EE3109 Applied Electromagnetics
2022/23 Semester B
Week 9: Tutorial Solution

Q1. Sol'n.

2.18 Using a slotted line, the voltage on a lossless transmission line was found to have a maximum magnitude of 1.5 V and a minimum magnitude of 0.5 V. Find the magnitude of the load's reflection coefficient.

Solution: From the definition of the Standing Wave Ratio given by Eq. (2.73),

$$S = \frac{|\widetilde{V}|_{\max}}{|\widetilde{V}|_{\min}} = \frac{1.5}{0.5} = 3.$$

Solving for the magnitude of the reflection coefficient in terms of S, as in Example 2-5,

$$|\Gamma| = \frac{S-1}{S+1} = \frac{3-1}{3+1} = 0.5.$$

Q2 Sol'n.

2.28 A lossless transmission line of electrical length $l = 0.35\lambda$ is terminated in a load impedance as shown in Fig. P2.28. Find Γ, S, and Z_{in}. Verify your results using CD Modules 2.4 or 2.5. Include a printout of the screen's output display.

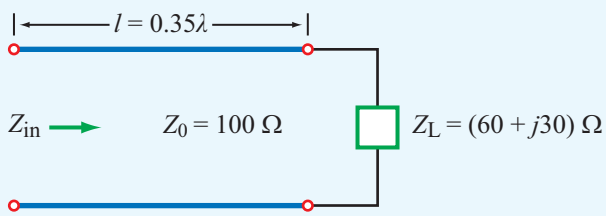

Figure P2.28 Circuit for Problem 2.28.

Solution: From Eq. (2.59),

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{(60 + j30) - 100}{(60 + j30) + 100} = 0.307 \exp j132.5^\circ.$$

From Eq. (2.73),

$$S = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 0.307}{1 - 0.307} = 1.89.$$

From Eq. (2.79)

$$Z_{in} = Z_0 \left(\frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \right)$$

$$= 100 \left[\frac{(60 + j30) + j100 \tan\left(\frac{2\pi \text{ rad}}{\lambda} 0.35\lambda\right)}{100 + j(60 + j30) \tan\left(\frac{2\pi \text{ rad}}{\lambda} 0.35\lambda\right)} \right] = (64.8 - j38.3)\ \Omega.$$

Q3. Sol'n.

2.29 At an operating frequency of 300 MHz, a lossless 50 Ω air-spaced transmission line 2.5 m in length is terminated with an impedance $Z_L = (40 + j20)$ Ω. Find the input impedance.

Solution: Given a lossless transmission line, $Z_0 = 50$ Ω, $f = 300$ MHz, $l = 2.5$ m, and $Z_L = (40 + j20)$ Ω. Since the line is air-filled, $u_p = c$ and therefore, from Eq. (2.48),

$$\beta = \frac{\omega}{u_p} = \frac{2\pi \times 300 \times 10^6}{3 \times 10^8} = 2\pi \text{ rad/m}.$$

Since the line is lossless, Eq. (2.79) is valid:

$$\begin{aligned} Z_{in} = Z_0 \left(\frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \right) &= 50 \left[\frac{(40 + j20) + j50 \tan (2\pi \text{ rad/m} \times 2.5 \text{ m})}{50 + j(40 + j20) \tan (2\pi \text{ rad/m} \times 2.5 \text{ m})} \right] \\ &= 50 \left[(40 + j20) + j50 \times 0 \right] 50 + j(40 + j20) \times 0 \\ &= (40 + j20) \ \Omega. \end{aligned}$$

Q4. Sol'n.

2.30 Show that at the position where the magnitude of the voltage on the line is a maximum, the input impedance is purely real.

Solution: From Eq. (2.70), $d_{\max} = (\theta_r + 2n\pi)/2\beta$, so from Eq. (2.61), using polar representation for Γ,

$$\begin{aligned} Z_{in}(d_{\max}) &= Z_0 \left(\frac{1 + |\Gamma| \exp j\theta_r \exp -j2\beta l_{\max}}{1 - |\Gamma| \exp j\theta_r \exp -j2\beta l_{\max}} \right) \\ &= Z_0 \left(\frac{1 + |\Gamma| \exp j\theta_r \exp -j(\theta_r + 2n\pi)}{1 - |\Gamma| \exp j\theta_r \exp -j(\theta_r + 2n\pi)} \right) = Z_0 \left(\frac{1 + |\Gamma|}{1 - |\Gamma|} \right), \end{aligned}$$

which is real, provided Z_0 is real.

