

Circuits II Outline

In Circuits I, we looked at the basic quantities in EE, definitions, terms and laws that allow us to analyze circuits.

In Circuits II, we will apply these concepts and laws to develop two powerful techniques to analyze circuits:

- Nodal (voltage) analysis
- Mesh (current) analysis

At the end, we will introduce source transformation as another powerful tool to simplify circuits.

Nodal analysis

Nodal analysis applies Ohm's law and KCL.

It uses node voltages as circuit variables (reduces number of variables to solve compared to using the voltage across elements)

Less variables means less equations to solve

Steps involved:

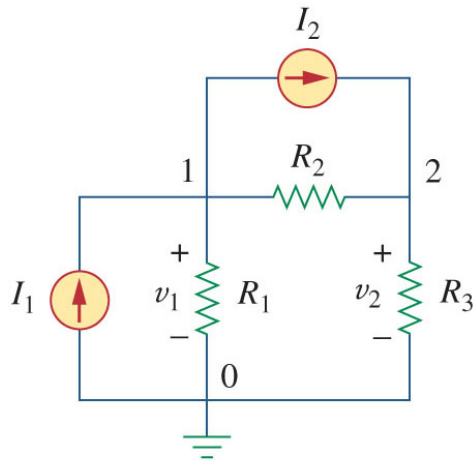
- 1) Select a convenient node as the reference node
- 2) Apply KCL to each of unknown voltage node to create equations
- 3) Solve the system of simultaneous equations (if there are more than one variable to solve)

Recall convention: $I_{AB} = \frac{V_A - V_B}{R_{AB}}$ I_{AB} : Current flowing from A → B through the resistor R_{AB} that is between A and B

Nodal analysis applies KCL to find unknown voltages

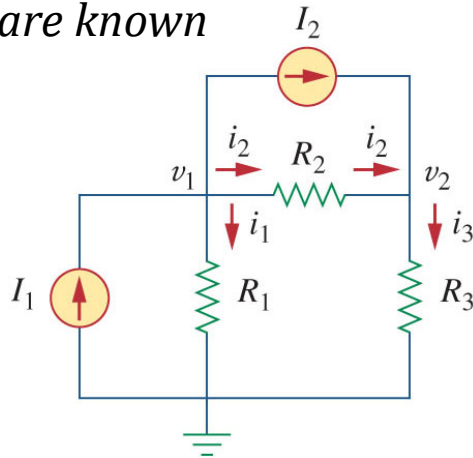
Nodal analysis

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(a)

I_1 and I_2 are known



(b)

In the example on the left, there are two unknown nodes that we need to solve.

These are label node 1 and 2.

The reference node (set to ground) has already been selected here.

We will write the node equations from a different view and show that they are the same:

First view: Assume directions of currents

Apply KCL at node 1: $I_1 - i_2 - i_1 - I_2 = 0$

Apply KCL at node 2: $i_2 + I_2 - i_3 = 0$

Nodal analysis

We then re-express the currents in terms of node voltages and resistors

$$i_1 = \frac{v_1 - 0}{R_1} \quad i_2 = \frac{v_1 - v_2}{R_2} \quad i_3 = \frac{v_2 - 0}{R_3}$$

Substitute these equations back into each node equation:

$$I_1 = I_2 + \frac{v_1}{R_1} + \frac{v_1 - v_2}{R_2} \quad \text{Node 1 (} v_1 \text{)}$$

$$I_2 + \frac{v_1 - v_2}{R_2} = \frac{v_2}{R_3} \quad \text{Node 2 (} v_2 \text{)}$$

When arranged on the same side of the equation, the defining node voltage variables must share the same sign.

Nodal analysis

But we do not have to first define current directions when applying nodal analysis. We can simply apply KCL directly to each node and write down node equations in terms of node voltages and resistors.

$$\text{Node 1 (} v_1 \text{)} \quad \frac{\boxed{} - v_1}{R_1} + \frac{\boxed{} - v_1}{R_2} \boxed{} I_1 \boxed{} I_2 = 0$$

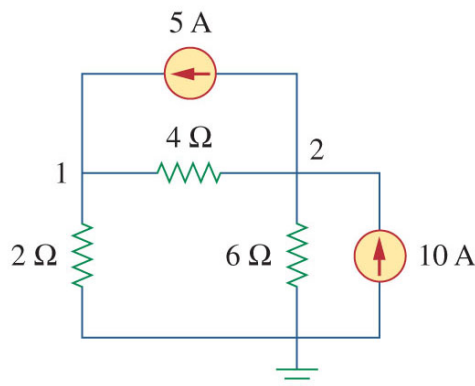
$$\text{Node 2 (} v_2 \text{)} \quad \frac{\boxed{} - v_2}{R_2} + \frac{\boxed{} - v_2}{R_3} \boxed{} I_2 = 0$$

Compare and see they are the same as the previous slide

Always count the number of terms to check if it corresponds to the number of branches around that particular node

Nodal analysis: Example 1

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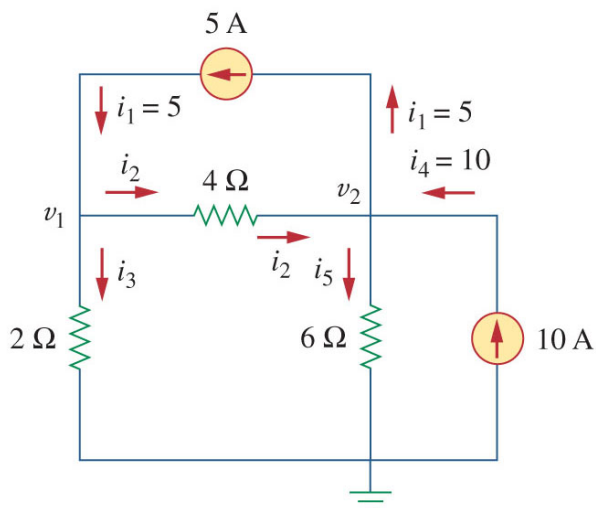


