

Complex Dynamical Networks:

Lecture 7a: Network Game Theory

EE 6605

Instructor: G Ron Chen



Two-Player Games



Human Cooperation and Competition

- ❖ Prisoner's Dilemma Game: two-player game
- ❖ Public Goods Game: multi-player game

DG Rand and MA Nowak, Human cooperation. Trends in Cogn. Sci. 2013, 17 (8): 413-425

Prisoners' Dilemma (PD)



Merrill Flood and Melvin Dresher (1950) proposed the idea

Albert W. Tucker (1950) formalized the game with prison sentence rewards and gave it the name “prisoners’ dilemma”

Two criminal persons are arrested and imprisoned

The two prisoners cannot talk to each other, and they are both selfish

Each prisoner is given the opportunity either to **betray** the other (testifying the other had committed the crime) or to **cooperate** with the other (remaining silent).

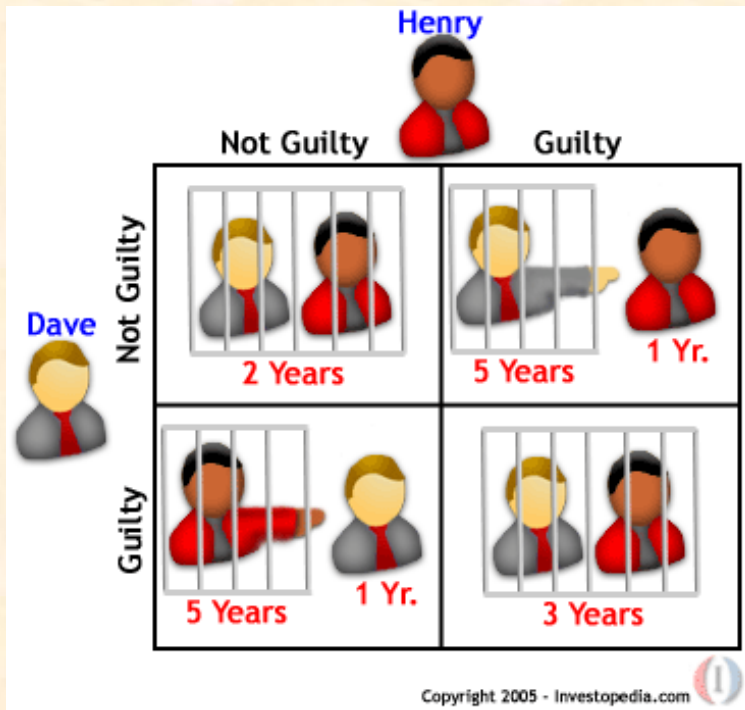
If A and B both betray the other, each of them serves 3 years in prison

If A betrays B but B remains silent, A will be serve 1 year but B will serve 5 years in prison (and vice versa)

If A and B both remain silent, both of them will only serve 2 year in prison (since the police cannot prove they committed the crime)

Prisoners' Dilemma (PD)

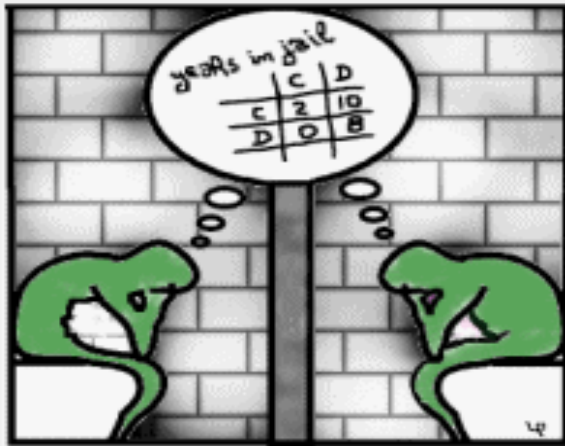
- ❖ **Cooperator**: helping the other at a cost to both
- ❖ **Defector**: receiving benefit without providing help



Payoff Matrix

	C	D
C	$(-2, -2)$	$(-5, -1)$
D	$(-1, -5)$	$(-3, -3)$

Observation: In this example, whatever another does, one does better by **defecting** ...



