

Write your answers on **paper** using the **following format**:

Answer to Question X

Scan your answers **or** take **photos** of them.

Submit **scanned** answers **or** their **photos** in a **single file** to Canvas.

Name your file in following format: **Quiz_Take home_SID_LastName_FirstName**

Write down all the steps in your calculations.

Q1) Consider the function $f(x) = \cos(x^2)$.

40 marks

a) Find $f'(x)$ and $f'(0)$.

$$f'(x) = -2x \sin(x^2) \text{ (6 marks)}$$

$$f'(0) = 0 \text{ (2 marks)}$$

b) Find $f''(x)$ and $f''(0)$.

$$f''(x) = -2 \sin(x^2) - 4x^2 \cos(x^2) \text{ (6 marks)}$$

$$f''(0) = 0. \text{ (2 marks)}$$

c) Find $f'''(x)$ and $f'''(0)$.

$$f'''(x) = 8x^3 \sin(x^2) - 12x \cos(x^2) \text{ (6 marks)}$$

$$f'''(0) = 0. \text{ (2 marks)}$$

d) Find $f^4(x)$ and $f^4(0)$.

$$f^4(x) = 16x^4 \cos(x^2) - 12 \cos(x^2) + 48x^2 \sin(x^2) \text{ (6 marks)}$$

$$f^4(0) = -12 \text{ (2 marks)}$$

e) Find the fourth-order Taylor approximation of $f(x)$ around $x_i=0$.

$$1 - \frac{x^4}{2} \text{ (8 marks)}$$

Write down all the steps in your calculations.

Q2) Solve the following system of linear equations using the LU decomposition method.

$$\begin{bmatrix} 3 & 2 & 1 \\ 6 & 7 & 4 \\ 9 & 9 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 27 \\ 39 \end{bmatrix}$$

20 marks

Write down all the steps in your calculations.

Answer:

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 1 & 1 \end{bmatrix} \quad (6 \text{ marks}) \quad U = \begin{bmatrix} 3 & 2 & 1 \\ 0 & 3 & 2 \\ 0 & 0 & 1 \end{bmatrix} \quad (8 \text{ marks})$$

Please deduct 0.5 mark for each wrong entry.

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 1 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 27 \\ 39 \end{bmatrix} \quad y_1 = 10; y_2 = 7, y_3 = 2 \quad (3 \text{ marks})$$

$$\begin{bmatrix} 3 & 2 & 1 \\ 0 & 3 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 7 \\ 2 \end{bmatrix} \quad x_1 = 2; x_2 = 1, x_3 = 2 \quad (3 \text{ marks})$$

Q3) Find the eigenvalues and eigenvectors of the following matrices.

$$A = \begin{bmatrix} 2.2 & -0.4 \\ -0.4 & 2.8 \end{bmatrix},$$

20 marks

Answer:

$$x^2 - 5x + 6 = 0 \quad (5 \text{ marks})$$

$$, \lambda_1 = 3, \lambda_2 = 2 \quad (5 \text{ marks})$$

For $\lambda_1 = 3$, we have $-0.8v_1 = 0.4v_2$. So, $v_2 = 2$ and the eigenvector is $\begin{bmatrix} 1 \\ -2 \end{bmatrix}$. (5 marks)

For $\lambda_2 = 2$, we have $0.2v_1 = 0.4v_2$. So, $v_2 = 0.5$ and the eigenvector is $\begin{bmatrix} 1 \\ 0.5 \end{bmatrix}$. (5 marks)

Find the eigenvectors of A . Remember each eigenvector will be of the form $\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$. Assume $v_1 = 1$ when you compute each eigenvector.

Write down all the steps in your calculations.

Q4) Consider the following matrix.

20 marks

$$A = \begin{bmatrix} -1 & 2 & 2 \\ 2 & 2 & 2 \\ -3 & -6 & -6 \end{bmatrix}$$

- a) Find the characteristic polynomial of A .
- b) Find the eigenvalues of A

Write down all the steps in your calculations.

a) $-x^3 - 5x^2 - 6x$ (14 marks)

c) The other eigenvalues are $0, -2, -3$. (6 marks)