

EE3101 Tutorial 3 Solutions

(1)

(a) Assume that the input is: $A \sin(\omega t) \rightarrow$ signal output power is: $A^2/2$

(b) Assume that the double-sided power spectral density of the Gaussian noise is: $N_0/2$;

$$\text{Signal O/P} = \frac{A \angle 0 \frac{1}{j\omega_0 C}}{R + \frac{1}{j\omega_0 C}}$$

$$\text{or Signal O/P} = \frac{A \sin(\omega_0 t + \theta)}{(1 + \omega_0^2 R^2 C^2)^{1/2}} = A' \sin(\omega_0 t + \theta) \quad \text{where } A' = \frac{A}{(1 + \omega_0^2 R^2 C^2)^{1/2}}$$

$$\text{Signal O/P Power} = \frac{A^2 / 2}{1 + \omega_0^2 R^2 C^2}$$

$$H(\omega) = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{1 + j\omega RC};$$

$$\text{Noise O/P Power} = \frac{N_0}{2} \int_{-\infty}^{\infty} |H(\omega)|^2 d\omega = \frac{\pi N_0}{2RC}$$

$$S/N = \frac{\frac{A^2 / 2}{1 + \omega_0^2 R^2 C^2}}{\frac{\pi N_0}{2RC}} = \frac{A^2 \tau}{\pi N_0 (1 + \omega_0^2 \tau^2)} \quad \text{where } \tau = RC$$

$$(S/N) = \max, \implies \frac{d(S/N)}{d\tau} = 0$$

$$\text{i.e. } \frac{\pi N_0 (1 + \omega_0^2 \tau^2)(A^2) - (A^2 \tau)(2\pi N_0 \omega_0^2 \tau)}{[\pi N_0 (1 + \omega_0^2 \tau^2)]^2} = 0$$

$$(1 + \omega_0^2 \tau^2) - 2\omega_0^2 \tau^2 = 0$$

$$\implies \omega_0 = 1/\tau; \text{ or } RC = 1/\omega_0$$

(2)

$$(a) Q(4.753) = 10^{-6}$$

$$\rightarrow \sqrt{\frac{2E_b}{N_0}} = 4.753, \quad \frac{E_b}{N_0} = 11.296$$

$$\text{if transmission power is doubled, } \rightarrow \frac{E_b}{N_0} = 11.296 \times 2 = 22.592$$

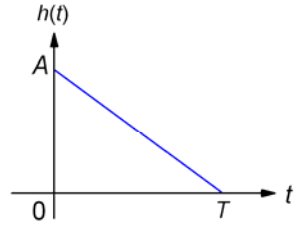
Prob of error is:

$$Q(\sqrt{2 \times 22.592}) = Q(6.722) = \frac{\exp(-6.722^2 / 2)}{6.722 \sqrt{2\pi}} \left(1 - \frac{1}{6.722^2}\right) = 8.95 \times 10^{-12}$$

Please note that the Prob of error is reduced by a factor of $\sim 10^5$ by simply doubling the transmission power.

$$(b) \text{ The prob of error for Unipolar NRZ is } Q\left(\sqrt{\frac{E_b}{N_0}}\right) = Q(3.361) = 3.8 \times 10^{-4}$$

(3) (a) The impulse response of the matched filter



Note: "A" can be replaced by any non-zero constant value k,

$$(b) \quad h(t) = A\left(1 - \frac{t}{T}\right) \quad 0 < t \leq T$$

$$= 0 \quad \text{other } t \text{ value}$$

Matched Filter O/P $z(t) = [s(t) + n(t)] \otimes h(t)$

$$= \int_0^t s(\tau) h(t - \tau) d\tau + \int_0^t n(\tau) h(t - \tau) d\tau$$

For $0 < t \leq T$, the signal component at the matched filter output is:

$$\frac{A^2}{T} \int_0^t \tau \left[1 - \frac{t-\tau}{T}\right] d\tau = \frac{A^2}{T} \int_0^t \left[\tau \left(1 - \frac{t}{T}\right) + \frac{\tau^2}{T}\right] d\tau$$

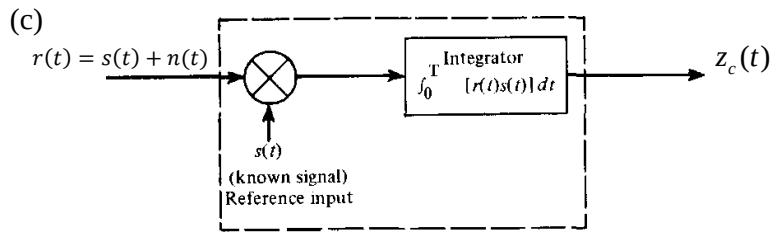
$$= \frac{A^2}{T} \left[\left(1 - \frac{t}{T}\right) \frac{\tau^2}{2} \Big|_0^t + \frac{\tau^3}{3T} \Big|_0^t \right] = \frac{A^2}{T} \left[\left(1 - \frac{t}{T}\right) \frac{t^2}{2} + \frac{t^3}{3T} \right] = \frac{A^2 t^2}{2T} \left[1 - \frac{t}{3T}\right]$$

i.e. $B = \frac{A^2}{2T}$

[Note: The value of B will be different if a different voltage level is used in $h(t)$]

For $0 < t \leq T$, the noise component at the matched filter output is:

$$\int_0^t n(\tau) A \left[1 - \frac{t-\tau}{T}\right] d\tau$$



$$z_c(t) = \int_0^t \left(\frac{A\tau}{T}\right)^2 d\tau + \int_0^t n(\tau) \frac{A}{T} \tau d\tau = \left(\frac{A}{T}\right)^2 \frac{t^3}{3} + \frac{A}{T} \int_0^t n(\tau) \tau d\tau$$

$$(d) \quad z(T) = \frac{A^2 T}{3} + \frac{A}{T} \int_0^T n(\tau) \tau d\tau$$

$$z_c(T) = \frac{A^2 T}{3} + \frac{A}{T} \int_0^T n(t) \tau d\tau$$

Ignore the noise component; the following are the plots of the Matched filter output and the Correlator output

