

### EE 3008. Receiver Design Tutorial Questions

**Q1)** Suppose we are transmitting a variable  $v$ , which can be either 1 or 0. Suppose during the transmission  $v$  is distorted by a Gaussian random variable  $n$  with zero mean and variance  $\sigma^2$ , so the receiver actually receives  $x=v+n$ . What is the probability of error at the receiver? (Hint: The receiver can use the rule that  $v'=1$ , if  $x>0.5$  and  $v'=0$ , if  $x<0.5$ ).

**Q2)** Find the BER for a binary PAM signal using rectangular pulses with amplitudes  $\pm 10^{-2}V$  at the receiver and the data rate is 10 kbps. The channel is AWGN channel with  $N_0=4 \times 10^{-9}$  V<sup>2</sup>/Hz. What is the BER if the transmission power is reduced by half?

**Q3)** Consider pulse duration  $\tau$  and integration receiver. Assume  $A>B$ . Let  $d$  be the data.

- When  $d=0$ , we transmit a pulse with amplitude  $A$ . Then the correlator output is  $y(\tau) = A\tau + \eta$ .
  - When  $d=1$ , we transmit a pulse with amplitude  $B$ . Then the correlator output is  $y(\tau) = B\tau + \eta$ .
- In the above,  $\eta$  is a Gaussian random variable with mean  $=0$  and variance  $=\sigma_\eta^2=\sigma^2\tau$ .

- (a) Prove that the BER performance of the system remains the same provided that  $A-B=C$ , where  $C$  is a constant, (meaning that  $A$  and  $B$  can vary but their difference is fixed).
- (b) Assume that  $A-B=C$  is a constant. Find  $A$  and  $B$  that lead to minimum average power. Express  $A$  and  $B$  in terms of the average transmission power  $P$ .
- (c) Assume that the system has fixed average power  $P$ . Find  $A$  and  $B$  that minimize BER.

**Q4)** Consider a transmission method based on a pulse  $f(t)$  with unit amplitude and limited duration  $\tau$ . Let the transmitted signal

- $s(t) = s_1(t) = Af(t)$  if information bit “1” is transmitted, and
- $s(t) = s_0(t) = Bf(t)$  if information bit “0” is transmitted,

where  $A$  and  $B$  are real constants. The probabilities of sending “0” and “1” are equal. The received signal is  $r(t)=s(t)+n(t)$ , where  $n(t)$  represents channel noise. Suppose that  $A > B$ . Let the decision threshold of the system be  $(A+B)\tau/2$ .

- (a) Write down the expression of BER in terms of  $A$ ,  $B$  and  $\tau$  for this system.
- (b) Show that the result of (a) is consistent with the BER expression of a bipolar signal.
- (c) Show that the result of (a) is consistent with the BER expression of on-off.

**Q5)** Consider a system with data rate 2.5kb/sec in an AWGN channel with  $N_0=10^{-7}$ W/Hz. If we want to achieve BER=0.023. (a) Find the required average transmission power for bipolar.

(Note:  $Q(2) \approx 0.023$ .) (b) Find the required average transmission power for on-off. (c) Compare your results of (a) and (b).

**Q6)** The random variable  $w$  is distributed according to the PDF:  $f_X(w) = Ke^{-\frac{w^2}{4}}$ .

- (a) Find the constant  $K$ .
- (b) Find  $\Pr(2.2 \leq w \leq 5)$  in terms of the Q-function.
- (c) Consider a signal model  $y=x+w$  where  $x=\pm 1$  with equal probability. Find  $\Pr(y \leq -0.5)$  and  $\Pr(y > 5)$ . (Hints: Consider conditional probability.)
- (d) What is the BER based on threshold decoding? Let the threshold be 0.
- (e) Let  $y=hx+w$  where  $h$  is a constant. What is the optimal decoding rule? What is the BER now?