EF3442 Intermediate Microeconomics

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To Succeed at EF3442

1. Read the syllabus.
2. Ask questions. Make use of the fact that the class is 'live'.
3. Start on the homeworks early.
4. Do not substitute pattern matching for thinking.
5. Make use of office hours.

Simple Selling Exercise

Seller: Sell the bag for as much as possible.

You are limited to a posted price.

Write a price and your name down on paper and pass it to the buyer.

No other communication is permitted.



Simple Selling Exercise

Buyer: You decide to accept or reject.

If reject, the end.

If you accept, you own the bag.

If you own the bag, I will buy it from you for $20.

The bag is worthless outside of this room.

Rational Buyer Model

- Reservation Price (RP) = Maximum amount a buyer is willing to pay
- SURPLUS = RP $-$Price
- Buyers choose that which gives them largest surplus

Rational Buyer Model

	Choice A	Choice B
RP	$10	$15
Price	$7	$11
Surplus	$3	$4

Rational Buyer Model

	Choice A	Choice B
RP	$10	$15
Price	$5	$11
Surplus	$5	$4

Demand Curve

Assume best alternative delivers zero surplus to everyone.

Name	Mr. A	Mr. B	Mr. C	Mr. D	Mr. E
RP	$10	$15	$12	$ 7	$ 17

Demand Curve

Assume best alternative delivers zero surplus to everyone.

Name	Mr. A	Mr. B	Mr. C	Mr. D	Mr. E
RP	\$10	\$15	\$12	\$ 7	\$ 17

Price	\$7	\$8	\$9	\$10	\$11	\$12	\$13	\$14	\$15	\$16
Demand	5	4	4	4	3	3	2	2	2	1

Demand Curve

A way to summarize how demand changes with unit price (*holding other things fixed*).

$D(p)$ is quantity demanded at unit price of p.

$D(p)$ goes down as p increases

$$\Rightarrow \frac{dD(p)}{dp} < 0$$

Demand Curve: Examples

$D(p) = 9 - 2p$ (linear demand curve)

$D(p) = \frac{10}{p^2}$

$\ln D(p) = \ln 10 - 2 \ln p$ (log-log demand curve)

$D(p) = 5e^{-2p}$

$\ln D(p) = 5 - 2p$ (semi-log demand curve)

Demand Curve: REMINDER

Consider the demand curve

$$D(p) = 9 - 2p$$

Strictly speaking should write this as $D(p) = \max\{9 - 2p, 0\}$.

Why? When $p > 4.5$, $D(p) < 0$ has no meaning.

Interpret demand at price p to be zero whenever $D(p) < 0$.

The smallest price p where $D(p) = 0$ is called the **choke price**.

Margin vs. Volume

A high price means a high margin (profit per unit), but a low likelihood of sale or low volume.

A low price means a low margin (profit per unit), but a high likelihood of sale or high volume.

Between the two is a 'sweet spot' and it is natural to ask what that sweet spot depends on.

To focus on the tradeoff between margin and volume it will be useful to ignore competition.

Margin vs. Volume

Monopolist faces a demand curve $9 - 2p$ and has unit costs of \$2 a unit.

What unit price should monopolist charge to maximize profit?

Monopolist does *not* mean absence of competition.

Short hand for assuming that competitors keep their prices fixed as you change your price.

Margin vs. Volume

Because we know demand at any price, we produce to exactly meet demand.

If set unit price of p, demand/volume will be $9 - 2p$.

Profit per unit (margin) will be $p - 2$.

Total profit, $\Pi(p)$ as a function of p will be $(p - 2)(9 - 2p)$.

Find p to maximize $(p - 2)(9 - 2p)$.

Margin vs. Volume

$$\Pi(p) = (p-2)(9-2p) = -2p^2 + 13p - 18$$

$$\frac{d\Pi(p)}{dp} = -4p + 13$$

Set derivative to zero and solve for p:

$$-4p + 13 = 0 \;\Rightarrow\; p = 3.25$$

To verify that $p = 3.25$ is profit maximizing check second derivative of profit wrt p is negative.

Elasticity of Demand

Total profit, $\Pi(p)$ as a function of p is $(p-2)(9-2p)$.

If $p < 3.25$, say $p = 3$, the monopolist can increase profits by raising price.

- As one raises price, margin increases while the volume drops.
- However, the profit per unit is rising faster than the demand is dropping.

How do we know this?

Elasticity of Demand

A measure of how sensitive demand is to a change in price.

% CHANGE in demand for an infinitesimally small % CHANGE in price.

APPROXIMATION: % CHANGE in demand for a 1 % CHANGE in price.

Elasticity of Demand

Demand as a function of unit price p is given by $D(p)$.

Demand decreases as p increases, so $\frac{dD(p)}{dp} < 0$ (also written as $D'(p) < 0$).

Elasticity of Demand is

$$-[\frac{p}{D(p)}][\frac{dD(p)}{dp}]$$

Also written as $-[\frac{p}{D(p)}]D'(p)$.

WARNING: Some books define elasticity as $[\frac{p}{D(p)}]D'(p)$, i.e., no negative sign.

Elasticity of Demand

EXAMPLE: Demand as a function of unit price p is given by $D(p) = 9 - 2p$.

Elasticity of demand is: $-[\frac{p}{D(p)}][\frac{dD(p)}{dp}]$

Now $\frac{dD(p)}{dp} = -2$, therefore elasticity of demand is $\frac{2p}{9-2p}$.

Elasticity of Demand: Derivation

1. Suppose current unit price is p. Current demand is $D(p)$.
2. Increase price by small amount h.
3. New demand is $D(p+h) < D(p)$.
4. % drop in demand is $100 \times \frac{D(p)-D(p+h)}{D(p)}$.
5. % increase in price is $100 \times \frac{(p+h)-p}{p}$.
6. Divide percentage change in demand by percentage change in price to get elasticity and let h go to zero.

Elasticity of Demand: Derivation

$$\frac{\frac{D(p)-D(p+h)}{D(p)}}{\frac{(p+h)-p}{p}} = \frac{D(p)-D(p+h)}{D(p)} \times \frac{p}{(p+h)-p} = \frac{D(p)-D(p+h)}{D(p)} \times \frac{p}{h}$$

$$= \frac{p}{D(p)} \times \frac{D(p)-D(p+h)}{h} = -\frac{p}{D(p)} \times \frac{D(p+h)-D(p)}{h}$$

Recall $\lim_{h\to 0} \frac{D(p+h)-D(p)}{h} = \frac{dD(p)}{dp}$.

Margin vs. Volume

Monopolist faces a demand curve $9 - 2p$ and has unit costs of \$2 a unit.

