

EE3114 HW-8 Solutions

8-1 Given the following system transfer function

$$\frac{Y(s)}{U(s)} = \frac{s+3}{s^2+3s+2}$$

- (a) Show its controllability canonical form **(1 points)**
- (b) Show its diagonal controllability canonical form **(1 points)**
- (c) Show its observability canonical form **(1 points)**

SOLUTION:

(a)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$
$$y = \begin{bmatrix} 3 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

(b)

$$\frac{Y(s)}{U(s)} = \frac{2}{s+1} + \frac{-1}{s+2}$$
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$$
$$y = \begin{bmatrix} 2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

(c)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 3 \\ 1 \end{bmatrix} u$$
$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

8-2 Are the following two LTI systems controllable? Show your verifications.

(a) **(1.5points)**

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 4 \\ 3 \end{bmatrix} u$$

(b) **(1.5points)**

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 4 & 2 \\ 0 & 0 \\ 3 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

SOLUTION:

(a)

$$[B \quad AB \quad A^2B] = \begin{bmatrix} 0 & 4 & -8 \\ 4 & -4 & -4 \\ 3 & -6 & 12 \end{bmatrix} \text{ is full row-rank, and then the LTI system is controllable.}$$

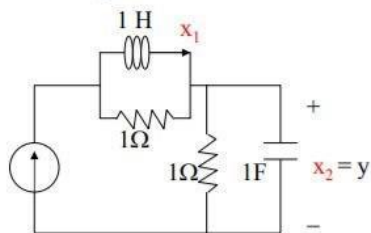
(b)

$$[B \quad AB \quad A^2B] = \begin{bmatrix} 4 & 2 & -4 & -2 & 4 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 0 & -6 & 0 & 0 & 0 \end{bmatrix} \text{ is not full row-rank, and then the LTI system is not controllable.}$$

8-3 Consider the following two circuits.

(a) Show derivation steps to verify their given state-space representations. **(2 points)**

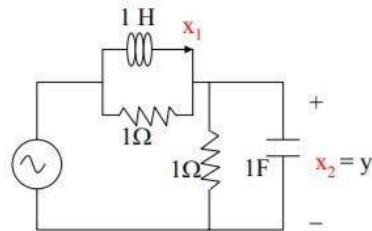
(b) Are these two systems observable? **(2 points)**



$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u,$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} x$$

(i)



$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u,$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} x$$

(ii)

SOLUTION:

(a) (i)

$$u = x_1 + \left(L \frac{di}{dt} \right) / R$$

$$\frac{di}{dt} = -\frac{R}{L}x_1 + \frac{R}{L}u$$

$$\dot{x}_1 = -x_1 + u$$

$$u = \frac{x_2}{R} + C\dot{x}_2$$

$$\dot{x}_2 = -x_2 + u$$

$$y = x_2$$

(ii)

$$u = x_2 + L \frac{di}{dt}$$

$$\frac{di}{dt} = -\frac{1}{L}x_2 + \frac{1}{L}u$$

$$\dot{x}_1 = -x_2 + u$$

$$u = x_2 + \left(C\dot{x}_2 + \frac{x_2}{R} - x_1 \right) R$$

$$\dot{x}_2 = x_1 - 2x_2 + u$$

$$y = x_2$$

(b)

(i)

$\begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix}$ is not full column-rank, and then the LTI system is not observable.

(ii)

$\begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & -2 \end{bmatrix}$ is full column-rank, and then the LTI system is observable.