mathbf{V}_C}{\mathbf{I}_C} = \frac{1}{j\omega C}.$$

AC Phasor Analysis: General Procedure

Step 1

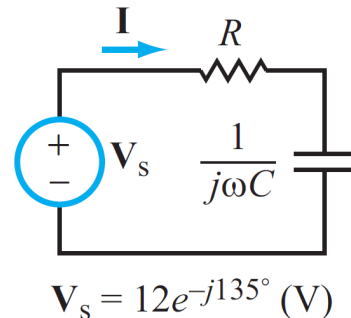
Adopt Cosine Reference
(Time Domain)



Step 2

Transfer to Phasor Domain

$i \rightarrow \mathbf{I}$
 $v \rightarrow \mathbf{V}$
 $R \rightarrow \mathbf{Z}_R = R$
 $L \rightarrow \mathbf{Z}_L = j\omega L$
 $C \rightarrow \mathbf{Z}_C = 1/j\omega C$



Step 3

Cast Equations in
Phasor Form

$$\mathbf{I} \left(R + \frac{1}{j\omega C} \right) = \mathbf{V}_s$$

Step 4

Solve for Unknown Variable
(Phasor Domain)

$$\mathbf{I} = \frac{\mathbf{V}_s}{R + \frac{1}{j\omega C}}$$

Step 5

Transform Solution
Back to Time Domain

$$i(t) = \Re[\mathbf{I}e^{j\omega t}] = 6 \cos(\omega t - 105^\circ) \text{ (mA)}$$

Example 6: RL Circuit

The voltage source of the circuit shown in Fig. 7-8(a) is given by

$$v_s(t) = 15 \sin(4 \times 10^4 t - 30^\circ) \text{ V.}$$

Also, $R = 3 \, \Omega$ and $L = 0.1 \text{ mH}$. Obtain an expression for the voltage across the inductor.

Solution:

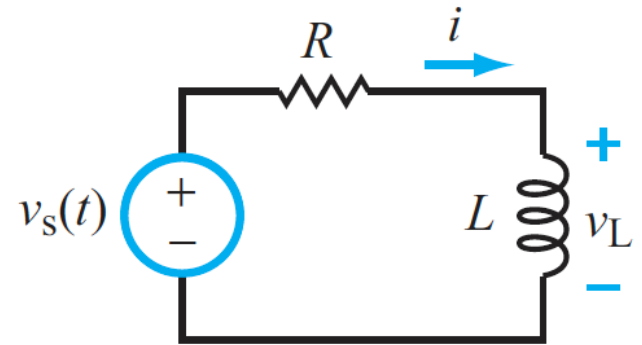
Step 1: Convert $v_s(t)$ to the cosine reference

$$\begin{aligned} v_s(t) &= 15 \sin(4 \times 10^4 t - 30^\circ) \\ &= 15 \cos(4 \times 10^4 t - 30^\circ - 90^\circ) \\ &= 15 \cos(4 \times 10^4 t - 120^\circ) \text{ V,} \end{aligned}$$

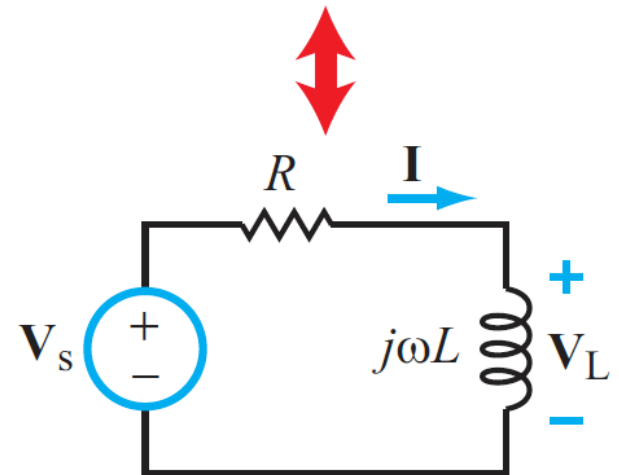
and its corresponding phasor $\mathbf{V}_s$ is given by

$$\mathbf{V}_s = 15e^{-j120^\circ} \text{ V.}$$

Step 2: Transform circuit to the phasor domain



(a) Time domain



(b) Phasor domain **Cont.**

Example 6: RL Circuit cont.

