

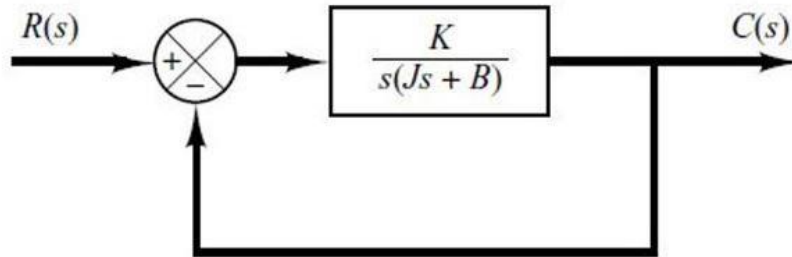
EE3114 Systems and Control

Tutorial - 2

Semester B, 2017-18

Exercise 1

Recall from Lecture-3:



Closed-loop Transfer Function:

$$\frac{C(s)}{R(s)} = \frac{K}{Js^2 + Bs + K}$$



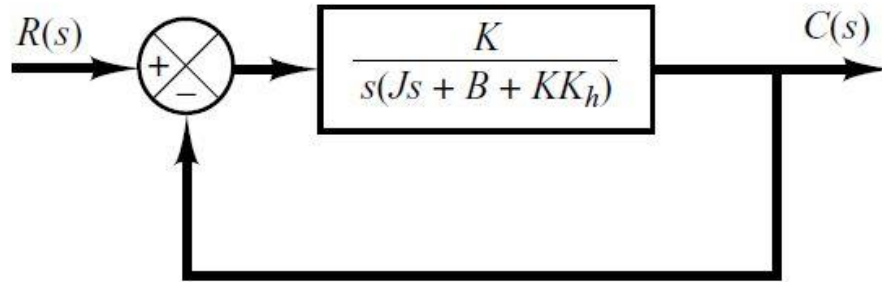
$$\frac{C(s)}{R(s)} = \frac{\frac{K}{J}}{\left[s + \frac{B}{2J} + \sqrt{\left(\frac{B}{2J}\right)^2 - \frac{K}{J}}\right]\left[s + \frac{B}{2J} - \sqrt{\left(\frac{B}{2J}\right)^2 - \frac{K}{J}}\right]}$$

$$p = -\sigma + j\omega_d \quad \text{and} \quad p^* = -\sigma - j\omega_d$$

$$\sigma = \zeta\omega_n, \quad \omega_d = \omega_n\sqrt{1-\zeta^2} \quad \zeta = \frac{B}{2\sqrt{JK}}$$

$$\frac{K}{J} = \omega_n^2, \quad \frac{B}{J} = 2\zeta\omega_n = 2\sigma$$

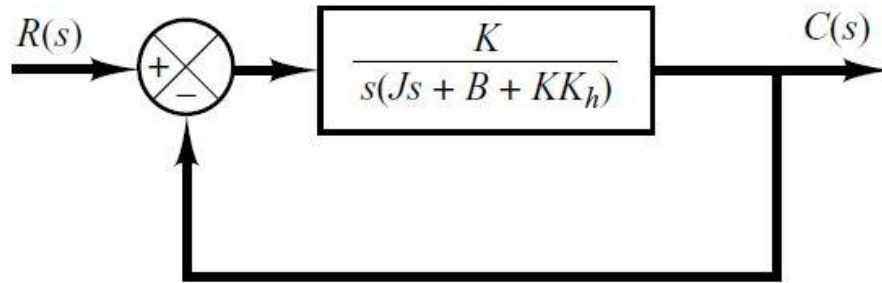
Now, consider:



(i) Find its closed-loop transfer function $H(s)$

5 minutes

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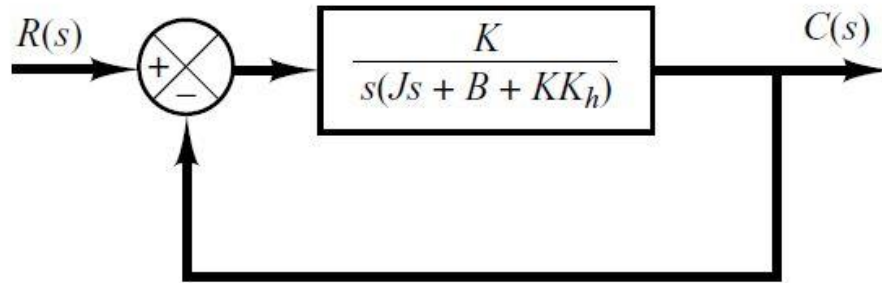
(i) Find its closed-loop transfer function $H(s)$