Payoff Matrix

B

A

	C	D
C	(-2, -2)	(-5, -1)
D	(-1, -5)	(-3, -3)

In general, regardless of what the other decides, each prisoner gets a higher payoff by betraying the other (“defecting” is better):

If **B** cooperates, **A** should defect, since -1 is better than -2

If **B** defects, **A** should also defect, since -3 is better than -5

So, either way, **A** should defect (symmetry: same for **B**)

But, both cooperating is reasonable, since -2 is better than -3

--- This is a **dilemma**

Nash Equilibrium

Nash Equilibrium is a situation where no player can improve his result by changing his strategy

Main Properties:

Nash equilibrium is generally stable

Every game has at least one Nash Equilibrium:

In **pure strategies**: Players only play a single strategy in equilibrium

In **mixed strategies**: Players play a combination of several strategies with a fixed probability

John Nash provided a mathematical theory

John F. Nash, Jr.

1994 Nobel Laureate in Economics

Movie: *A Beautiful Mind*

Key Paper has only 28 lines

Nash, Jr., John F., "Equilibrium points in n -person games,"
PNAS (1950) 36:48-49



(1928 -)

MIT, Princeton

EQUILIBRIUM POINTS IN N-PERSON GAMES

BY JOHN F. NASH, JR.*

PRINCETON UNIVERSITY

Communicated by S. Lefschetz, November 16, 1949

One may define a concept of an n -person game in which each player has a finite set of pure strategies and in which a definite set of payments to the n players corresponds to each n -tuple of pure strategies, one strategy being taken for each player. For mixed strategies, which are probability

City University Distinguished Lecture Series

Honorary
Doctorate

Speaker

Professor John F. Nash, Jr.

1994 Nobel Laureate in Economic Sciences



Professor Nash will present his current research on contemporary topics

on

Monday, 7 November 2011 at 4:30 pm

at

Connie Fan Multi-media Conference Room

4/F, Cheng Yick-chi Building

City University of Hong Kong

Tat Chee Avenue, Kowloon

Distinguished

Generalized PD Game

Rules:

If both **A** and **B** cooperate, they both receive the same *reward*, **R**

If **A** defects while **B** cooperates, then **A** receives *temptation*, **T**, while **B** receives *sucker*, **S**

If **A** cooperates while **B** defects, then **A** receives sucker **S** while **B** receives temptation **T**

If both **A** and **B** defect, then they both receive *punishment*, **P**

$$T > R > P > S$$

B

A

	C	D
C	(R, R)	(S, T)
D	(T, S)	(P, P)

Generalized PD Game

$T > R > P > S$

B

	C	D
A C	(R, R)	(S, T)
D	(T, S)	(P, P)

Dilemma:

Dominant strategy: Payoffs $T > R$ and $P > S$ imply that defecting is the dominant strategy for both players (Nash equilibrium)

Rational strategy: Payoffs $R > P$ implies that both cooperating is better than both defecting

Mixed strategy equilibria

Payoff Matrix

		B	
		C	D
A	C	(1, 0)	(0, 1)
	D	(0, 1)	(1, 0)

Strategy for both players:

Play C and D *randomly* with a probability of 0.5

Thus: Each outcome (1 or 0) has a probability of 0.25



Check if this is a Nash equilibrium:

- ❖ **Q:** **A** plays **C** and **D** randomly with probability 0.5.
Can **B** increase his score by changing his strategy?
- ❖ **Try:** **B** plays **C** with probability 0.6 and plays **D** with probability 0.4. Then, his new success rate is
$$(0.6 \times 0.5) + (0.4 \times 0.5) = (0.3) + (0.2) = 0.5$$
- ❖ So, by choosing 0.6/0.4, **B** scores more on **C** but less on **D**. Thus, his success rate remains the same:
His situation has not been improved
- ❖ Actually, for any choice of probabilities for **B**: 0.a/0.b, his success rate:
$$(0.a \times 0.5) + (0.b \times 0.5) = (0.a + 0.b) \times 0.5 = 0.5$$
- ❖ This corresponds to a Nash equilibrium

Iterated Prisoners' Dilemma Game

Two players play PD game more than once in succession and each remembers previous actions of his opponent thereby changing his strategy accordingly

In addition to $T > R > P > S$, it also requires $2R > T + S$, which can prevent that the strategy of “alternating cooperation and defection” gains a greater reward than “both cooperation”

If the game is played exactly N times and both players know this, then it is always best to defect in all rounds. In this case, the only possible Nash equilibrium is to always defect

Inductive Proof: One certainly will defect on the last turn, since the opponent will not have a chance to punish him. Therefore, both will defect on the last turn. Thus, one might as well defect on the second-to-last turn, since the opponent will defect on the last turn, no matter what was done, and so on

Stochastic Strategies

Deterministic Strategies:

“always cooperate” or “always defect”