(a)

Write down all node equations

Node 1:

$$\frac{\boxed{} - v_1}{2} + \frac{\boxed{} - v_1}{4} \boxed{} 5 = 0$$



(b)

Node 2:

$$\frac{\boxed{} - v_2}{6} + \frac{\boxed{} - v_2}{4} \boxed{} 5 \boxed{} 10 = 0$$

Once you have solved for all node voltages, we can use these to solve for all branch currents

System of equations

The most critical step is obtaining the node questions correctly

Once you have the derived the node equations, you can rearrange the variables to obtain a system of equations

Based on the previous example, the system of equations for each node are:

$$\begin{array}{lcl} \text{Node 1:} & 3v_1 - v_2 = 20 & \\ \text{Node 2:} & -3v_1 + 5v_2 = 60 & \end{array} \left. \vphantom{\begin{array}{l} \\ \end{array}} \right\} \begin{array}{l} \text{Method 1: Solve by elimination} \\ v_1 = 40/3 = 13.3\text{V}, v_2 = 20\text{V} \end{array}$$

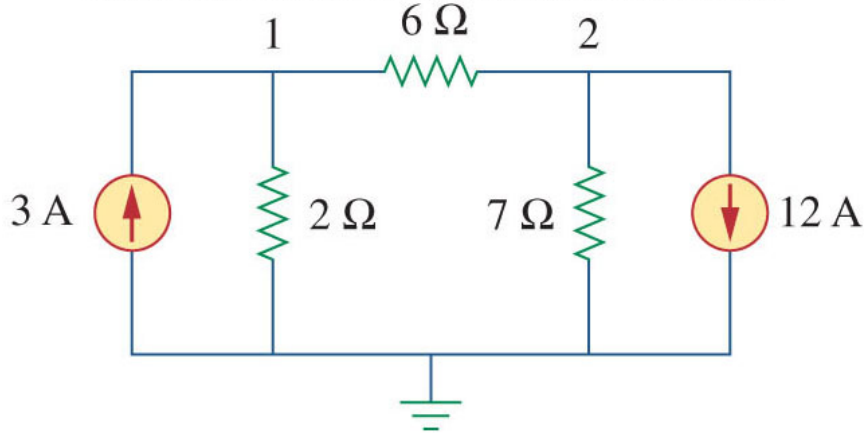
Method 2: Put the system of equations into matrix form

$$\begin{bmatrix} 3 & -1 \\ -3 & 5 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 60 \end{bmatrix}$$

Method 2 is preferred when there are large number of variables to solve
Computers are very good at solving matrices

Nodal analysis: Example 2

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Write down all node equations

Node 1:

$$\frac{\boxed{} - v_1}{2} + \frac{\boxed{} - v_1}{6} \boxed{} 3 = 0$$

Node 2:

$$\frac{\boxed{} - v_2}{6} + \frac{\boxed{} - v_2}{7} \boxed{} 12 = 0$$

System of equations

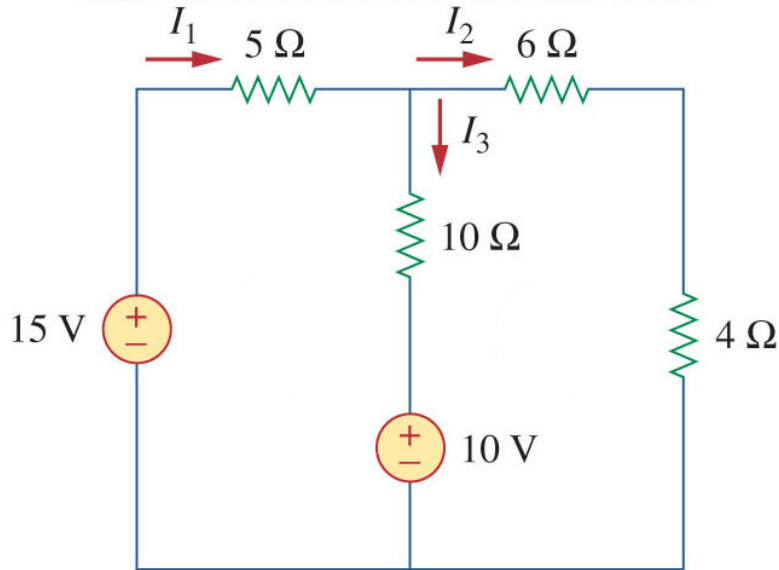
$$\text{Node 1: } 4v_1 - v_2 = 18$$

$$\text{Node 2: } 7v_1 - 13v_2 = 504$$

$$\left. \begin{array}{l} 4v_1 - v_2 = 18 \\ 7v_1 - 13v_2 = 504 \end{array} \right\} v_1 = -6\text{V}, v_2 = -42\text{V}$$

Nodal analysis: Example 3

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Write down all node equations

First: Select a convenient node as the reference node

Node 1:

$$\frac{\boxed{} - v_1}{5} + \frac{\boxed{} - v_1}{6 + 4} + \frac{\boxed{} - v_1}{10} = 0$$