Answer:
$$\frac{C(s)}{R(s)} = \frac{K}{Js^2 + (B + KK_h)s + K}$$

(ii) Find: **5 minutes**

$$\sigma = ? \quad \omega_d = ? \quad \zeta = ? \quad \omega_n = ?$$

Now, consider:



(i) Find its closed-loop transfer function $H(s)$

Answer:
$$\frac{C(s)}{R(s)} = \frac{K}{Js^2 + (B + KK_h)s + K}$$

(ii) Find:

Answer:
$$\zeta = \frac{B + KK_h}{2\sqrt{KJ}}, \quad \omega_n = \sqrt{\frac{K}{J}}$$

$$\sigma = \zeta\omega_n, \quad \omega_d = \omega_n\sqrt{1-\zeta^2}$$

Exercise 2

Given a transfer function from input to output:

$$H(s) = \frac{2s + 1}{s^2 + 3s + 2}$$

(i) Find the partial-fraction expansion of $H(s)$ **5 minutes**

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$$H(s) = \frac{2s + 1}{s^2 + 3s + 2}$$

(i) Find the partial-fraction expansion of $H(s)$

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$$H(s) = -\frac{1}{s + 1} + \frac{3}{s + 2}$$

(ii) Find the impulse response $h(t)$ (use Table) **5 minutes**

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Answer:

$$h(t) = \begin{cases} -e^{-t} + 3e^{-2t} & t \geq 0 \\ 0 & t < 0 \end{cases}$$

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(iii) Find $h(0) = ?$ **5 minutes**

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(iii) Find $h(0) = ?$

Answer: $h(0) = \lim_{s \rightarrow \infty} sH(s) = 2$

(iv) Find $h(\infty) = ?$ **5 minutes**

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(v) Sketch $h(t), t \geq 0$ **10 minutes**

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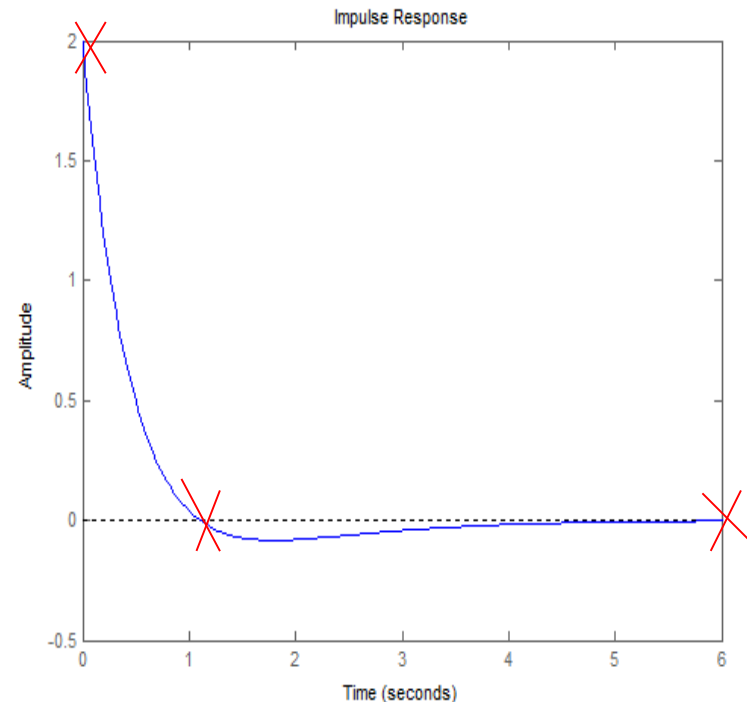
(v) Sketch $h(t), t \geq 0$

Answer:

$$-e^{-t} + 3e^{-2t} = 0$$

$$\rightarrow t = \ln 3 = 1.1$$

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num = [2 1];
den = [1 3 2];
impz(num,den);
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END