Stochastic Strategies: depend on some probabilities

- 1) Tit-For-Tat strategy decides its moves based on the opponent's last move
- 2) Memory-One strategies decide its moves based on four probabilities of both players:

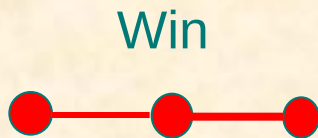
CC (p_1), CD (p_2), DC (p_3), DD (p_4)

Tit-For-Tat Strategy

TFT decides its moves based on opponent's last move

In a two-player game, one player may unilaterally set the other player's score

For example: If red goes first, he can set the result be always tie



●	●	●
●	●	●
●	●	●

Tie

Zero-Determinant Game Theory

One player can determine to achieve a fixed linear relationship between its own payoff with that of its opponent

Recall: Memory-One strategies decide their moves based on four probabilities of both players:

CC (p_1), CD (p_2), DC (p_3), DD (p_4)

By setting

$$p_2 = \frac{p_1(T - P) - (1 + p_4)(T - R)}{R - P}, \quad p_3 = \frac{(1 - p_1)(P - S) + p_4(R - S)}{R - P}$$

W. Press and F.J. Dyson (PNAS 109, 2012) show that the opponent's payoff depends only on two probabilities, p_1 and p_4 , and, consequently, that if the opponent defects then it will hurt himself by getting an even lower payoff

Multi-Player Games



Multi-Player Game

Recall: Iterated PD game is fundamental to the theory of human cooperation and trust

With the assumption that the game can model transactions between two persons requiring trust, cooperative behavior in a population may be modeled by a **multi-player game**



Multi-Player Games

Multiplayer game theory first emerged in the 1970s

In a multiplayer game, the players might be independent opponents forming into teams, or just be individuals against all the others in the game

Games with many independent players are difficult to analyze, especially when some players form coalitions

If coalitions are not allowed, **John Nash** proved that even for multi-player games, there exists an equilibrium (**Nash equilibrium**). But the equilibrium may not be optimal regarding players' individual benefits

Multi-Player Game on a Network

Public Goods Game (PGG)

PGG is a standard for experimental economics

In the basic game, players secretly choose how many of their private tokens to put into a public pool

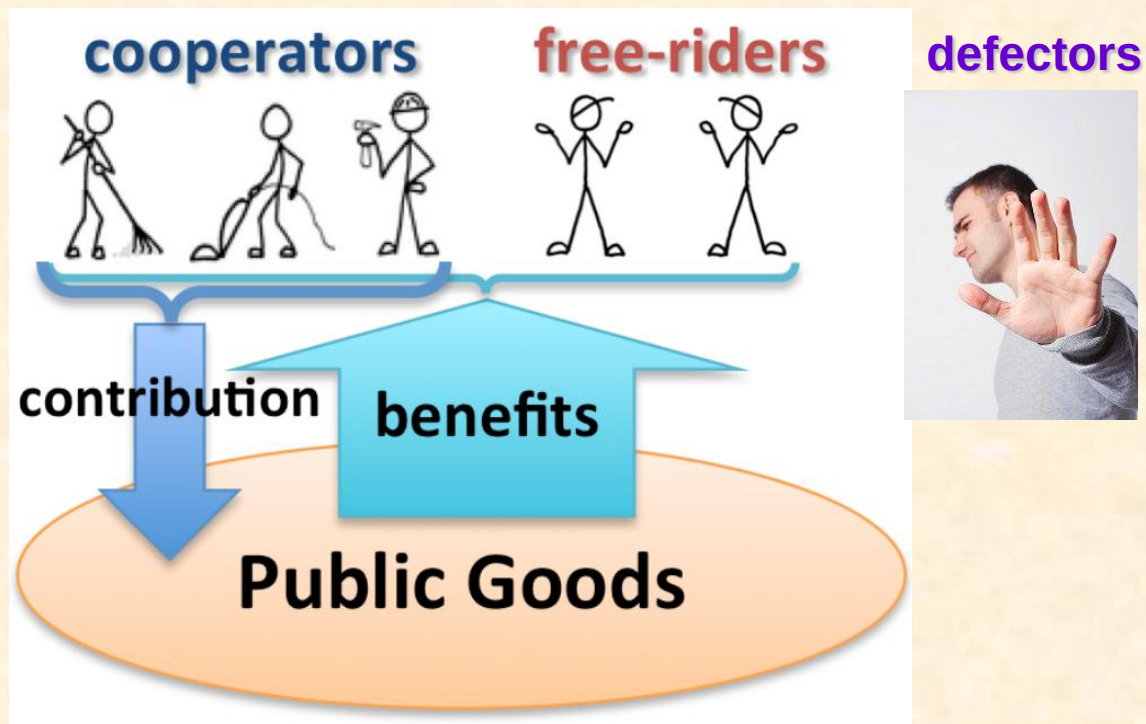
The tokens in the pool are multiplied by a factor (greater than one but less than the number of players) and the “public good” payoff is evenly divided among players