Price	\$2	\$2.25	\$2.5	\$2.75	\$3	\$ 3.25	\$3.5	\$3.75	\$4
Demand	5	4.5	4	3.5	3	2.5	2	1.5	1
Margin	0	0.25	0.5	0.75	1	1.25	1.5	1.75	2
Profit	0	1.125	2	2.625	3	3.125	3	2.625	2
Elasticity	0.8	1	1.25	1.8333	2	2.6	3.5	5	8

In general, if the elasticity of demand at a particular price is *smaller than 1*, it is always profitable to increase price.

Markup Formula

$D(p)$ demand curve

c is unit cost of production

$e(p)$ elasticity of demand at price p

If p^* is profit maximizing price, then,

$$\frac{p^* - c}{p^*} = \frac{1}{e(p^*)}$$

LHS = **relative markup**,
RHS = **reciprocal of the elasticity of demand**

Markup formula: Derivation

A monopolist's problem is

$$\max_p (p-c)D(p)$$

The first-order condition is

$$\begin{aligned} & D(p) + (p-c)D'(p) = 0, \\ \Rightarrow & \frac{p-c}{p} = -\frac{D(p)}{pD'(p)}, \\ \Rightarrow & \frac{p-c}{p} = \frac{1}{e(p)}. \end{aligned}$$

Revenue vs. Profit

Author is compensated based on % revenue.

Author could set price of book or publisher could set price of book.

Who would prefer a higher price for the book?

Revenue vs. Profit

Author is focused on maximizing revenue.

Publisher cares about maximizing profit.

Revenue is not the same as profit.

Revenue vs. Profit

If $c > 0$, according to markup formula publisher will choose a price p^* where $e(p^*) > 1$.

$$\frac{1}{e(p^*)} = \frac{p^* - c}{p^*} < 1 \;\Rightarrow\; e(p^*) > 1$$

Author ignores c, i.e., treats it as zero. So, author chooses p^* so

$$\frac{p^* - 0}{p^*} = \frac{1}{e(p^*)} \;\Rightarrow\; e(p^*) = 1$$

Amazon vs. Hachette

Moreover, e-books are highly price elastic. This means that when the price goes down, customers buy much more. We've quantified the price elasticity of e-books from repeated measurements across many titles. For every copy an e-book would sell at $14.99, it would sell 1.74 copies if priced at $9.99.

From Amazon letter to authors sent Summer 2014

Amazon vs. Hachette

Drop in price from \$14.99 to \$9.99 is a $\frac{14.99-9.99}{14.99} \times 100 = 33\%$ decrease.

Increase in demand from 1 to 1.74 is a 74% increase.

Estimated elasticity is $\frac{74}{33} = 2.24$

Amazon vs. Hachette

So, for example, if customers would buy 100,000 copies of a particular e-book at $14.99, then customers would buy 174,000 copies of that same e-book at $9.99. Total revenue at $14.99 would be $1,499,000. Total revenue at $9.99 is $1,738,000. The important thing to note here is that the lower price is good for all parties involved: the customer is paying 33% less and the author is getting a royalty check 16% larger and being read by an audience that's 74% larger. The pie is simply bigger.

From Amazon letter to authors sent Summer 2014

Choosing Quantity

Instead of choosing profit maximizing price we can equivalently choose profit maximizing output/volume/quantity.

EXAMPLE: Demand, D is given by the demand curve: $D = 9 - 2p$.

Inverse demand curve shows price as a function of demand: $p = \frac{9-D}{2}$.

As talking about price as a function of demand seems odd, we change notation and terminology and speak of price as being determined by quantity. So, D is replaced by q and the inverse demand function is $p = \frac{9-q}{2}$.

Choosing Quantity

Consider monopolist facing inverse demand function $p = \frac{9-q}{2}$ with unit costs of production of \$2 a unit (i.e. constant marginal costs of \$2 a unit).

What value of q will maximize profit?

$\text{REVENUE} = q \times \frac{9-q}{2}$

$\text{COST} = 2q$

$\text{PROFIT } (\Pi(q)) = q \times \frac{9-q}{2} - 2q$

Choosing Quantity

$$\Pi(q) = q \times \frac{9-q}{2} - 2q$$

$$\frac{d\Pi(q)}{dq} = 4.5 - q - 2 = 2.5 - q$$

$$\frac{d\Pi(q)}{dq} = 0 \;\Rightarrow\; 2.5 - q = 0 \;\Rightarrow\; q = 2.5$$

Profit maximizing level of output is 2.5 (make sure to check second derivative).

Profit maximizing price will be $\frac{9-2.5}{2} = 3.25$

Price and Cost

If a monopolist's unit cost of production rise by \$1 a unit, should it pass along the full cost increase to its customers, i.e. raise price by \$1?

Price and Cost

Suppose $D(p) = 9 - 2p$ and unit costs of production are c per unit.

$$\Pi(p) = (p - c)(9 - 2p) = -2p^2 + (9 + 2c)p - 9c$$

$$\frac{d\Pi(p)}{dp} = -4p + (9 + 2c)$$

Set derivative to zero and solve for p

$$-4p + (9 + 2c) = 0 \;\Rightarrow\; p = \frac{9 + 2c}{4}$$

Check second derivative is negative.

Price and Cost

$$p = \frac{9 + 2c}{4}$$

Raise c by \$1. Then,

$$p = \frac{9 + 2(c + 1)}{4} = \frac{9 + 2c}{4} + \frac{2}{4} = \frac{9 + 2c}{4} + \frac{1}{2}$$

Production Costs

Let $C(q)$ be the TOTAL cost to produce q units of output.

$C(q)$ increases as q increases, i.e., $C'(q) > 0$.

Average Cost

$C(q)$ is the TOTAL cost to produce q units of output.

If r units are produced, the AVERAGE cost (per unit) is:

$$\frac{C(r)}{r}.$$

Is this the right way to measure 'variable' cost?

Marginal Cost

Approximation: Increase in total cost from production of an additional unit (incremental cost).

Exact: Increase in cost from an infinitesimal increase in production.

Marginal cost function is $\frac{dC(q)}{dq}$.

Marginal Costs

Example: $C(q) = 2q^2 + q + 2$.

Marginal cost function is

$$\frac{dC(q)}{dq} = 4q + 1.$$

Marginal cost at 5 units of output is $4 \times 5 + 1 = 21$.

Interpretation: Once we have produced 5 units of output, the increase in cost from producing an additional unit will be (approx) \$21.

Returns to Scale

Efficiency of a production technology is determined by the slope of the marginal cost function.

Derivative of marginal cost is second derivative of $C(q)$

$$\frac{d^2 C(q)}{dq^2}$$

Constant Returns to Scale

Marginal Cost is Constant

$$\frac{d^2C(q)}{dq^2} = 0$$

Example: $C(q) = 2q$, this corresponds to a \$2 a unit cost of production.

