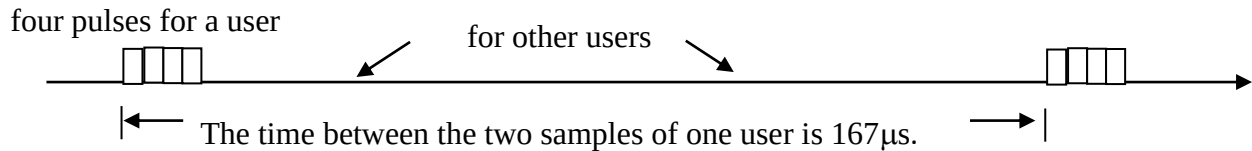


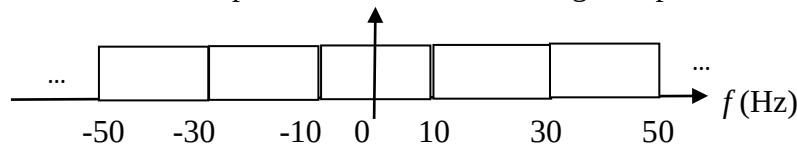
EE 3008. Chapter 6 Tutorial Solutions

Q1) For pulse duration of $1\mu\text{s}$ we can support a maximum rate of $1/10^{-6} = 10^6$ bits/s. Since the input bandwidth is 3kHz, the Nyquist sampling rate = $2 \times 3\text{k} = 6\text{kHz}$. For 16 quantization levels, we have $k = 4\text{bits}$. Therefore the rate of each user is 24×10^3 bits/s. Thus the number of users that can be supported is $10^6 / (24 \times 10^3) = 41.6666 \approx 41$ users.



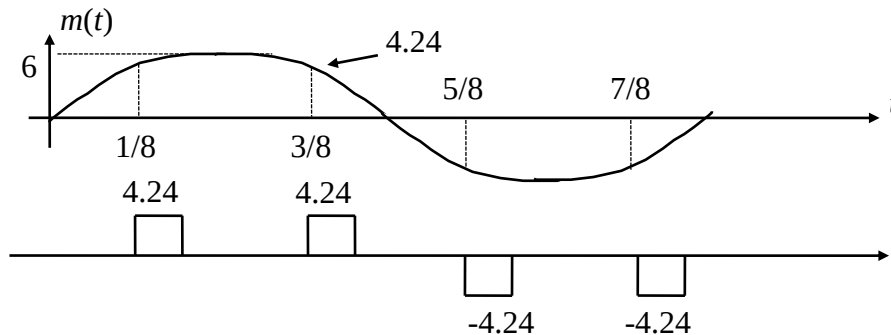
Q2) (a) The minimum rate (i.e., Nyquist rate) is 20Hz.

(b) (You need to read "Extra Reading" before attempting this question.) The impulse sampled spectrum is shown below. It is a repeated extension of the original spectrum.



(c) Use a low-pass filter of bandwidth 10Hz.

Q3) For $m(t) = 6\sin(2\pi t)$, the sampled values at $t = \pm 1/8\text{s}, \pm 3/8\text{s}, \pm 5/8\text{s}, \pm 7/8\text{s}$ are $\{+4.24, +4.24, -4.24, -4.24\}$.



The quantization levels are $\pm 0.5, \pm 1.5, \pm 2.5, \pm 3.5, \pm 4.5, \dots$. The binary representations can be

0.5 → 0000	1.5 → 0001	2.5 → 0010	3.5 → 0011	4.5 → 0100	5.5 → 0101
-0.5 → 1000	-1.5 → 1001	-2.5 → 1010	-3.5 → 1011	-4.5 → 1100	-5.5 → 1101

The closest levels of ± 4.24 are ± 4.5 . Therefore the quantized outputs are 0100, 0100, 1100, 1100, ...

Q4) 8-bit PCM → 256 levels, i.e., $n = 256$. According to the lecture notes, for a maximum input range A and step size a , we should use $n = A/a$.

In most practical engineering problems, we can assume that $Q_{\max} \approx A/2$ and $Q_{\min} = a/2$. Thus we also have $n = Q_{\max}/Q_{\min}$. Therefore dynamic range = $10\log_{10}(n^2) \text{ dB} = 20\log_{10}(256) = 48 \text{ dB}$.

Q5) (a) The maximum quantization error is $1/8$.

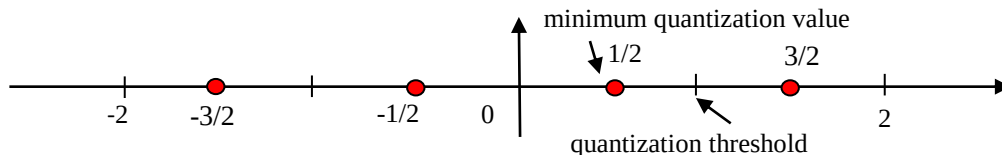
(b) The maximum quantization error is $1/64$ when the $k=7$. Hence the error power reduction is

$$10 \log \frac{(1/8)^2}{(1/64)^2} = 18 \text{dB} .$$

Q6) To use μ -law formula, the system should be first normalized into the amplitude range $(-1V, +1V)$. In this normalized μ -law system, the minimum quantization value $|x| = Q_{\min}/2 = 1/15$. Substitute $|x|$ into the μ -law formula, then we have the minimum quantization value of the normalized uniform quantization

$$|Q_{\min, \text{uniform}}| = \frac{\log(1 + \mu |Q_{\mu\text{-law}}|)}{\log(1 + \mu)} = \frac{1}{4}$$

Therefore, the minimum quantization value of the uniform quantizer is $1/4$ for the normalized range $(-1V, +1V)$. The quantization (without normalization) value is $Q_{\min-u} = 2 * 1/4 = 1/2$ for the original range $(-2V, +2V)$. See the figure below. The bit number required is 2.