Q5 Sol'n.

2.20 A 300-Ω lossless air transmission line is connected to a complex load composed of a resistor in series with an inductor, as shown in Fig. P2.20. At 5 MHz, determine: (a) Γ, (b) S, (c) location of voltage maximum nearest to the load, and (d) location of current maximum nearest to the load.

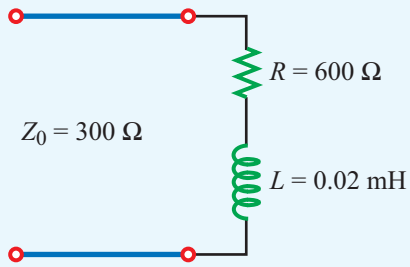

Figure P2.20 Circuit for Problem 2.20.

Solution:

(a)

$$Z_L = R + j\omega L$$
$$= 600 + j2\pi \times 5 \times 10^6 \times 2 \times 10^{-5} = (600 + j628)\ \Omega.$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$
$$= \frac{600 + j628 - 300}{600 + j628 + 300}$$
$$= \frac{300 + j628}{900 + j628} = 0.63e^{j29.6^\circ}.$$

(b)

$$S = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 0.63}{1 - 0.63} = 4.405.$$

(c)

$$l_{max} = \frac{\theta_r \lambda}{4\pi} \qquad \text{for } \theta_r > 0.$$
$$= \left(\frac{29.6^\circ \pi}{180^\circ}\right)\frac{60}{4\pi}, \qquad \left(\lambda = \frac{3 \times 10^8}{5 \times 10^6} = 60 \text{ m}\right)$$
$$= 2.46 \text{ m}$$

(d) The locations of current maxima correspond to voltage minima and vice versa. Hence, the location of current maximum nearest the load is the same as location of voltage minimum nearest the load. Thus

$$l_{\text{min}} = l_{\text{max}} + \frac{\lambda}{4}\,, \qquad \left(l_{\text{max}} < \frac{\lambda}{4} = 15 \text{ m}\right)$$
$$= 2.46 + 15 = 17.46 \text{ m}.$$

Q6. Sol'n.

2.26 A 50 Ω lossless transmission line is connected to a load composed of a 75 Ω resistor in series with a capacitor of unknown capacitance (Fig. P2.26). If at 10 MHz the voltage standing wave ratio on the line was measured to be 3, determine the capacitance C.

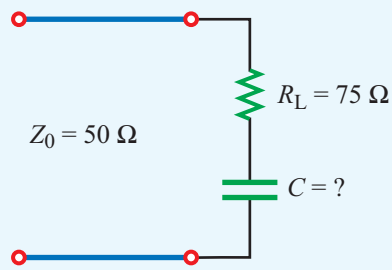

Figure P2.26 Circuit for Problem 2.26.

Solution:

$$|\Gamma| = \frac{S-1}{S+1} = \frac{3-1}{3+1} = \frac{2}{4} = 0.5$$

$$Z_L = R_L - jX_C, \qquad \text{where } X_C = \frac{1}{\omega C}.$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$|\Gamma|^2 = \left[\left(\frac{Z_L - Z_0}{Z_L + Z_0}\right)\left(\frac{Z_L^* - Z_0}{Z_L^* + Z_0}\right)\right]$$

$$|\Gamma|^2 = \frac{Z_L Z_L^* + Z_0^2 - Z_0(Z_L + Z_L^*)}{Z_L Z_L^* + Z_0^2 + Z_0(Z_L + Z_L^*)}$$

Noting that:

$$Z_L Z_L^* = (R_L - jX_C)(R_L + jX_C) = R_L^2 + X_C^2,$$

$$Z_0(Z_L + Z_L^*) = Z_0(R_L - jX_C + R_L + jX_C) = 2Z_0 R_L,$$

$$|\Gamma|^2 = \frac{R_L^2 + X_C^2 + Z_0^2 - 2Z_0 R_L}{R_L^2 + X_C^2 + Z_0^2 + 2Z_0 R_L}.$$