Increasing Returns to Scale

Marginal Cost is Decreasing

$$\frac{d^2C(q)}{dq^2} < 0$$

Example: $C(q) = q - q^2$ for $q \leq 1$.

Decreasing Returns to Scale

Marginal Cost is Increasing

$$\frac{d^2C(q)}{dq^2} > 0$$

Example: $C(q) = q^2$

Marginal Revenue

Approximation: Increase in revenue from an additional unit of output (incremental revenue).

Exact: Increase in revenue from an infinitesimal increase in output.

If $R(q)$ is revenue as a function of quantity/output, marginal revenue is $\frac{dR(q)}{dq}$.

Marginal Revenue

Example: Monopolist facing inverse demand function $p = \frac{9-q}{2}$.

$$R(q) = q \times \frac{9-q}{2} = \frac{9q}{2} - \frac{q^2}{2}$$

$$\frac{dR(q)}{dq} = 4.5 - q$$

Marginal Revenue

Monopolist facing inverse demand function $p(q)$.

Revenue is $R(q) = qp(q)$

Marginal revenue is

$$MR(q) = \frac{dR(q)}{dq} = p(q) + q\frac{dp(q)}{dq}$$

Marginal revenue decreases as q increases. Why? To sell more you must lower prices.

Marginal Revenue

Monopolist facing inverse demand function $p(q)$ with cost function $C(q)$.

Profit: $\Pi(q) = R(q) - C(q)$

$\frac{d\Pi(q)}{dq} = 0$ is a necessary condition for profit maximization.

$$\frac{d\Pi(q)}{dq} = 0 \;\Rightarrow\; \frac{dR(q)}{dq} - \frac{dC(q)}{dq} = 0$$

Marginal Revenue = **Marginal Cost** is necessary condition for profit maximization.

Marginal Revenue

We have

$$\frac{d\Pi(q)}{dq} = 0 \Rightarrow \frac{dR(q)}{dq} - \frac{dC(q)}{dq} = 0$$

To actually be profit maximizing, need second derivative to be negative.

If $C(q)$ has constant or decreasing returns to scale, this is the case. Why?

Marginal Revenue

Suppose inverse demand decreases with quantity (price goes down as quantity increases).

Suppose production costs exhibit constant or decreasing returns to scale.

Profit maximizing output q^*, is determined where marginal revenue = marginal cost.

Marginal Revenue

Revenue is the area under the marginal revenue curve, $MR(q)$.

If $R(q)$ is the revenue from q units of output, then, $MR(q) = \frac{dR(q)}{dq}$. Therefore,

$$R(q) = \int_0^q \frac{dR(x)}{dx} dx$$

Integral = 'Area'

Marginal Revenue

Total cost is the area under the marginal cost curve, $MC(q)$ (short explanation).

If $C(q)$ is total cost to produce q units, then, $MC(q) = \frac{dC(q)}{dq}$.

Therefore,

$$C(q) = \int_0^q \frac{dC(x)}{dx} dx$$

Marginal Revenue

Profit from producing q^* units of output is the area under the marginal revenue curve above the marginal cost curve:

$$\int_0^{q^*} MR(x)dx - \int_0^{q^*} MC(x)dx$$

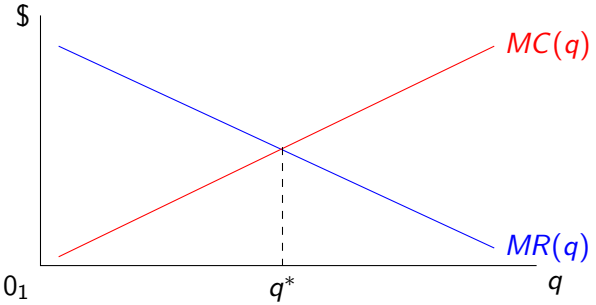
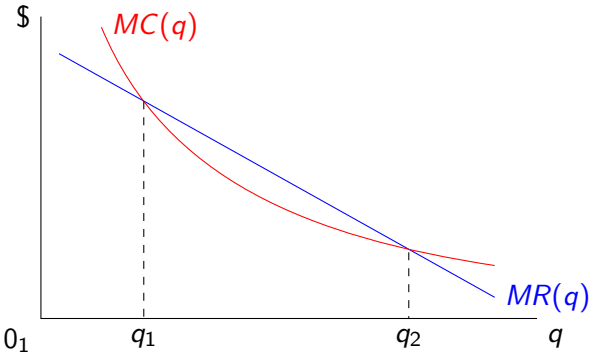


Figure: Profit maximization

Increasing Returns to Scale

Profit maximizing output occurs where marginal revenue = marginal cost.

However, there can me more than one such point.



Where Do Cost Functions Come From?

Cost function is determined by production technology.

Production function: specifies *maximum* amount of an output, q, that can be produced from a particular combination of inputs.

$$q = f(L, K)$$

- Labor (L): hours of work (managers, workers)
- Raw material (K): land, rubber, oil, hours of machine time

Examples of Production Functions

- Linear (perfect substitutes): $q = \alpha L + \beta K$
- Fixed proportions (perfect complements): $q = \min(\alpha L, \beta K)$
- Cobb-Douglas: $f(L, K) = AL^{\alpha}K^{\beta}$
- CES: $q = [K^{\rho} + L^{\rho}]^{1/\rho}$

Marginal Productivity

Marginal product of labor:

$$\frac{\partial f}{\partial L} \geq 0$$

Marginal product of raw material:

$$\frac{\partial f}{\partial K} \geq 0$$

These have the obvious interpretation of the incremental output from an infinitesimal change in labor and raw material, respectively.

Where Do Cost Functions Come From?

w = price per unit of labor (usual called wage)

r = price per unit of raw material

$C(q)$ is the minimum total cost incurred to produce q units of output using K and L.

$$C(q) = \min wL + rK$$
$$\text{s.t. } f(L, K) = q$$
$$L, K \geq 0$$

Example: Cost Minimization

Firm's production function is $q = L^{1/2}K^{1/2}$.

$$w = 4\ , r = 1$$

$$C(q) = \min 4L + 1K$$
$$\text{s.t. } L^{1/2}K^{1/2} = q$$
$$L, K \geq 0$$

Example: Cost Minimization

Use substiution:

Use equality constraint to write L in terms of K:

$$L^{1/2}K^{1/2} = q \;\Rightarrow\; L^{1/2} = qK^{-1/2} \;\Rightarrow\; L = q^2K^{-1}.$$

$$C(q) = 4L + K = 4q^2K^{-1} + K.$$

Now choose K to minimize this expression.

FOC is

$$4q^2(-K^{-2}) + 1 = 0 \;\Rightarrow\; K = 2q.$$

Example: Cost Minimization

Verify that this value of K is non-negative.