Each player also keeps the tokens he did not contribute

The group's **total payoff is maximized if** everyone contributes all of their tokens to the public pool

PGG

Depending on the experiment, those who contribute below average (including nothing) are called "**defectors**" or "**free riders**", as opposed to the all or above-average contributors who are called "**cooperators**".



Recall: The group's total payoff is maximized if everyone contributes all of their tokens to the public pool

However, the **Nash equilibrium** in this game is simply zero contributions by all; because any rational agent does best by contributing zero, regardless of whatever anyone else does

In fact, the Nash equilibrium is rarely seen in experiments

Expecting beneficial returns, people do tend to add something into the pool, but typically below the average

Real Examples

- ❖ **Commons** refers to the cultural and natural resources accessible to all members of a society, including natural materials such as air, water, and a habitable earth, which are held in common but not owned privately.

- ❖ **Tragedy of the Commons**

Garrett Hardin, The tragedy of the commons, *Science* 1968

It describes a situation in which multiple individuals, acting solely and rationally, will ultimately **deplete** a shared limited resource even if it is not in anyone's long-term interest for this to happen.

- ❖ Preserving the global climate is the biggest tragedy of the commons today

Milinski, Sommerfeld, Krambeck, Reed, Marotzke, The collective-risk social dilemma and the prevention of simulated dangerous climate change, *PNAS* 2008

Tavonia, Dannenberg, Kallis, Löschelb, Inequality, communication, and the avoidance of disastrous climate change in a public goods game, *PNAS* 2011

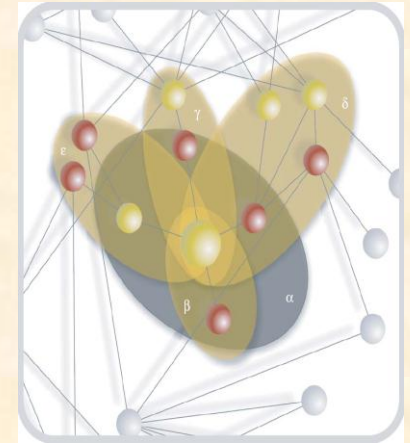
Francisco. Santos, Pacheco, Risk of collective failure provides an escape from the tragedy of the commons, *PNAS* 2011

Networked PGG

- ❖ Each player i takes part in $k_i + 1$ games centered on it and its neighbors (k_i is the node degree of player i)
- ❖ A **cooperator** (C) invests a fixed cost c , distributed equally among all games it participates, and a **defector** (D) invests nothing
- ❖ Payoff of i centered on j is

$$P_i^j = \frac{r}{k_j + 1} \sum_{x \in \Gamma_j} \frac{c}{k_x + 1} s_x - \frac{c}{k_i + 1} s_i$$