Upon substituting $|\Gamma_L| = 0.5$, $R_L = 75\ \Omega$, and $Z_0 = 50\ \Omega$, and then solving for X_C, we have

$$X_C = 66.1\ \Omega.$$

Hence

$$C = \frac{1}{\omega X_{\mathrm{C}}} = \frac{1}{2\pi \times 10^7 \times 66.1} = 2.41 \times 10^{-10} = 241 \text{ pF}.$$

Q7 Sol'n

7. (Adapted from Ulaby 2.63) A 50 Ω lossless line 0.6l long is terminated in a load with $Z_L = (50 + j25)$ Ω. At 0.3λ from the load, a resistor with resistance $R = 30$ Ω is connected as shown in Fig. P2.63.
   (a) Find Z_{in}.
   (b) If a voltage source with generator resistance 75 Ω feeds 1 W of power into this circuit, what is the generator voltage?
   (c) What percentage of the input power is dissipated by the load?

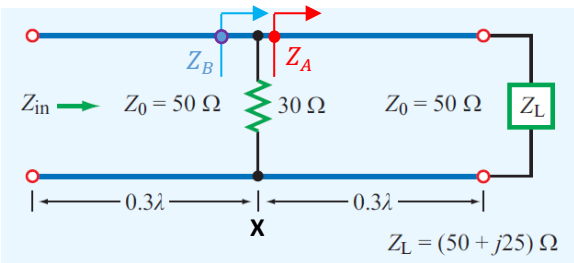

Figure P2.63 Circuit for Problem 2.63.

(a) We define Z_A and Z_B as shown in the picture above.

$$\beta l = \left(\frac{2\pi}{\lambda}\right)(0.3\lambda) = 0.6\pi \Rightarrow \tan\beta l = \tan 0.6\pi = -3.078$$

$$Z_A = Z_0\frac{Z_L + jZ_0\tan\beta l}{Z_0 + jZ_L\tan\beta l} = (50)\frac{(50 + j25) + j(50)(-3.078)}{(50) + j(50 + j25)(-3.078)} = 32.89 - j10.89\ [\Omega]$$

$$Z_B = \left(\frac{1}{Z_A} + \frac{1}{R}\right)^{-1} = 16.1059 - j2.4059\ [\Omega]$$

$$Z_{in} = Z_0\frac{Z_B + jZ_0\tan\beta l}{Z_0 + jZ_B\tan\beta l} = (50)\frac{(16.11 - j2.406) + j(50)(-3.078)}{(50) + j(16.11 - j2.406)(-3.078)} = 98.72 - j68.58\ [\Omega]$$

(b)

$$P_{in} = \frac{1}{2}Re\{V_{in}I_{in}^*\} = \frac{1}{2}Re\left\{\frac{|V_{in}|^2}{Z_{in}^*}\right\} = \frac{|V_{in}|^2}{2Re\{Z_{in}^*\}} = 1\ [\mathrm{W}]$$

$$V_{in} = V_g\left(\frac{Z_{in}}{R_g + Z_{in}}\right) \Rightarrow |V_{in}|^2 = V_g^2\left|\frac{Z_{in}}{R_g + Z_{in}}\right|^2$$

$$\Rightarrow V_g = \sqrt{(1)(2Re\{Z_{in}\})\left|\frac{R_g + Z_{in}}{Z_{in}}\right|^2} = 21.83\ [\mathrm{V}]$$

(c) Consider the equivalent circuit when Z_L and the TL section that connects it to node **X** is replaced by the equivalent impedance Z_A. At node **X**, the power flows either flows through and gets dissipated by R, or through Z_A and gets dissipated by the load (b/c the TL is lossless). The power dissipated by both are:

$$P_R = \frac{1}{2}Re\left\{\frac{|V_x|^2}{R}\right\} = \frac{1}{2}\frac{|V_x|^2}{R};\ P_{load} = \frac{1}{2}Re\left\{\frac{|V_x|^2}{Z_A^*}\right\} = \frac{1}{2}\frac{|V_x|^2}{Re\{Z_A\}}$$

The percentage of power dissipated by the load is:

$$\frac{P_{load}}{P_{load} + P_R} = \frac{R}{R + Re\{Z_A\}} = \frac{30}{30 + 32.89} = 47.70\%$$