Verify that this is indeed a minimum using SOC.

$$C(q) = 4L + K = 4q^2K^{-1} + K = 2q + 2q = 4q$$

Constant returns to scale.

Example: Cost Minimization

Write down the Lagrangean (we are ignoring the non-negativity constraints):

$$\mathcal{L} = 4L + 1K + \lambda(q - L^{1/2}K^{1/2})$$

First order conditions:

1. $\frac{\partial \mathcal{L}}{\partial L} = 4 - \lambda(\frac{1}{2})L^{-1/2}K^{1/2} = 0$
2. $\frac{\partial \mathcal{L}}{\partial K} = 1 - \lambda(\frac{1}{2})L^{1/2}K^{-1/2} = 0$
3. $\frac{\partial \mathcal{L}}{\partial \lambda} = q - L^{1/2}K^{1/2} = 0$

Example: Cost Minimization

First order conditions:

1. $0 = \frac{\partial \mathcal{L}}{\partial L} = 4 - \lambda(\frac{1}{2})L^{-1/2}K^{1/2} = 4 - (\frac{1}{2})\lambda\frac{q}{L}$
2. $0 = \frac{\partial \mathcal{L}}{\partial K} = 1 - \lambda(\frac{1}{2})L^{1/2}K^{-1/2} = 1 - (\frac{1}{2})\lambda\frac{q}{K}$
3. $\frac{\partial \mathcal{L}}{\partial \lambda} = q - L^{1/2}K^{1/2} = 0$

So, $4 = \frac{K}{L} \Rightarrow 4L = K$.

$$q = L^{1/2}(4L)^{1/2} = 2L \Rightarrow L = \frac{q}{2}, K = 2q$$

These satisfy the omitted non-negativity constraints.

$$C(q) = w(\frac{q}{2}) + 2rq = (\frac{w}{2} + 2r)q$$

Constant returns to scale.

Example: Cost Minimization

Checking the 2nd order condition.

$$\min g(x, y)$$
$$\text{s.t. } f(x, y) = q$$

Set up Lagrangean:

$$\mathcal{L}(x, y, \lambda) = g(x, y) + \lambda[q - f(x, y)]$$

If (x^*, y^*, λ^*) is the solution from the FOCs, then need to evaluate the Hessian at (x^*, y^*, λ^*).

Cost Minimization

What is effect on input combination if w goes up holding r and q fixed?

What is effect on input combination if r goes up holding w fixed?

$$\begin{aligned} C(q) &= \min wL + rK \\ \text{s.t. } & L^{1/2}K^{1/2} = q \\ & L, K \geq 0 \end{aligned}$$

Isoquants

All *combinations of inputs* that produce a *given* quantity q of output.

The set of all $L, K \geq 0$ such that $f(L, K) = q$.

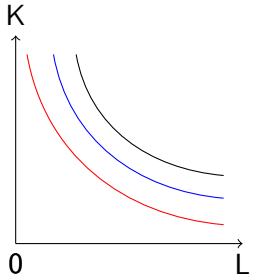


Figure: Isoquant at various values of q

Isocost Lines

Possible *combinations of inputs* that *cost* the *same* amount.

For a given C, the set of (L, K) such that

$$wL + rK = C$$

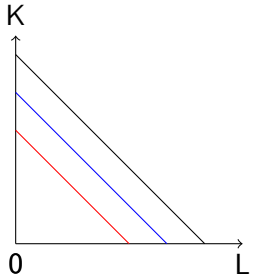


Figure: Isocost at various values of C

A Geometrical View of the Cost Minimization Problem

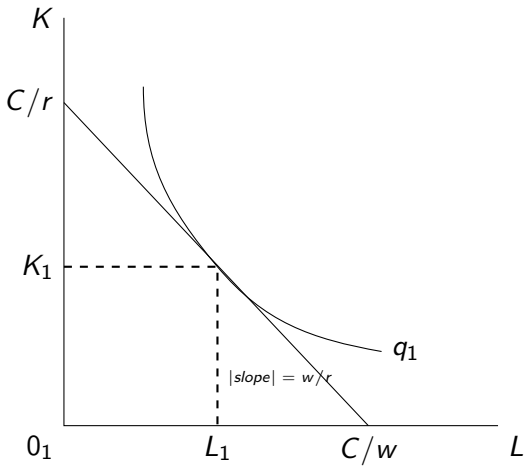


Figure: Cost minimization

Marginal Rate of Technical Substitution (MRTS)

MRTS is

$$-\frac{\frac{\partial f}{\partial L}}{\frac{\partial f}{\partial K}}$$

Same as *slope* of an isoquant.

Shows the rate at which inputs may be substituted while the output level remains constant.

The incremental amount of raw material that is needed to replace one unit of labour if one wishes to maintain the same level of output is $|-\frac{\frac{\partial f}{\partial L}}{\frac{\partial f}{\partial K}}|$

Diminishing MRTS

Typically assume that $|MRTS|$ is *diminishing*:

- If a firm keeps decreasing an input
- Holding total output and technology fixed
- Corresponding increases in the other input (to remain on the same isoquant) will become larger

$$\frac{\partial |MRTS|}{\partial L} < 0$$

Automation

Will automation displace labor?

Case 1: Automation is a substitute for labor.

$$f(L, K) = L + K$$

$L =$ the number of units of labor acquired

$K =$ the number of units of automation deployed

If one reduces labor by 1 unit and increases automation by 1 unit, total output remains unchanged.

Automation

$$\min rK + wL$$

$$K + L = q$$

$$K, L \geq 0$$

The optimal solution will depend on the sign of $(w - r)$.

1. If $w \leq r$, $L = q$ and $K = 0$.
2. If $w > r$, $L = 0$ and $K = q$.

Automation

Case 2: Automation is a complement to labor.

$f(K, L) = \min\{K, L\}$

$$\begin{aligned} &\min rK + wL \\ \text{s.t.}\quad &\min\{K, L\} = q \\ &K, L \geq 0 \end{aligned}$$

$K = L = q$.

Double Marginalization

Upstream manufacturer (M) with constant marginal costs of \$1 (think publisher)

M is sole supplier to a downstream firm, retailer (R) (think Amazon) and charges a wholesale price of w per unit

R faces a demand curve of $100 \times (10 - p)$, where p is the unit price set by R

Double Marginalization

How M should price depends on R's reaction.

Thus, M must anticipate R's behavior, before she can determine what price w to charge R.

Therefore, we first examine how R chooses p as function of w, which is R's unit cost.