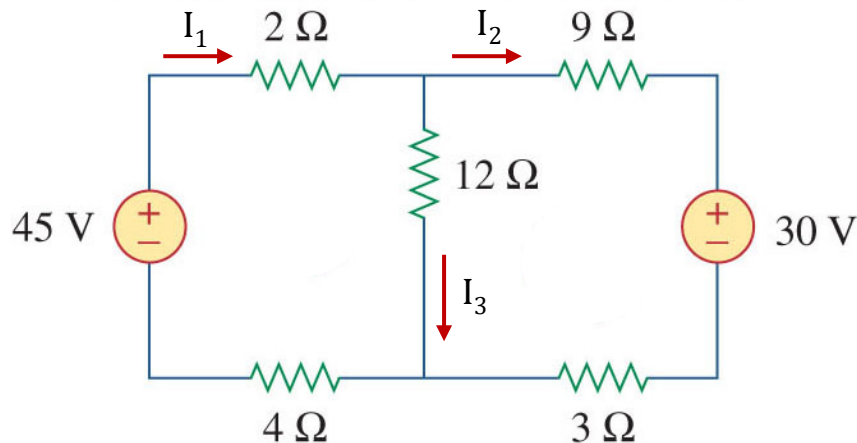
Rearranging terms: $v_1 = 10\text{V}$

Once v_1 is known, v_1 can be used to find all branch currents:

$$I_1 = 1\text{A}, I_2 = 1\text{A}, I_3 = 0\text{A}$$

Nodal analysis: example 4

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Write down all node equations

First: Select a convenient node as the reference node

Node 1:

$$\frac{\boxed{} - v_1}{12} + \frac{\boxed{} - v_1}{2 + 4} + \frac{\boxed{} - v_1}{9 + 3} = 0$$

Rearranging terms: $v_1 = 30\text{V}$

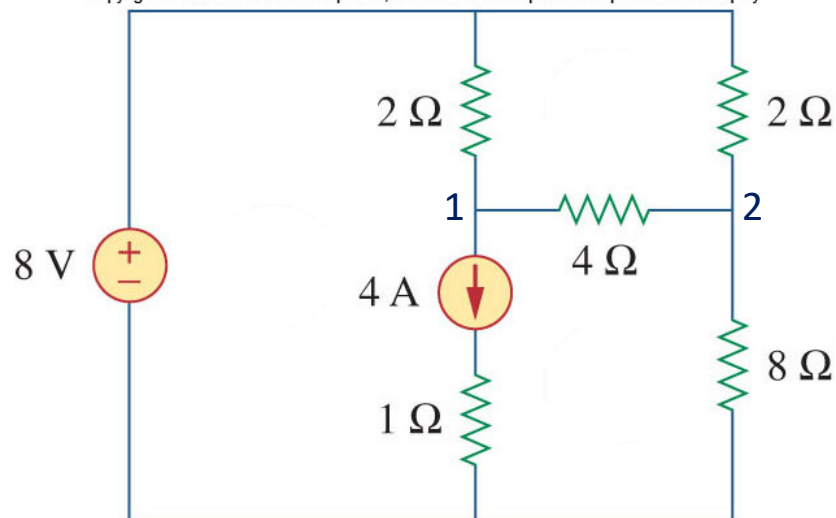
Once v_1 is known, v_1 can be used to find all branch currents:

$$I_1 = 2.5\text{A}, I_2 = 0\text{A}, I_3 = 2.5\text{A}$$

Sequence of elements in the same branch can be rearranged

Nodal analysis: Example 5

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Write down all node equations

First: Select a convenient node as the reference node

Node 1:

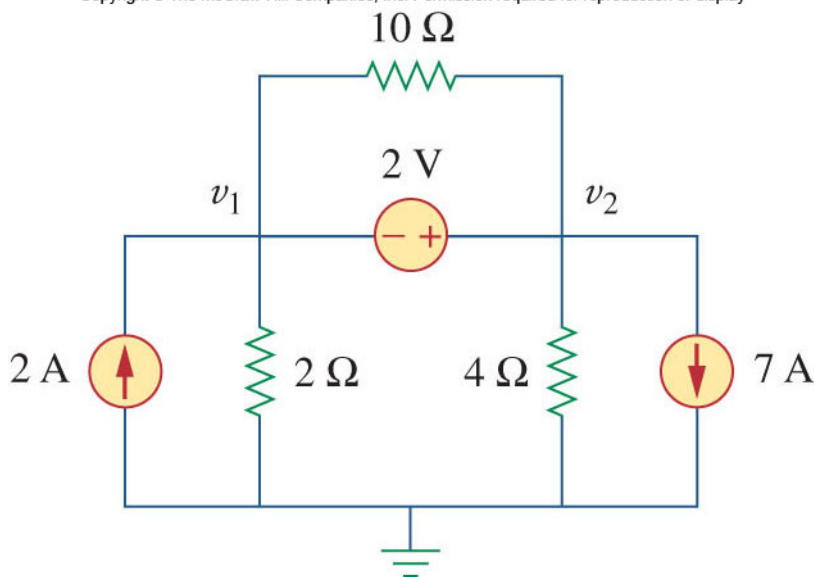
$$\frac{\boxed{} - v_1}{2} + \frac{\boxed{} - v_1}{4} \boxed{} \boxed{} = 0$$

Node 2:

$$\frac{\boxed{} - v_2}{2} + \frac{\boxed{} - v_2}{4} + \frac{\boxed{} - v_2}{8} = 0$$

Nodal analysis: Example 6

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Write down all node equations

Node 1:

$$\frac{\boxed{} - v_1}{2} + \frac{\boxed{} - v_1}{10} \boxed{} 2 \boxed{} \boxed{} = 0$$