Q7) You may skip this question if you find it difficult.

(a) Consider $f_k = x_{k-1}$. We have $\Delta_k = x_k - x_{k-1}$.

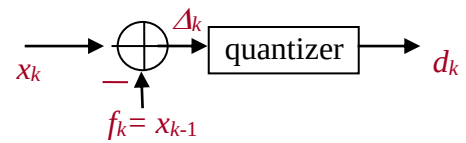
After quantization, a quantization error ε_k is produced, so

$$d_k = \Delta_k + \varepsilon_k .$$

The receiver restores the original signal using

$$\sum_{i=1}^k d_i = \sum_{i=1}^k \Delta_i + \sum_{i=1}^k \varepsilon_i = x_k + \sum_{i=1}^k \varepsilon_i$$

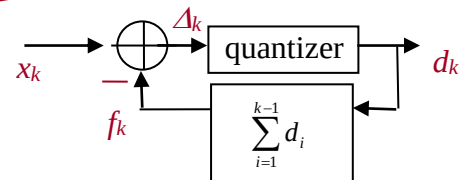
Here $\sum \varepsilon_i$ represents the accumulated error. Such accumulation generally enhances error effect.



(b) Now consider $f_k = \sum_{i=1}^{k-1} d_i$ and $d_k = \Delta_k + \varepsilon_k$. We have $d_k = (x_k - f_k) + \varepsilon_k = x_k - \sum_{i=1}^{k-1} d_i + \varepsilon_k$. The receiver still restores the original signal using

$$\sum_{i=1}^k d_i = x_k + \varepsilon_k$$

Clearly, we can avoid error accumulation in this way.



(c) The main advantage of DPCM is that it is more efficient for signals with very slow change (the lowpass signals). Such signals have smaller dynamic range. Using DPCM can reduce the number of bits required for a given QSNR, or equivalently increase QSNR for a given number of quantization bits.

Q8) (a) The maximum quantization error is $\pm a/2$, $\frac{a/2}{A} \leq 1\% \Rightarrow \frac{A}{a} \geq 50$ (Recall that $n=A/a$.)

Number of bits required to quantize this signal is $2^k \geq 50 \Rightarrow k \geq 5.64$, i.e., $k = 6$ bits.

(b) The minimum sampling rate is $2f_m$, i.e., 6 k samples/second.

(c) For BPSK modulation, the symbol rate $f_{BPSK} = \text{data rate} = 36$ k.

Thus, the bandwidth of the transmitted signal is $B_1 = 2f_{BPSK} = 72$ kHz.

(d) For 8PSK modulation, the symbol rate $f_{8PSK} = \text{data rate}/3 = 12$ k.

Thus, the bandwidth of the transmitted signal is $B_2 = 2f_{8PSK} = 24$ kHz.

Q9) (a) sampling rate = 8kHz. Quantization bit number = $32/8=4$.

(b) Symbol rate = 32k symbols per second. Transmission bandwidth = $32k \times 2 = 64$ kHz.

(c) Sampling rate and quantization bits unchanged. Symbol rate = 16k symbols per second. Transmission bandwidth = $16k \times 2 = 32$ kHz.

Q10) (a) Nyquist rate is 5kHz.

(b) Since each sample is quantized into 256 levels, the number of digits for representation of each quantized sample is 8. Therefore, we have the bit rate $R = 8 \times 5000 = 40$ k bit/sec.

(c) Since a baseband bipolar waveform scheme is used, each digit is mapped to a bipolar pulse. Thus the time duration for one rectangular bipolar waveform is $T = 1/R = 2.5 \times 10^{-5}$ sec. Thus, the required bandwidth is $W = 1/T = 40$ kHz.

Q11) (a) $n = 2^k = 4$, so $k = 2$ and quantization SNR = $n^2 = 16 = 12$ dB

(b) $B = 5$ kHz, so the minimum sampling rate is $R_s = 2B = 10$ kHz. The minimum bit rate of the PCM output signal is $kR_s = 20$ kHz. For BPSK, the required bandwidth is $2kR_s = 40$ kHz. Then, the required bandwidth of QPSK modulation is half of that of BPSK, i.e., $2kR_s/2 = 20$ kHz.

(c) $(-6/4, -2/4, +2/4, +6/4)$.

Q12) (a) Non-uniform quantization uses smaller intervals for signals with small amplitudes and vice versa. It reduces the dynamic range of the quantization SNR and average quantization SNR.

(b) $k = 4$, so $n = 2^k = 16$ and the quantization step of uniform quantization is

$$a = [1 - (-1)]/n = 2/n = 1/8.$$

The maximum amplitude of the quantized value is $y = 1 - a/2 = 15/16$

As a consequence, the corresponding maximum amplitude of the quantized value with μ -law quantization x is given by

$$|y| = \frac{\log_2(1 + \mu |x|)}{\log_2(1 + \mu)}$$

$$|x| = \frac{2^{(|y|\log_2(1+\mu))} - 1}{\mu} = \frac{(1 + \mu)^{|y|} - 1}{\mu} = \frac{(1 + 10)^{\frac{15}{16}} - 1}{10} = 0.8469$$

so $x = 0.8469$.

(c) 3×0.8469 .

Q13) (a) From a tutorial solution in Chapter 3, average signal power = variance of a uniform random variable over $[-0.5A, +0.5A]$ = by $A^2/12$. (Notice that mean = 0 here.) (b) Quantization error is uniformly distributed over $[-0.5A/n, +0.5A/n]$. Thus average error power = $A^2/(12n^2)$. (c) Average signal to quantization error power ratio = n^2 (same as that for the maximum error case).