Double Marginalization

By the markup formula, R's profit maximizing price, p^* would be set so that

$$\frac{p^* - w}{p^*} = \frac{1}{e(p^*)}$$

Substituting in the relevant expressions:

$$\frac{p^* - w}{p^*} = \frac{10 - p^*}{p^*} \Rightarrow p^* = \frac{10 + w}{2}$$

Therefore downstream demand will be

$$100 \times (10 - p^*) = 100 \times \left(10 - \frac{10 + w}{2}\right) = 500 - 50w$$

Double Marginalization

Downstream demand is M's demand: $500 - 50w$

The elasticity of demand is $\frac{50w}{500-50w} = \frac{w}{10-w}$

By the markup formula, M's profit maximizing price, w^* would be set so that

$$\frac{w^* - 1}{w^*} = \frac{10 - w^*}{w^*}$$

The optimal value of w is \$5.5.

Double Marginalization

If M sets a wholesale price of $w = \$5.5$, then R will set a price equal to $\frac{10+w}{2} = \$7.75$.

$$\text{DEMAND} = 100 \times (10 - 7.75) = 225$$

$$\text{R's MARGIN} = \$(7.75 - 5.5) = \$2.25$$
$$\text{R's PROFIT } (\Pi_R) = \$2.25 \times 225 = \$506.25$$

$$\text{M's MARGIN} = \$(5.5 - 1) = \$4.5$$
$$\text{M's PROFIT } (\Pi_M) = \$4.5 \times 225 = \$1,012.5$$

$$\text{TOTAL PROFIT} = \Pi_R + \Pi_M = \$1,518.75.$$

Double Marginalization

Scenario #2: M sells directly into the market.

It has three interpretations.

1. M simply cuts R out as a middleman and sells directly to the end user.
2. M and R merge into a single firm.
3. M and R agree to coordinate so as maximize their combined profit.

Double Marginalization

M has constant marginal costs of \$1.

M faces a demand curve of $100 \times (10 - p)$, where p is the unit price set by M.

What value of p will M pick to maximize profit?

By the markup formula, M's profit maximizing price, p^* would be set so that

$$\frac{p^* - 1}{p^*} = \frac{10 - p^*}{p^*} \Rightarrow p^* = 5.5$$

Double Marginalization

M's profit maximizing price is $p^* = 5.5$

DEMAND $= 100 \times (10 - 5.5) = 450$

MARGIN $= \$(5.5 - 1) = \$4.5$

PROFIT(Π) $= \$4.5 \times 450 = \$2,025$

Double Marginalization

$\Pi > \Pi_R + \Pi_M$. Firms, collectively, are better off.

The downstream price is lower in scenario #2 than scenario #1. Consumers are better off.

Double Marginalization

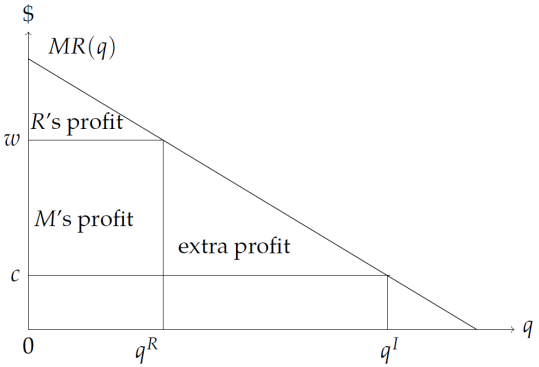


Figure: Double Marginalization

Double Marginalization

How might M and R recover those 'lost' profits?

1. **Integration**: M and R merge into a single firm, also called **vertical integration**.
2. **Resale Price Maintenance**: M places a ceiling on the price set by R.
3. M introduces a competitor to R in the downstream market. (discussed later).

Consumer Surplus

Measure benefit to consumers by consumer surplus.

A buyer with an RP of v who pays p for the good, enjoys a benefit of $v - p$.

(Inverse) Demand curve is a summary of the RPs of all buyers in the market.

Consumer Surplus & Inverse Demand Curve

Assume best alternative delivers zero surplus to everyone.

Name	Mr. A	Mr. B	Mr. C	Mr. D	Mr. E
RP	\$10	\$15	\$12	\$ 7	\$ 17

Order the buyers by decreasing RP.

Name	Mr. E	Mr. B	Mr. C	Mr. A	Mr. D
RP	\$17	\$15	\$12	\$ 10	\$ 7

Consumer Surplus (using inverse demand)

Order the buyers by decreasing RP. We can think of $p(q)$, the inverse demand curve, as the RP of the qth buyer on the list.

Suppose we are currently at quantity q. If we increase quantity to $q + \Delta$, we pick up Δ extra buyers.

The RP of these buyers is approximately $p(q + \Delta)$. The *increase* in total RP of all buyers is $p(q + \Delta)\Delta$.

Consumer Surplus (using inverse demand)

Let q^* be the quantity chosen by the monopolist.

Look at $[0, \Delta]$, $[\Delta, 2\Delta] \ldots [q^* - \Delta, q^*]$.

Let $\Delta \to 0$.

Total RP will be roughly

$$p(\Delta)\Delta + p(2\Delta)\Delta + \ldots p(q^*)\Delta$$

Total RP for quantity q^* is area under the inverse demand curve: $\int_0^{q^*} p(x)dx$.

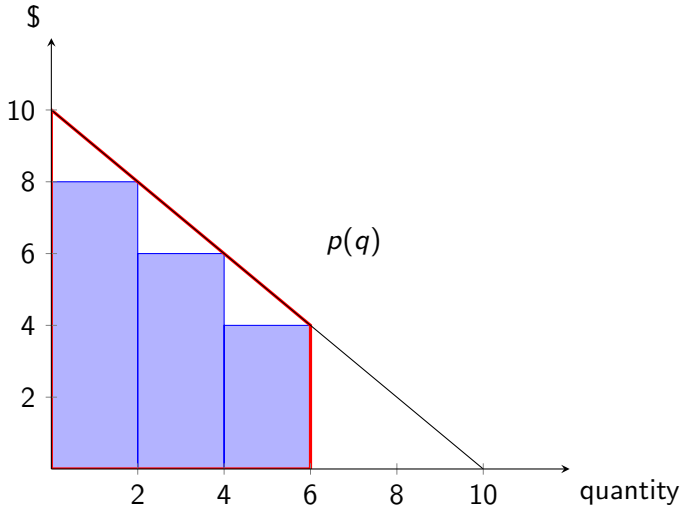


Figure: Total RP ($q^* = 6, \Delta = 2$)

Consumer Surplus (using inverse demand)

If monopolist chooses a quantity q^*, the price will be $p(q^*)$.

