CITY UNIVERSITY OF HONG KONG
DEPARTMENT OF ELECTRICAL ENGINEERING

Homework #9

Solutions:

Course: *EE3114 Systems and Control*
Date:

Problem 1

Solution.

$$A = \begin{bmatrix} 0 & 1 \\ -2 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

Defining the desired state feedback gain matrix K

$$K = [k_1 \quad k_2].$$

Equating

$$\det(sI - (A - BK)) = \left| \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -2 & -2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} [k_1, k_2] \right| = \begin{vmatrix} s & -1 \\ 2 + k_1 & s + 2 + k_2 \end{vmatrix}$$
$$= s^2 + (k_2 + 2)s + (k_1 + 2)$$

The desired characteristic equation is

$$(s - (-3 + 3j))(s - (-3 - 3j)) = s^2 + 6s + 18.$$

Then,

$$\begin{cases} k_2 + 2 = 6 \\ k_1 + 2 = 18 \end{cases} \Rightarrow \begin{cases} k_2 = 4 \\ k_1 = 16 \end{cases}$$

So,

$$K = [16 \quad 4].$$

Problem 2

Solution. (a)

$$A = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}.$$

Then,

$$\begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix}.$$

It is full rank matrix. So, it is observable.
(b)

$$\det(A - LC) = s^2 + s(L_1 - 6) + (11 - 4L_1 + 3L_2),$$

so,

$$\begin{cases} L_1 - 6 = 2\varsigma\omega_n \\ 11 - 4L_1 + 3L_2 = \omega_n^2 \end{cases} \Rightarrow \begin{cases} L_1 = 2 + 2\varsigma\omega_n \\ L_2 = \frac{\omega_n^2 + 8\varsigma\omega_n + 13}{3} \end{cases}$$

(c)When $\varsigma = 1$ and $\omega_n = 6$, from (b)

$$\begin{cases} L_1 = 18 \\ L_2 = 32.34 \end{cases}.$$

Then,

$$L = \begin{bmatrix} 18 \\ 32.34 \end{bmatrix}$$

Problem 3

Solution. Yes, one can transform $g(s)$ to $g_f(s)$ by state feedback. This is possible since one can always find a controllable realization $(A,\ B,\ C, D)$ of $G(s)$, and design a state feedback gain matrix K such that the eigenvalues of $A + BK$ are placed $-3,\ -2,\ 1$. Since the zeros of $g(s)$ are invariant under state feedback, the zero -2 and 1 of $g(s)$ will be canceled by the pole -2 and 1. thus resulting

$$\hat{g}_f(s) = \frac{1}{s+3}.$$

The system is BIBO stable but not asymptotically stable. (Note that the state is still possible to diverge, so the zero-pole cancellation in right-half-plane is invalid.)