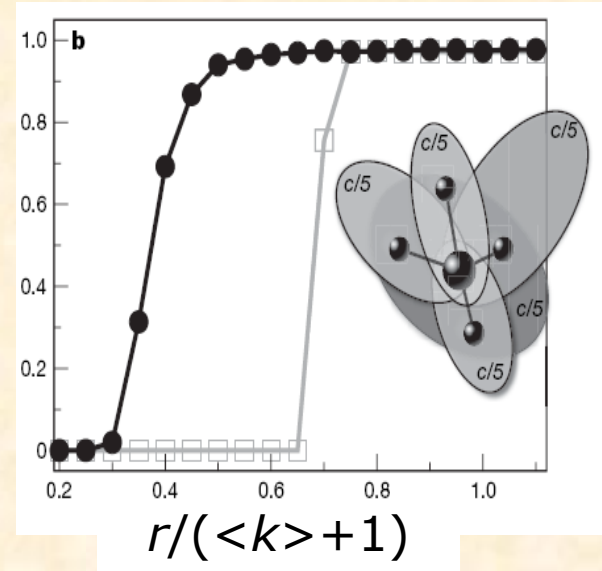
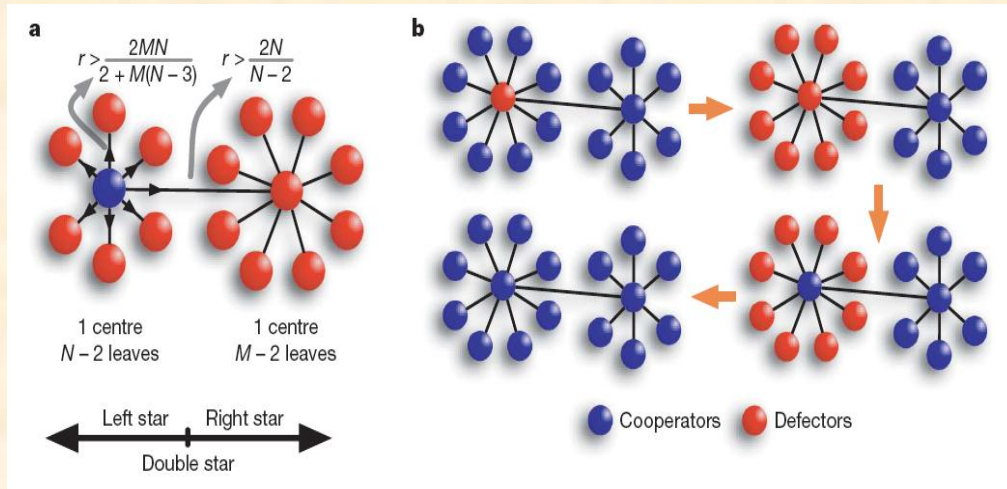
where $s_i = 1$ for C and $s_i = 0$ for D, r is the degree correlation, Γ_j is neighborhood of j



Santos F C, Santos M D, Pacheco J M, Social diversity promotes the emergence of cooperation in public goods games, *Nature* 2008

PGG on BA Scale-Free Network

- ❖ The heterogeneity of the scale-free network means the social diversity



- ❖ Cooperation is promoted by the diversity associated with both number and size of the PGG groups

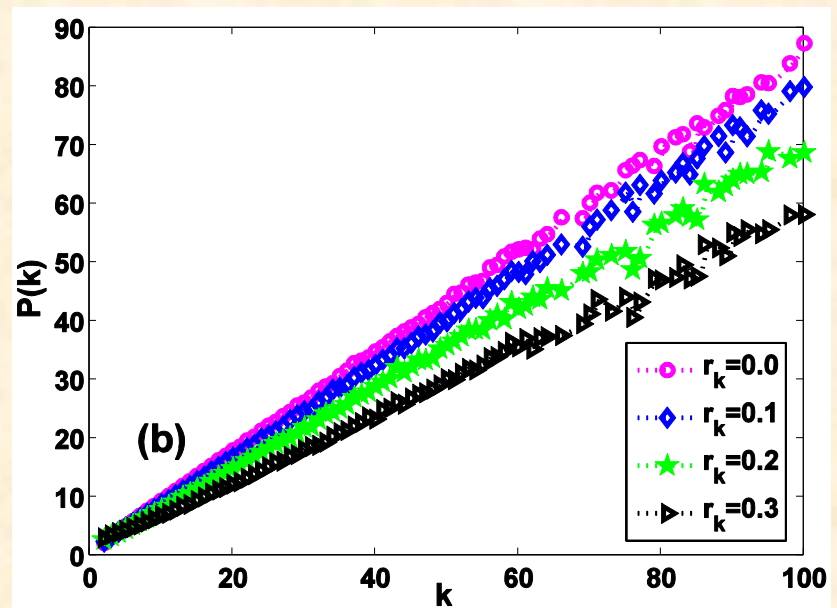
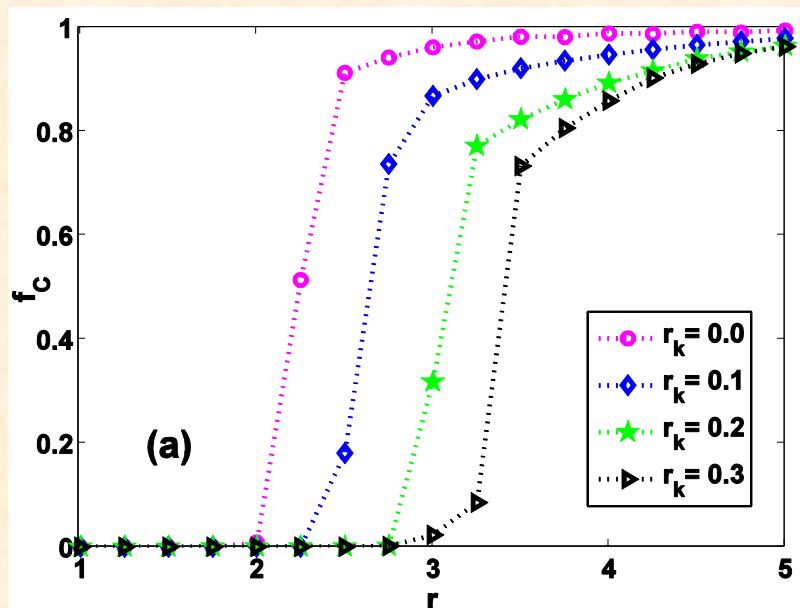
Santos F C, Santos M D, Pacheco J M, Social diversity promotes the emergence of cooperation in public goods games, *Nature* 2008