The total RP of all consumers who buy will be

$$\int_0^{q^*} p(x)dx$$

The total consumer surplus will be

$$\Sigma(q^*) = \int_0^{q^*} p(x)dx - p(q^*)q^*$$

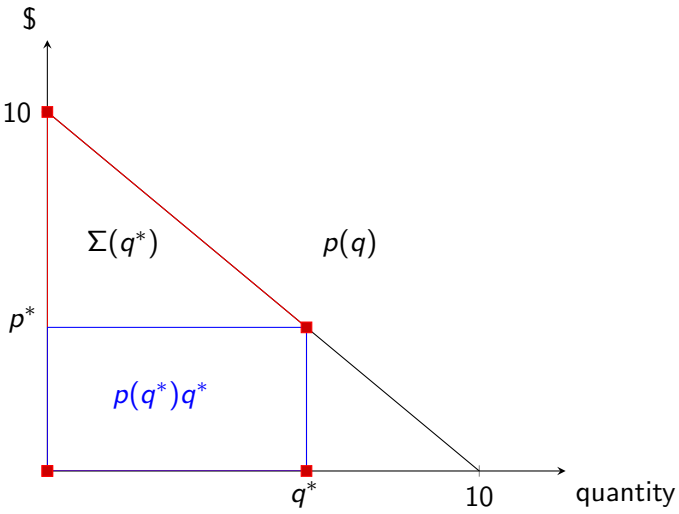


Figure: Total Consumer Surplus

Consumer Surplus (using demand)

We can think of $D(p)$ as the number of buyers with an RP that exceeds p.

Let p^* be the price chosen by the monopolist.

Number of buyers with an RP in interval $[p^*, p^* + \Delta]$ is roughly $D(p^*) - D(p^* + \Delta)$.

Each has a surplus at most Δ.

Total consumer surplus for buyers in $[p^*, p^* + \Delta]$ is approximately $[D(p^*) - D(p^* + \Delta)]\Delta$

Consumer Surplus (using demand)

Now look at $[p^* + \Delta, p^* + 2\Delta]$, $[p^* + 2\Delta, p^* + 3\Delta]$, etc.

Total consumer surplus, $\Sigma(p^*)$ will be roughly

$$\begin{aligned}&[D(p^*) - D(p^* + \Delta)]\Delta + [D(p^* + \Delta) - D(p^* + 2\Delta)]2\Delta + \dots \\ =&D(p^*)\Delta + D(p^* + \Delta)\Delta + D(p^* + 2\Delta)\Delta + \dots\end{aligned}$$

Let $\Delta \to 0$ and we have an integral:

$$\Sigma(p^*) = \int_{p^*}^{\bar{p}} D(x)dx$$

where $\bar{p}$ is such that $D(\bar{p}) = 0$, also known as **choke** price.

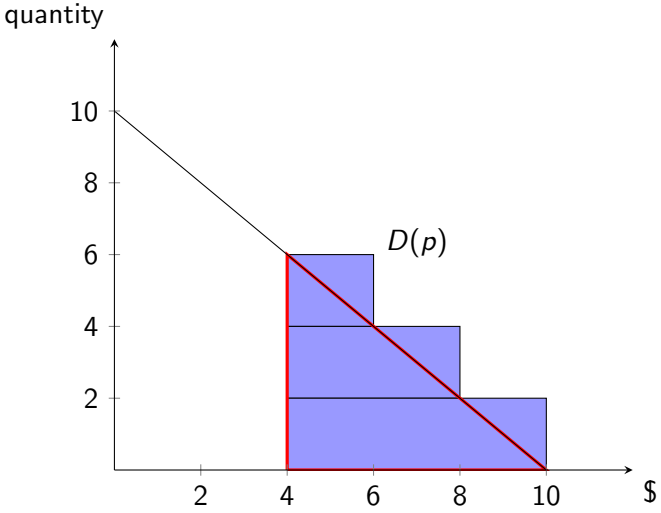


Figure: Total consumer surplus ($p^* = 4, \overline{p} = 10, \Delta = 2$)

Regulating a Monopolist

Should one regulate the price set by a monopolist?

If the price is lowered from the optimal price, consumers benefit.

If the price is lowered from the optimal price, the monopolist loses.

Does the benefit enjoyed by consumers exceed the loss suffered by the monopolist?

Total Surplus

Total surplus at quantity q is total consumer surplus at q plus monopoly profit at q.

$$\Sigma(q) + \Pi(q)$$

Suppose monopolist has constant marginal cost of c.

$$\Pi(q) = p(q)q - cq$$

Pick q to maximize: $[\int_0^q p(x)dx - p(q)q] + [p(q)q - cq]$

Total Surplus

$$\max_q [\int_0^q p(x)dx - p(q)q] + [p(q)q - cq]$$

$$\max_q [\int_0^q p(x)dx - cq]$$

Differentiate wrt to q and set to zero: $p(q) - c = 0$ (check sign of second derivative).

In words, choose quantity that corresponds to a price of c. Force monopolist to price at cost.

Monopsony

Literally, sole buyer, i.e., monopoly buyer rather than seller.

Each potential supplier (buyer) has a cost (RP). If price offered exceeds (is below) its costs (RP), then supplier (buyer) will offer to supply (buy).

Buyer (seller) faces supply (demand) curve that increases (decreases) with increase in unit price offered e.g. $S(w) = w$ (e.g. $D(p) = 100 - p$).

Monopsony

Monopsonist facing supply curve of $S(w)$.

If monopsonist offers price of w^*, a supplier with cost *below* w^* will offer to supply.

Therefore, the $S(w^*)$ suppliers who step forward must all have costs below w^*.

Monopsony

Weyland-Utami Corp. is monopsony on the input side and monopoly on the output side.

On input side faces a supply curve for input of $S(w) = w$.

It takes one unit of supply and converts it into one unit of output.

On output side it faces an inverse demand curve of $p(q) = 100 - 2q$.

What output level q and input price w should Weyland-Utami set to maximize profit?

Monopsony

$$\max qp(q) - wS(w)$$

$$\text{s.t. } q = S(w)$$

$$q, w \geq 0$$

Substituting in the relevant expressions:

$$\max q(100 - 2q) - w^2$$

$$\text{s.t. } q = w$$

$$q, w \geq 0$$

Monopsony

$$\max q(100 - 2q) - w^2$$

$$\text{s.t. } q = w$$

$$q, w \geq 0$$

Eliminate q. Substitute in:

$$\max w(100 - 2w) - w^2$$

$$\text{s.t. } w \geq 0$$

Optimal value of $w = \frac{100}{6}$.

Supplier Surplus (using supply)

We can think of $S(w)$ as the number of suppliers with a cost that is below w.

Let w^* be the price chosen by the monopsony.

Number of supplier with a cost in interval $[w^* - \Delta, w^*]$ is roughly $S(w^*) - S(w^* - \Delta)$.

Each has a surplus at most Δ.

Total supplier surplus for suppliers in $[w^* - \Delta, w^*]$ is approximately $[S(w^*) - S(w^* - \Delta)]\Delta$

Supplier Surplus (using supply)

Now look at $[w^* - 2\Delta, w^* - \Delta]$, $[w^* - 3\Delta, w^* - 2\Delta]$, etc.