Node 2:

$$\frac{\boxed{} - v_2}{10} + \frac{\boxed{} - v_2}{4} \boxed{} 7 \boxed{} \boxed{} = 0$$

$$\text{Node 1: } \frac{-v_1}{2} + \frac{2}{10} + 2 = I$$

$$\text{Node 2: } \frac{-2}{10} + \frac{-(v_1+2)}{4} - 7 + I = 0 \quad \left. \vphantom{\frac{-2}{10}} \right\} \text{Sub. } V_2 - V_1 = 2$$

Mesh analysis

Mesh analysis applies Ohm's law and KVL

It uses mesh currents as circuit variables (reduces number of variables to solve compared to using branch currents in every element)

Recall: Loop is a closed path with no node passed more than once

Recall: Mesh is a loop that does not contain other loops

Steps involved:

- 1) Assign mesh currents
- 2) Apply KVL to each mesh (using Ohm's law to express of the voltages in terms of mesh currents)
- 3) Solve the system of simultaneous equations (if there are more than one variable to solve)

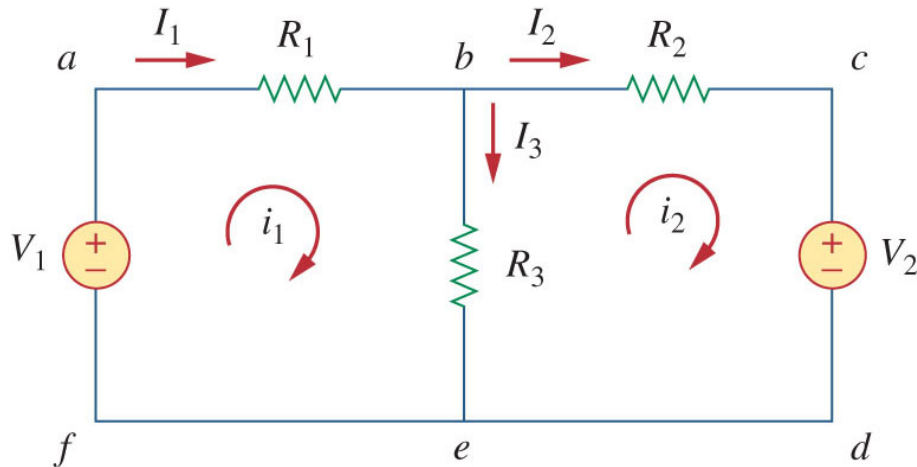
Mesh analysis applies KVL to find unknown currents

Mesh analysis

In writing the mesh equations, we ignore the branch currents

When we have solved for the mesh current we will use the mesh currents to obtain the branch currents

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Mesh currents merge the branch currents to minimize number of variables

3 branch currents VS 2 mesh currents

$$I_1 = i_1$$

$$I_2 = i_2$$

$$I_3 = i_1 - i_2$$

Suggested approach in summing voltages around a mesh when applying KVL:

Follow the direction of the mesh current

Element with voltage increase: use + sign

Element with voltage drop: use - sign

Mesh analysis

Apply KVL to mesh 1 (CW)

$f \rightarrow a \rightarrow b \rightarrow e \rightarrow f$

$$\boxed{} V_1 - R_1 i_1 - R_3 \left(i_1 \boxed{} \boxed{} \right) = 0$$

Voltage drop across R_3

Voltage drop across R_1

Apply KVL to mesh 2 (CW)

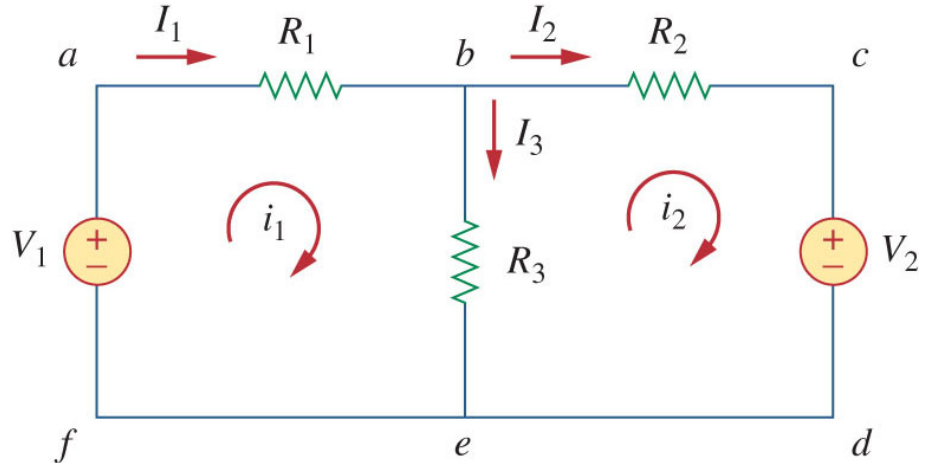
$e \rightarrow b \rightarrow c \rightarrow d \rightarrow e$

$$-R_3 \left(i_2 \boxed{} \boxed{} \right) - R_2 i_2 \boxed{} V_2 = 0$$

Voltage drop across R_2

Voltage drop across R_3

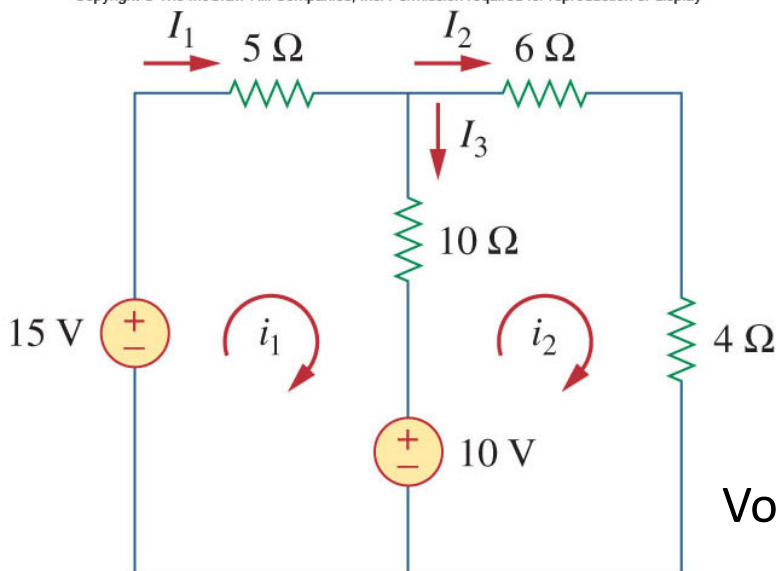
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Always count and check that you have the correct number of terms in each equations (must be equal to number of elements in the mesh)

