

Homework 3

Due: **Wednesday, 6 November**

IMPORTANT NOTE: Please submit your assignment online through Canvas by the due date or deadline. You may write your answers in MS Word or use any other processor (e.g., LaTeX). However, the final file you upload on Canvas must be a pdf document. It is also fine to write your answers on paper and then scan them or take a high-quality image using your camera/phone and create a PDF document to upload on Canvas.

Other Notes: *Please note that the questions are based on the material covered in Lecture 6. There are five questions in this assignment. You must show all steps to get full credit for an answer.*

Question 1: Consider the following function

$$f(x, y) = y^{\sqrt{x}}$$

(a) (**10 points**) Show that $\frac{\partial f}{\partial x} = \left(\frac{\ln y}{2\sqrt{x}}\right)y^{\sqrt{x}}$

(b) (**10 points**) Show that $\frac{\partial f}{\partial y} = \sqrt{x}y^{\sqrt{x}-1}$

Question 2: Consider the following equation

$$g(x, y) = e^{xy} + 2(x + y) = 5$$

(a) (**15 points**) Find $\frac{dy}{dx}$.

(b) (**5 points**) Find the derivative of $y(x)$ when $x = 0$.

Question 3: (**20 points**) Find the degree of homogeneity of the following function

$$h(x, y) = x(x^2 + xy)^{1/5} + y^{3/5}x^{4/5}$$

Question 4: (**20 points**) Find and classify (i.e., determine the nature of) the critical points of the function

$$f(x, y) = x^2 - 2xy + 2y^2 - 2y + 2$$

Question 5: (*20 points*) A monopoly manufactures two goods, X and Y , with demand functions

$$x = 12 - p^X \qquad y = 18 - p^Y$$

The firm's cost function is $C(x, y) = x^2 + y^2 + 2xy$. Find the maximum profit achievable, and the quantities produced of each of X and Y in order to achieve this.