Total supplier surplus, $\Sigma(w^*)$ will be roughly

$$\begin{aligned}&[S(w^*) - S(w^* - \Delta)]\Delta + [S(w^* - \Delta) - S(w^* - 2\Delta)]2\Delta + \dots \\ =&S(w^*)\Delta + S(w^* - \Delta)\Delta + S(w^* - 2\Delta)\Delta + \dots\end{aligned}$$

Let $\Delta \to 0$ and we have an integral:

$$\Sigma(w^*) = \int_0^{w^*} S(x)dx.$$

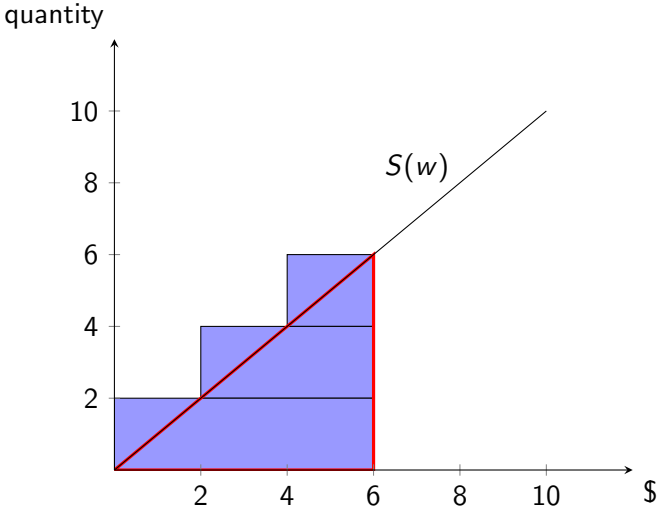


Figure: Total supplier surplus ($w^* = 6, \Delta = 2$)

Regulating Monopsony

Should one regulate the wage set by a monopsony?

If the wage is raised from the optimal wage, suppliers benefit.

If the wage is raised from the optimal wage, the monopsony loses.

Does the benefit enjoyed by suppliers exceed the loss suffered by the monopsony?

Regulating Monopsony

Get revenue of $R(q)$ from q units of output.

On input side face a supply curve for input of $S(w)$.

$$\max R(q) - wS(w)$$
$$\text{s.t. } q = S(w)$$

Profit to firm is

$$\max R(S(w)) - wS(w)$$

Regulating Monopsony

Choose w to maximize sum of benefit to firm PLUS input suppliers

$$\max R(S(w)) - wS(w) + \int_0^w S(x)dx$$

$$\text{s.t. } w \geq 0$$

$$\max w(100 - 2w) - w^2 + \int_0^w xdx = w(100 - 2w) - \frac{w^2}{2}$$

Optimal value of $w = \frac{100}{5}$.

Minimum Wage

Minimum wage makes the employer worse off.

Does it make workers better off?

If the employer reduces its workforce, then, the workers let go are clearly worse off and those who remain are better off.

If the employer increases hiring, then workers are clearly better off.

Minimum Wage

OmniCorp, a producer of Soma, is the only employer in the city of Detroit.

Faces an inverse demand curve of $p(q) = 100 - q$.

Production technology converts one unit of labor into one unit of Soma.

Minimum Wage

If OmniCorp offers a wage of w per unit of labor, other things held fixed, it can expect w units of labor to be supplied in Detroit.

Supply curve is $S(w) = w$.

If OmniCorp wishes to produce q units of output it must purchase q units of labor.

Minimum Wage

To attract q units of labor, the wage needs to be set at q.

Total cost is $C(q) = q^2$.

Hence, profit will be:

$$\Pi(q) = R(q) - C(q) = 100q - 2q^2.$$

Profit-maximizing quantity is $q^* = 25$ and profit-maximizing wage is $w^* = 25$.

Minimum Wage

Suppose a minimum wage is set *slightly* above 25.

Minimum wage is $25 + \Delta$.

OmniCorp is not obliged to hire all labor prepared to work for that wage.

Minimum Wage

Option #1: No change in output and no change in labor.

$$25 \times (100 - 25) - 25 \times (25 + \Delta) = [25 \times 75 - 25 \times 25] - 25\Delta.$$

Option #2 Reduce output and hire fewer workers.

$$\max q(100 - q) - q(25 + \Delta)$$
$$\text{s.t. } 0 \leq q \leq 25$$

Optimal value of $q = 25$.

Option #3: Increase output to $25 + \Delta$

$$(25 + \Delta)(100 - 25 - \Delta) - (25 + \Delta) \times (25 + \Delta) = [25 \times 75 - 25 \times 25] - 2\Delta^2.$$

Minimum Wage

For Δ sufficiently small, Option #3 is better than Options #1 and #2.

Hence, OmniCorp would actually increase the amount of labor hired with the passage of the minimum wage regulation

Price Discrimination

Same product or service sold to at least two different buyers at different prices.

1. **First Degree (or Perfect):** Sell to each buyer at a price just under their RP
2. **Third Degree (or Group):** Price according to an observable characteristic linked to RP
3. **Second Degree (or Nonlinear):** Present a menu and have buyers select

Second Degree Price Discrimination

1. Versioning
2. Bundling
3. Two-part tariff

Versioning

Two Printers; FAST & SLOW

Marginal cost of each is zero.

Two buyers, Mr. A & Mr. B (equivalently, two equal sized segments)

3 Choices

Price and sell only SLOW.

Price and sell only FAST.

Price and sell both.

Which of the three maximizes revenue?

Versioning

	FAST	SLOW
A's RP	$100	$40
B's RP	$49	$30

Versioning

Price and sell only FAST

	FAST
A's RP	\$100
B's RP	\$49

Price	\$49	\$100
Demand	2	1
Revenue	\$98	\$100

Price FAST at \$100 generates maximum revenue \$100

Versioning

Price and sell only SLOW

	SLOW
A's RP	\$40
B's RP	\$30

Price	\$30	\$40
Demand	2	1
Revenue	\$60	\$40

Price SLOW at \$30 generates maximum revenue \$60

Versioning

Sell FAST & SLOW

	FAST	SLOW
A's RP	\$100	\$40
B's RP	\$49	\$30

	FAST	SLOW
Price	?	\$30
Demand	?	1
Revenue	?	\$30

Mr. A purchases FAST if $\$100 - P_{FAST} \geq \$40 - \$30 \Rightarrow P_{FAST} \leq \$90$.

Versioning

Sell FAST & SLOW

	FAST	SLOW
A's RP	\$100	\$40
B's RP	\$49	\$30

	FAST	SLOW
Price	\$90	\$30
Demand	1	1
Revenue	\$90	\$30

Price FAST at \$90 and SLOW at \$30 generates revenue \$120.