❖ Payoff of i centered on j :

$$P_i^j = \frac{r}{k_j + 1} \sum_{x \in \Gamma_j} \frac{c}{k_x + 1} s_x - \frac{c}{k_i + 1} s_i$$

❖ $k_j \nearrow \rightarrow P_i^j \searrow$

❖ $r_k \nearrow \rightarrow f_c \searrow$ f_c -- cooperator frequency



Effect of the degree correlation in public goods game on scale-free network

Z. Rong, Z.-X. Wu, *European Physics Letters* 2009

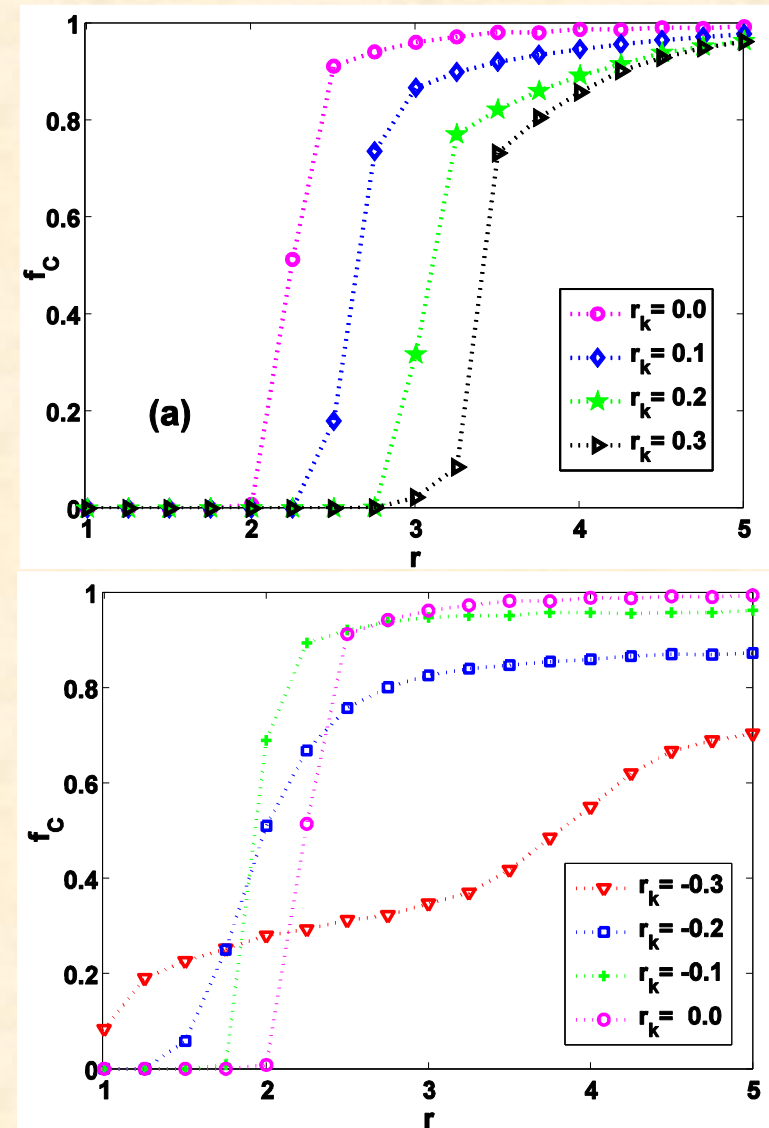
PGG on Corrected Scale-Free Networks

Degree correlations:

$r > 0$ assortative

$r < 0$ disassortative

- ❖ The assortative mixing inhibits cooperation to emerge and to sustain
- ❖ In disassortative networks, the absence of communications among the hubs actually is helpful for cooperator to persist



Social Networks of Hunter-Gatherers

The social networks of the **Hadza**, a population of hunter-gatherers in **Tanzania**, ..., have important properties, including a **skewed degree distribution**, **assortativity**, **transitivity**, **reciprocity** and **homophily**.