Mesh analysis: Example 1

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This the same circuit as Example 3 (nodal analysis) but this time we apply mesh analysis

Mesh 1 (CW):

$$-5i_1 - 10 \left(i_1 \boxed{} \boxed{} \right) \boxed{} 10 \boxed{} 15 = 0$$

Voltage drop across 5Ω Voltage drop across 10Ω

Mesh 2 (CW):

$$-10 \left(i_2 \boxed{} \boxed{} \right) - 6i_2 - 4i_2 \boxed{} 10 = 0$$

Voltage drop across 10Ω

Voltage drop across 6Ω and 4Ω

Notice the sign of the 10V source term in each of the mesh equations

Mesh analysis: Example 1

Rearrange terms in each mesh equation:

$$\begin{array}{lcl} \text{Mesh 1 (CW):} & 3i_1 - 2i_2 = 1 & \\ \text{Mesh 2 (CW):} & -i_1 + 2i_2 = 1 & \end{array} \quad \left. \vphantom{\begin{array}{l} \\ \end{array}} \right\} \begin{array}{l} i_1 = 1\text{A} \\ i_2 = 1\text{A} \end{array}$$

In matrix form: $\begin{bmatrix} 3 & -2 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

In terms of branch currents:

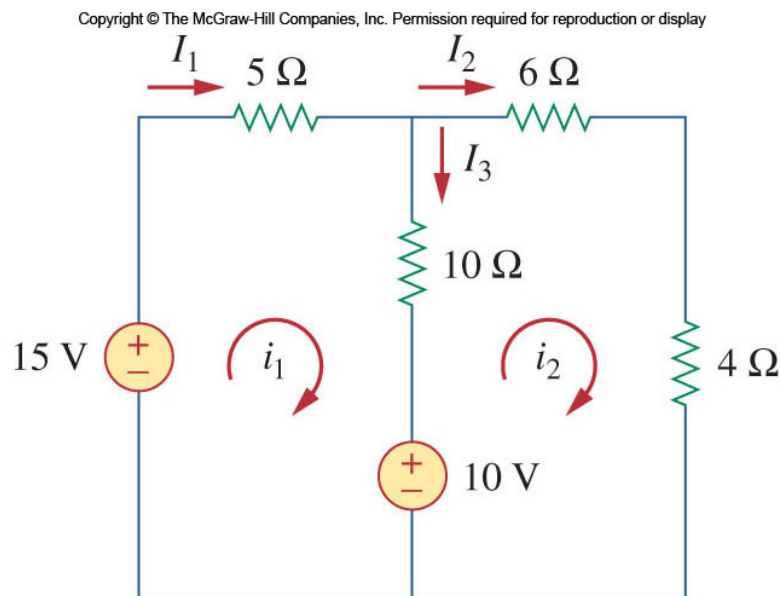
$$I_1 = i_1, I_2 = i_2, I_3 = i_1 - i_2$$

Solving:

$$I_1 = 1\text{A}, I_2 = 1\text{A}, I_3 = 0\text{A}$$

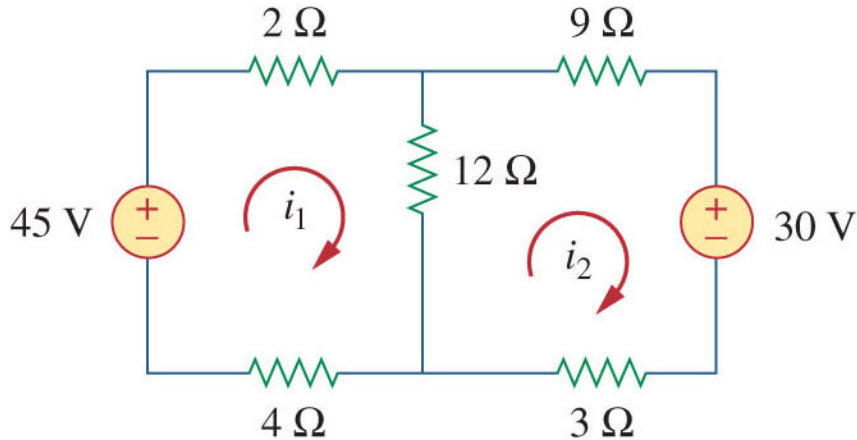
Compare with solution by nodal analysis

Compare with nodal analysis the number of variables involved



Mesh analysis: Example 2

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This the same circuit as Example 4 (nodal analysis) but this time we apply mesh analysis

Compare with nodal analysis the number of variables involved

$$\text{Mesh 1 (CW): } -2i_1 - 12 \left(i_1 \square \square \right) - 4i_1 \square 45 = 0$$

