

Practice Problem Set 1 Solutions

Question 1: Suppose that the supply and demand functions for a good are

$$q^S(p) = bp - a \quad q^D(p) = c - dp,$$

where a, b, c, d are positive constants. Find the equilibrium price (p^*). Next, if an excise tax of T per unit is imposed ($T \neq 0$), find the resulting market price p^T and show that $p^T < p^* + T$.

The equilibrium price p^* (in the absence of any tax) is found by solving the equations $q^* = bp^* - a = c - dp^*$, which give

$$p^* = \frac{a + c}{b + d}$$

When an excise tax of T per unit is imposed, the effective price from the supplier's point of view is $p^T - T$. The quantity sold, q^T , satisfies

$$q^T = b(p^T - T) - a \quad \text{and} \quad q^T = c - dp^T$$

Eliminating q^T , we obtain

$$p^T = \frac{a + c}{b + d} + \frac{b}{b + d}T = p^* + \frac{b}{b + d}T$$

Thus the selling price rises by $(b/(b + d))T$ to its new equilibrium value. Since $b, d > 0$, $\frac{b}{b + d} < 1$, and hence $p^T < p^* + T$.

Question 2: Let $Q = \{(x, y) | y = x^2 + 3x + 4\}$ and $L = \{(x, y) | y = x + 1\}$. Find $Q \cap L$.

The intersection $Q \cap L$ is the set of (x, y) which belong both to Q and to L , i.e.,

$$y = x^2 + 3x + 4 \quad \text{and} \quad y = x + 1$$

Eliminating y , we get $x^2 + 3x + 4 = x + 1$, which reduces to $x^2 + 2x + 3 = 0$.

The discriminant, $b^2 - 4ac$, of the above quadratic equation (when written in the standard form $ax^2 + bx + c$) is -8 . Therefore, there is no solution. Since there are no possible values of x , there can be no points (x, y) in $Q \cap L$, or $Q \cap L = \emptyset$. (Note: $\emptyset$ is the notation for an empty set, i.e., a set with no elements.)

Question 3: Find the solution of the recurrence equation

$$3y_t = 2y_{t-1} + 10$$

when $y_0 = 25$, and describe its behavior as $t \rightarrow \infty$. Find the condition (in terms of t) that needs to be satisfied to make y_t differ from the time-independent solution by less than 0.5.

Dividing the equation by 3, we have

$$y_t = \frac{2}{3}y_{t-1} + \frac{10}{3}$$

Comparing the above with the standard form $y_t = ay_{t-1} + b$, we have

$$a = \frac{2}{3} \quad \text{and} \quad b = \frac{10}{3}$$

The time-independent solution is

$$y^* = \frac{b}{1-a} = \frac{10/3}{1-2/3} = 10$$

The general solution is

$$y_t = y^* + (y_0 - y^*)a^t = 10 + 15\left(\frac{2}{3}\right)^t$$

Since $2/3$ is positive and less than 1, $(2/3)^t \rightarrow 0$ as $t \rightarrow \infty$. Hence $y_t \rightarrow y^* = 10$ as $t \rightarrow \infty$.

To make y_t differ by less than 0.5 from y^* , we need

$$y_t - 10 < 0.5 \implies 15\left(\frac{2}{3}\right)^t < 0.5 \implies \left(\frac{2}{3}\right)^t < \frac{1}{30}$$

Question 4: An amount of \$2000 is invested and attracts interest at a rate equivalent to 8% per annum. Find the total after 1 year if the interest is compounded (a) annually (once a year), (b) quarterly (every three months), and (c) daily. (Assume that the year is not a leap year.)

(a) $2000(1+r) = 2000(1+0.08) = 2160$

(b) $\left\{ \left(\left(2000\left(1 + \frac{r}{4}\right) \right) \left(1 + \frac{r}{4}\right) \right) \left(1 + \frac{r}{4}\right) \right\} \left(1 + \frac{r}{4}\right) = 2000\left(1 + \frac{r}{4}\right)^4 = 2000(1.02)^4 = 2164.86$

(c) $2000\left(1 + \frac{r}{365}\right)^{365} = 2000\left(1 + \frac{0.08}{365}\right)^{365} = 2166.56$

Question 5: Consider the cobweb model. Suppose the supply and demand equations are given by the equations $q = p - 2$ and $q = 7 - 2p$, respectively. Derive the recurrence equation,

and suppose that $p_0 = 3.5$. Does the market approach the equilibrium price when $t \rightarrow \infty$?

Suppliers operate according to the cobweb model so that if p_t and q_t are (respectively) the price and quantity in year t , then $p_t = p^D(q_t)$ and $q_t = q^S(p_{t-1})$. Thus

$$\begin{aligned} q_t &= p_{t-1} - 2 & p_t &= \frac{7 - q_t}{2} \\ \implies 2p_t &= -p_{t-1} + 9 & \implies p_t &= -0.5p_{t-1} + 4.5 \end{aligned}$$

The equilibrium price is given by

$$p^* - 2 = 7 - 2p^* \implies p^* = 3$$

The general solution is

$$p_t = 3 + (p_0 - 3)(-0.5)^t$$

With $p_0 = 3.5$, the prices oscillate around the equilibrium value, and because $(-0.5)^t \rightarrow 0$ as $t \rightarrow \infty$. Thus, $p_t \rightarrow p^*$ as $t \rightarrow \infty$.