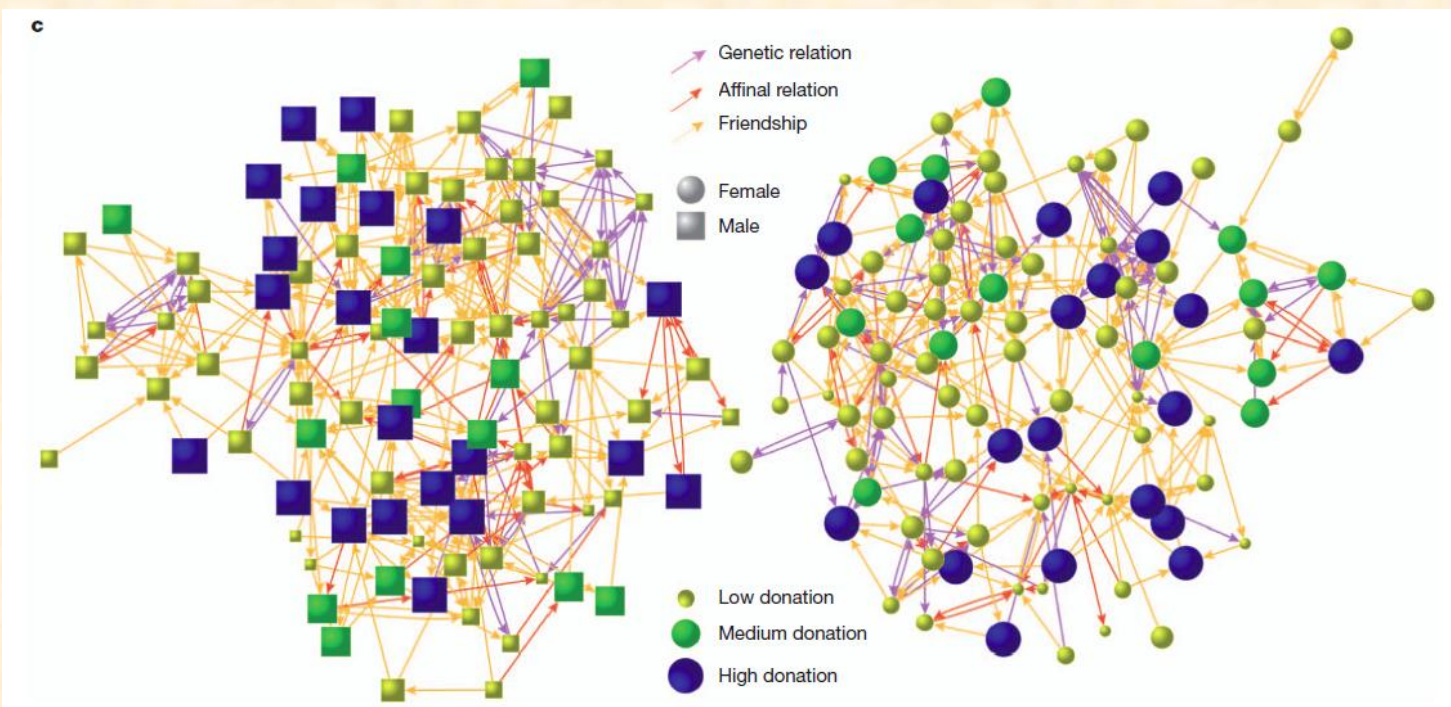
The individuals play the **public goods game** within each camp.



Cooperative Social Networks of Hunter-Gatherers

“Our results suggest that certain elements of social network structure may have been present at an early point in human history. ... Social networks may thus have contributed to the emergence of cooperation.”

Apicella, Marlowe, Fowler and Christakis, *Nature* 2012



Cooperative Game Theory

Cooperative game is a game where groups of players ("coalitions") may enforce cooperative behavior, hence the game is a **competition between coalitions** of players, rather than between individual players.

Cooperative game-theoretic analysis can capture the effect of competition among the players in a **bargaining process**.