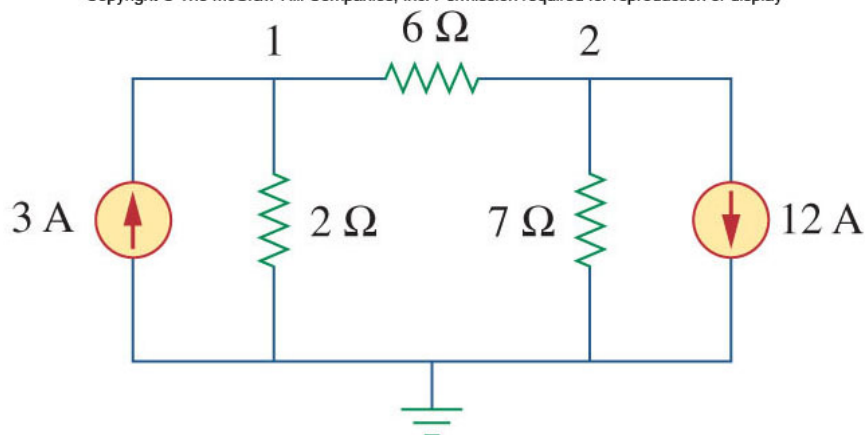
$$\text{Mesh 2 (CW): } -3i_2 - 12 \left(i_2 \square \square \right) - 9i_1 \square 30 = 0$$

$$i_1 = 2.5\text{A}$$

$$i_2 = 0\text{A}$$

Mesh analysis: Example 3

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This the same circuit as Example 2 (nodal analysis) but this time we apply mesh analysis

There are 3 meshes in this circuit, but only the mesh current in the middle is unknown

Mesh currents and the left and right are defined by the current sources:

$$i_1 = 3A, i_3 = 12A$$

In this case where the mesh currents are not defined in the question, we choose to define the direction of i_1 and i_3 according the current sources

The only unknown mesh current to solve for is i_2

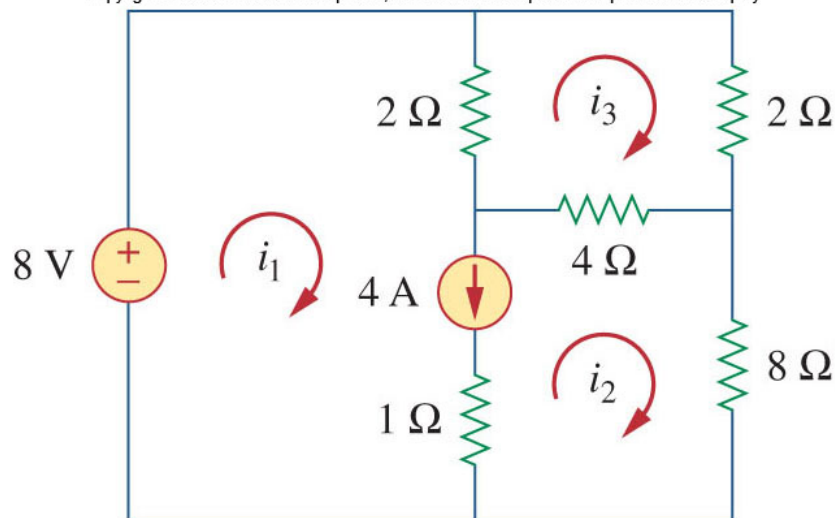
$$\text{Mesh 2: } -6i_2 - 7(i_2 \square 12) - 2(i_2 \square 3) = 0$$

$$i_2 = 6A$$

Compare with nodal analysis the number of variables involved

Mesh analysis: Example 4

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This the same circuit as Example 5 (nodal analysis) but this time we apply mesh analysis

Mesh 3:

$$-2i_3 - 4 \left(i_3 \square \square \right) - 2 \left(i_3 \square \square \right) = 0$$

$$\text{Mesh 1: } -2 \left(i_1 \square \square \right) \square \square - 1 \left(i_1 \square \square \right) \square 8 = 0$$

$$\text{Mesh 2: } -4 \left(i_2 \square \square \right) - 8i_2 - 1 \left(i_2 \square \square \right) \square \square = 0$$

Mesh analysis: Example 4

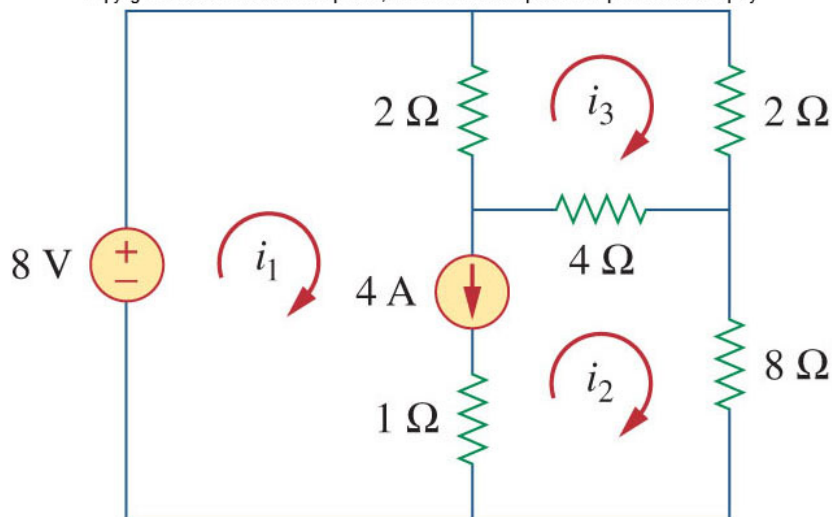
Super mesh

We have seen that the 4A current source introduces yet another unknown variable (the voltage across the 4A source)

To avoid creating another variable into our system of equations, we can combine the two meshes where the current source intersects