Step 3: Cast KVL in phasor domain

$$R\mathbf{I} + j\omega L\mathbf{I} = \mathbf{V}_s.$$

Instead of Steps 3-4: Solve Resistor divider problem!

$$\frac{V_L}{V_s} = \frac{j\omega L}{R + j\omega L} \Rightarrow V_L = \left(\frac{j\omega L}{R + j\omega L} \right) V_s$$

Step 4: Solve for unknown variable

$$V_s = 15e^{-j120^\circ} \text{ V}; \quad R = 3\Omega; \quad j\omega L = j(4 \times 10^4)(0.1 \times 10^{-3}) = j4\Omega;$$

$$\begin{aligned} \mathbf{I} &= \frac{\mathbf{V}_s}{R + j\omega L} = \frac{15e^{-j120^\circ}}{3 + j4 \times 10^4 \times 10^{-4}} \\ &= \frac{15e^{-j120^\circ}}{3 + j4} = \frac{15e^{-j120^\circ}}{5e^{j53.1^\circ}} = 3e^{-j173.1^\circ} \text{ A.} \end{aligned}$$

$$\begin{aligned} \Rightarrow V_L &= \left(\frac{j4}{3 + j4} \right) (15e^{-j120^\circ}) = \left(\frac{4e^{j90^\circ}}{5e^{j53.1^\circ}} \right) (15e^{-j120^\circ}) \\ &= \frac{(4)(15)}{(5)} e^{j(-120^\circ + 90^\circ - 53.1^\circ)} = 12e^{-j83.1^\circ} \text{ V.} \end{aligned}$$

The phasor voltage across the inductor is related to $\mathbf{I}$ by

Step 5: Transform solution to the time domain

$$\begin{aligned} \mathbf{V}_L &= j\omega L\mathbf{I} \\ &= j4 \times 10^4 \times 10^{-4} \times 3e^{-j173.1^\circ} \\ &= j12e^{-j173.1^\circ} \\ &= 12e^{-j173.1^\circ} \cdot e^{j90^\circ} = 12e^{-j83.1^\circ} \text{ V,} \end{aligned}$$

The corresponding time-domain voltage is

$$\begin{aligned} v_L(t) &= \Re[\mathbf{V}_L e^{j\omega t}] \\ &= \Re[12e^{-j83.1^\circ} e^{j4 \times 10^4 t}] \\ &= 12 \cos(4 \times 10^4 t - 83.1^\circ) \text{ V.} \end{aligned}$$

where we replaced j with e^{j90° .

Table 1-5: Time-domain sinusoidal functions $z(t)$ and their cosine-reference phasor-domain counterparts $\tilde{Z}$, where $z(t) = \Re e [\tilde{Z}e^{j\omega t}]$.

$z(t)$		$\tilde{Z}$
$A \cos \omega t$	$\longleftrightarrow$	A
$A \cos(\omega t + \phi_0)$	$\longleftrightarrow$	$Ae^{j\phi_0}$
$A \cos(\omega t + \beta x + \phi_0)$	$\longleftrightarrow$	$Ae^{j(\beta x + \phi_0)}$
$Ae^{-\alpha x} \cos(\omega t + \beta x + \phi_0)$	$\longleftrightarrow$	$Ae^{-\alpha x} e^{j(\beta x + \phi_0)}$
$A \sin \omega t$	$\longleftrightarrow$	$Ae^{-j\pi/2}$
$A \sin(\omega t + \phi_0)$	$\longleftrightarrow$	$Ae^{j(\phi_0 - \pi/2)}$
$\frac{d}{dt}(z(t))$	$\longleftrightarrow$	$j\omega \tilde{Z}$
$\frac{d}{dt}[A \cos(\omega t + \phi_0)]$	$\longleftrightarrow$	$j\omega Ae^{j\phi_0}$
$\int z(t) dt$	$\longleftrightarrow$	$\frac{1}{j\omega} \tilde{Z}$
$\int A \sin(\omega t + \phi_0) dt$	$\longleftrightarrow$	$\frac{1}{j\omega} Ae^{j(\phi_0 - \pi/2)}$

“Examples 7 – N”

Please go through this week’s problem set carefully!

Summary

- In this class we:
 - Introduced the course
 - Reviewed some background mathematics required for this course, which include
 - Vector Analysis
 - Vector Calculus Operations
 - Complex Numbers
 - Phasor Analysis