Example

Consider three players: $N = \{1, 2, 3\}$

Player 1 is a seller; Player 2 and Player 3 are potential buyers

Player 1 has a single product to sell, for a price of \$4

Each buyer is interested in buying at most one product

Player 2 is willing to bid by paying \$ 9 for the product

Player 3 is willing to bid by paying \$11 for the product

Define a characteristic function (gain value) $v(\cdot)$ for this game:

$$v(\{1, 2\}) = \$ 9 - \$4 = \$5$$

$$v(\{1, 3\}) = \$11 - \$4 = \$7$$

$$v(\{2, 3\}) = \$0$$

$$v(\{1\}) = v(\{2\}) = v(\{3\}) = \$0$$

$$v(\{1, 2, 3\}) = \$7 \text{ [Player 1 will not sell the product to Player 2]}$$

Example (continued)

Recall the characteristic function (gain value) $v(\cdot)$ for this game:

$$v(\{1, 2\}) = \$9 - \$4 = \$5$$

$$v(\{1, 3\}) = \$11 - \$4 = \$7$$

$$v(\{2, 3\}) = \$0$$

$$v(\{1\}) = v(\{2\}) = v(\{3\}) = \$0$$

$$v(\{1, 2, 3\}) = \$7 \text{ [Player 1 will not sell the product to Player 2]}$$

Marginal Contributions of the three players (= captured money):

$$MC1 = v(\{1, 2, 3\}) - v(\{2, 3\}) = \$7 - \$0 = \$7$$

$$MC2 = v(\{1, 2, 3\}) - v(\{1, 3\}) = \$7 - \$7 = \$0$$

$$MC3 = v(\{1, 2, 3\}) - v(\{1, 2\}) = \$7 - \$5 = \$2$$

Example (continued)

Recall that the overall value created was \$7

Now, $(x_1, x_2, x_3) = (MC_1, MC_2, MC_3) = (\$7, \$0, \$2)$

Player 2 will capture \$0 value

Player 3 can capture no more than \$2

Player 1 will capture a minimum of $\$7 - \$2 = \$5$

The remaining value of \$2 shows that Player 1 may expect to sell \$2 more but Player 2 may expect to save \$2 instead

Bargaining: A reasonable solution is for them to split –
\$1 for each – So, comparing to the beginning,
Player 1 feels that he can sell for \$1 more;
Player 3 feels that he saves \$1

This cooperative agreement leads to

Result of This Cooperative Game

Recall: Consider three players, $N = \{1, 2, 3\}$

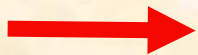
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Player 1 has a single product to sell, for a price of \$4

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Player 3 is willing to bid by paying \$11 for the product



Now, Consider three cooperative players, $N = \{1, 2, 3\}$

Player 1 is a seller; Player 2 and Player 3 are potential buyers

Player 1 has a single product to sell, for a price of \$5 [increased \$1]

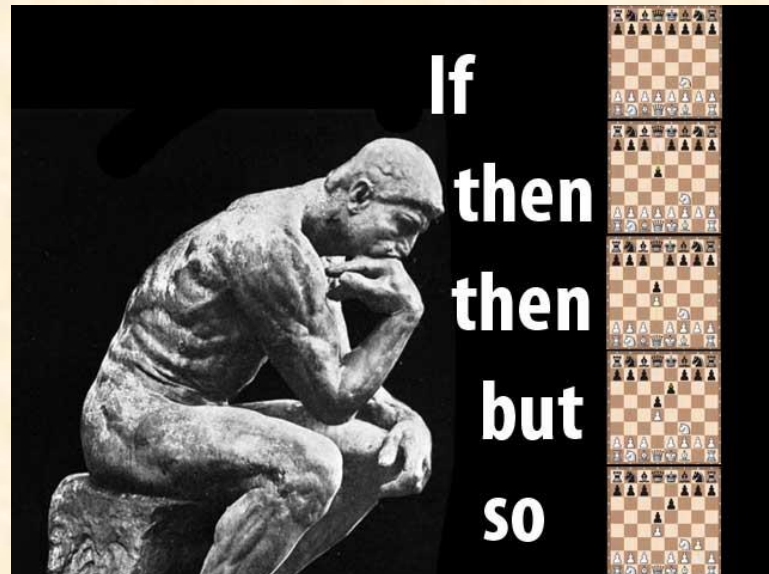
Each buyer is interested in buying at most one product

Player 2 is willing to bid by paying \$ 9 for the product

Player 3 is willing to bid by paying \$10 for the product [reduced \$1]

Now, the setting becomes more reasonable

So, the game may continue to iterative (for example, Player 2 may change his mind to offer a high price for further bidding) ...



BREAK

10 minutes