In this case, mesh 1 and 2 are merged to create a super mesh

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$$8 - 2(i_1 - i_2) - 4(i_2 - i_3) - 8i_2 = 0$$

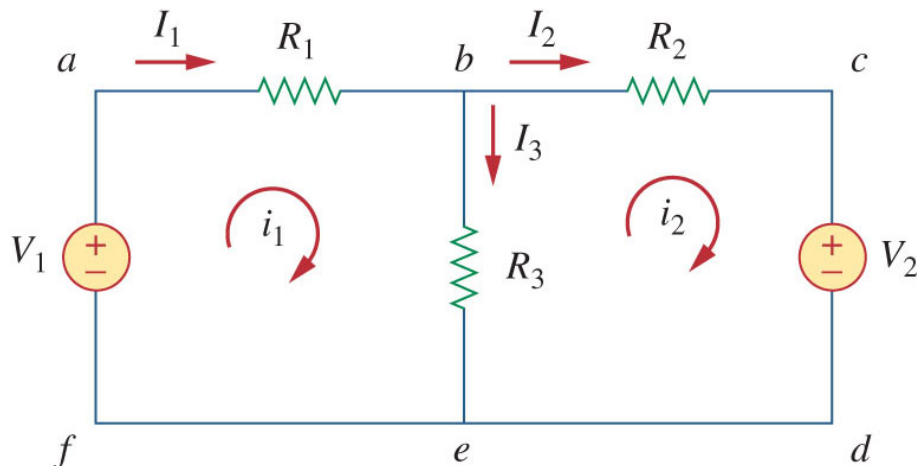
We have thus bypassed 4A and 1Ω

By KVL, the two paths have the same voltage difference

Mesh analysis: By inspection

Master the basics of mesh analysis before you try this!

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Recall the basic equations:

Mesh 1:

$$V_1 - R_1 i_1 - R_3 (i_1 - i_2) = 0$$

Mesh 2:

$$-R_3 (i_2 - i_1) - R_2 i_2 - V_2 = 0$$

Group variables and rearrange

$$(R_1 + R_3) i_1 - R_3 i_2 = V_1$$

$$(R_3 + R_2) i_2 - R_3 i_1 = -V_2$$

Add up in series resistors in the mesh

For resistors with neighboring mesh currents, using minus if opposite in direction or plus if same direction

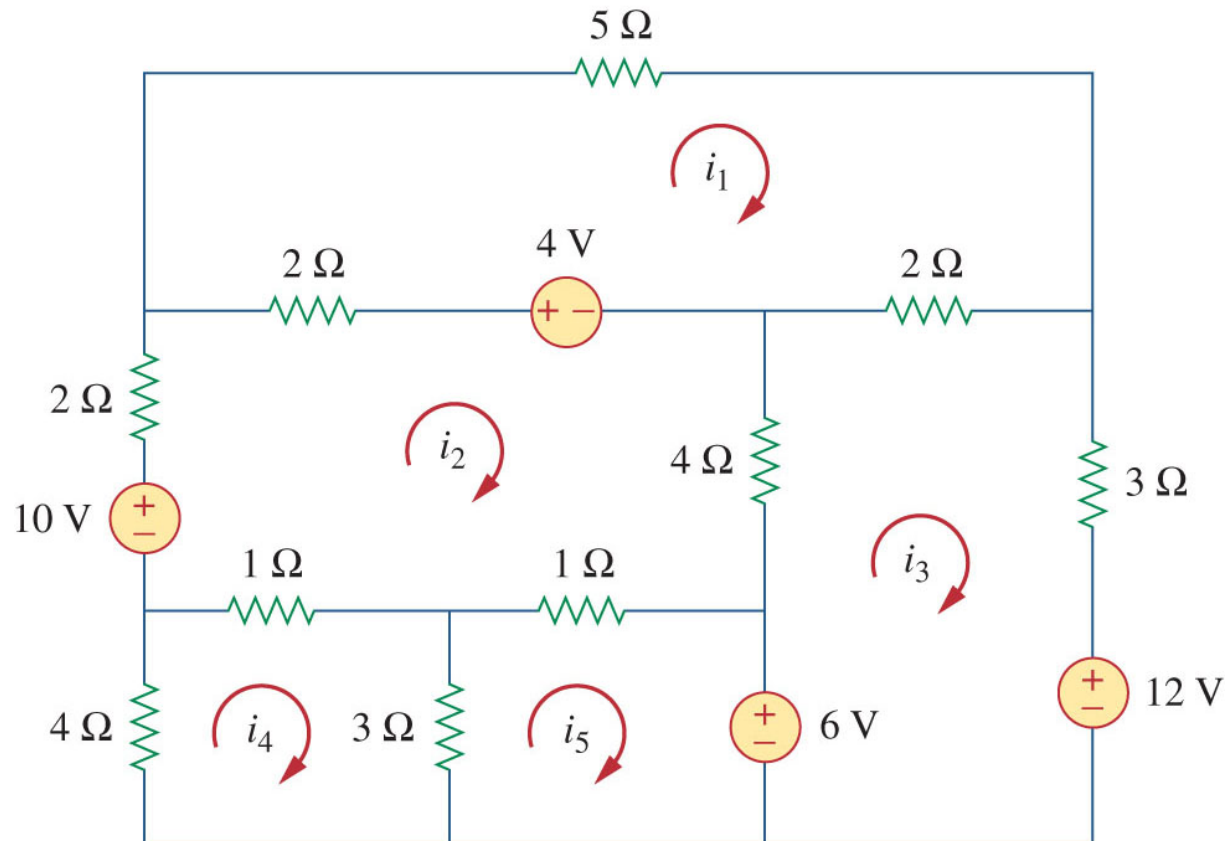
Voltage sources: use plus for voltage rise and use minus for voltage drop

Mesh analysis by inspection example

Try deriving mesh equations by inspection on the previous examples and then try this

Derive all 5 mesh equations

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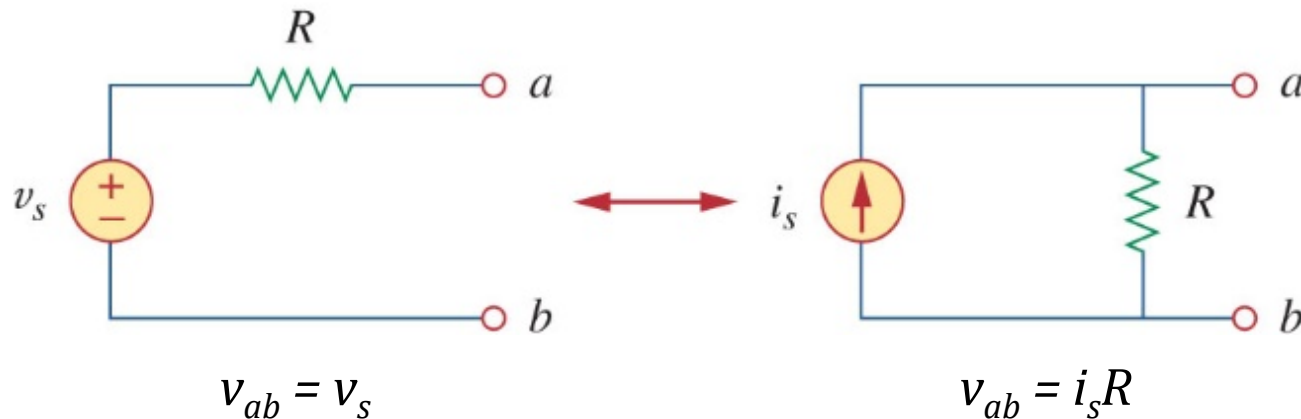


Source transformation

This technique involves substituting a voltage source in series with a resistor for a current source in parallel with a resistor and vice versa

This allows us to simplify the circuit to reduce the number of variables in our system of equations (where possible). Sometimes, it is possible to reduce the circuit down to one variable

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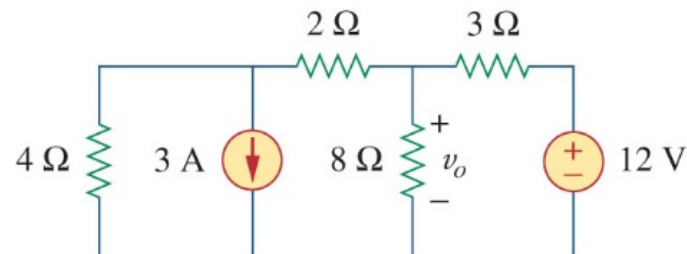


Equivalence: $v_s = i_s R$

Source transformation

We will apply source transformation to the circuit on the right to see how useful the method can be in simplifying our analysis to find v_o .

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First (see (a)): Transform

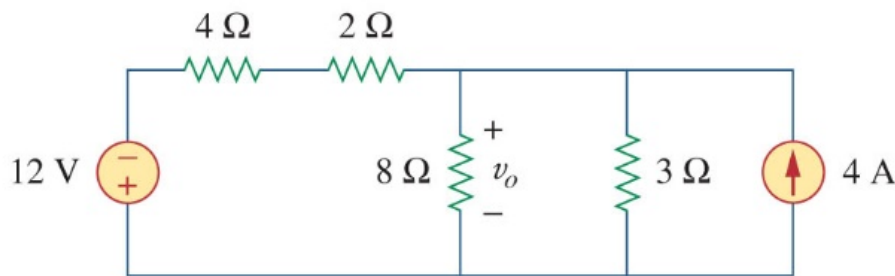
$3A \parallel 4\Omega \rightarrow 12V$ series 4Ω

$(3A \cdot 4\Omega = 12V)$

$12V \parallel 3\Omega \rightarrow 4A$ 3Ω

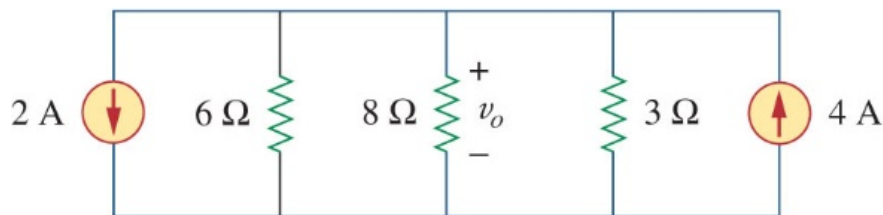
$(12V/3\Omega = 4A)$

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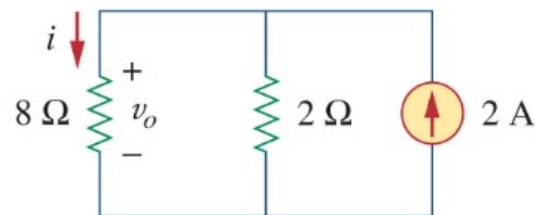


(a)

Note that 4Ω now appears in series with 2Ω in (a)



(b)



(c)

Source transformation

4Ω and 2Ω combined = 6Ω

Next: Transform

12V series $6\Omega = 2\text{A} \parallel 6\Omega$

$(12\text{V}/6\Omega = 2\text{A})$

Combine:

Parallel current sources

Parallel resistors

See (c)

Notice now how all the elements have become parallel to each other in (b)

No simultaneous equations required

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