Introducing a variant of a product (SLOW) produces two effects.

- POSITIVE EFFECT: the variant allows one to cater to a segment not previously being served (Mr. B).
- NEGATIVE EFFECT: the variant competes with the original product. (The price of FAST is lower.)

In the example above, the positive effect outweighed the negative.

The next example illustrates this is not always the case.

Versioning

	FAST	SLOW
A's RP	\$100	\$65
B's RP	\$49	\$30

Sell only FAST at \$100 generates revenue \$100.

Sell only SLOW at \$65 generates revenue \$ 65.

Versioning

Sell FAST & SLOW

	FAST	SLOW
A's RP	\$100	\$65
B's RP	\$49	\$30

	FAST	SLOW
Price	?	\$30
Demand	?	1
Revenue	?	\$30

Mr. A purchases FAST if $\$100 - P_{FAST} \geq \$65 - \$30 \Rightarrow P_{FAST} \leq \$65$.

Versioning

Sell FAST & SLOW

	FAST	SLOW
A's RP	\$100	\$65
B's RP	\$49	\$30

	FAST	SLOW
Price	\$65	\$30
Demand	1	1
Revenue	\$65	\$30

Price FAST at \$65 and SLOW at \$30 generates revenue \$95.

Bundling

Sell tickets to two concert series

There are three ways in which tickets can be sold:

1. Price and sell each ticket as an individual item.
2. Price and sell the tickets in a bundle of two only (pure bundling). The option of buying a ticket to a single concert is not available.
3. Price and sell a bundle as well as the tickets individually (mixed bundling).

Which will generate the most revenue?

Bundling

	STONES	JOHN	Bundle
A's RP	\$80	\$40	\$120
B's RP	\$40	\$80	\$120
C's RP	\$90	\$10	\$ 100
D's RP	\$10	\$90	\$100

Bundling: Individual Tickets

Price, Demand and Revenue for the Stones Concert

Price	Demand	Revenue
\$10	4	\$40
\$40	3	\$120
\$80	2	\$160
\$90	1	\$90

Revenue maximizing price for the Stone's concert is \$80

Similar analysis applies to the pricing of John concert

Generates a total revenue of \$320.

Bundling: Pure Bundling

Price, Demand and Revenue for the Bundle

Price	Demand	Revenue
$100	4	$400
$120	2	$240

Generates total revenue of $400.

Bundling: Mixed Bundling

	STONES	JOHN	Bundle
A's RP	$80	$40	$120
B's RP	$40	$80	$120
C's RP	$90	$10	$100
D's RP	$10	$90	$100

Bundling: Mixed Bundling

	Price	Demand	Revenue
STONES	$90	C	$90
JOHN	$90	D	$90
BUNDLING	$120	A & B	$240

Generates total revenue of $420

Two-Part Tariff

Buyer interested in purchasing multiple units of same product.

Unit	1st	2nd	3rd	4th	5th
Incremental RP	\$17	\$14	\$12	\$9	\$5

Constant marginal costs of \$6 a unit

Two-Part Tariff

Unit	1st	2nd	3rd	4th	5th
Incremental RP	\$17	\$14	\$12	\$9	\$5

Constant marginal costs of \$6 a unit

Price	\$6	\$9	\$12	\$14	\$17
Margin	\$0	\$3	\$6	\$8	\$11
Demand	4	4	3	2	1
Profit	\$0	\$12	\$18	\$16	\$11

The optimal price is \$12.
Profit: \$18
Buyer surplus: $(17-12)+(14-12)+(12-12)=7$

Two-Part Tariff

Charged fixed fee F (cover charge, entry fee, franchise fee)

Charge p for each unit bought

If purchase Q units, total price paid is $F + pQ$

Price per unit is $\frac{F+pQ}{Q} = \frac{F}{Q} + p$

Price per unit declines with volume

Two-Part Tariff

Set $F = 28$

Set $p =$ unit cost of \$6 a unit

Unit	1st	2nd	3rd	4th	5th
Incremental RP	\$17	\$14	\$12	\$ 9	\$ 5

Buyer will purchase 4 units. Total consumer surplus excluding F is \$28.

Profit will be : $\$28 > 18$

Two-Part Tariff

Buyer with inverse demand curve of $p(q)$.

Suppose charge p^* per unit.

Buyer will purchase q^* units where $p(q^*) = p^*$.

Buyer will enjoy consumer surplus of

$$\int_0^{q^*} p(x)dx - p^*q^*$$

Seller should set $F = \int_0^{q^*} p(x)dx - p^*q^*$

Two-Part Tariff

Seller's profit (assuming constant marginal costs of c)

$$\int_0^{q^*} p(x)dx - p^*q^* + (p^* - c)q^*$$

$$\int_0^{q^*} p(x)dx - cq^*$$

Seller wants to choose p^* to maximize profit.

p^* does not appear in profit, but q^* does which depends on p^*

Two-Part Tariff

Solve:

$$\max_{q^*} \int_0^{q^*} p(x)dx - cq^*$$

Find optimal q^*.

Plug back into inverse demand curve to find the price.

Two-Part Tariff

Solve:

$$\max_{q^*} \int_0^{q^*} p(x)dx - cq^*$$

Differentiate wrt q^* and set to zero (check second derivative):

$$p(q^*) - c = 0$$

$$p(q^*) = c$$

In words, choose q^* so that inverse demand equals marginal cost, i.e., $p^* = c$

Two-Part Tariff

Buyer with inverse demand curve of $p = 9 - q$ (equivalent to $q = 9 - p$).

Constant marginal costs of \$1 a unit.

Set unit price to \$1. At this price demand will be 8.

$$F = \int_0^8 (9 - x)dx - 8 \times 1 = \left[9x - \frac{x^2}{2}\right]_0^8 - 8 = 32$$

Two-Part Tariff

Two interpretations

Fixed fee: access fee, for example, to a fitness club or warehouse stores

Per unit price: price per session or good prices

Fixed fee: cost of a durable good like a razor or e-book reader

Per unit price: cost of a consumable good like blade or e-book

The analysis just conducted suggests: high priced durable, low priced consumable

Razors & Blades

"As everyone knows, safety razor manufacturers derive the bulk of their profit, not from razors, but from the replaceable blades." Time Magazine, 1929

"Gillette made its real profit from the high margin on the blades." FREE by Chris Anderson

Razors & Blades

The razor and blade strategy is a strategy in which the durable good is sold at a low price (or given away for free) in order to increase sales of consumable supplies.

It is frequently credited to King Camp Gillette, the inventor of the disposable razor.

Gillette did not in fact follow this strategy. The razor and blade strategy was not adopted by the company until *after* 1920 when it faced competition.

Reference: Harvard Buesness Review, "*Gillette's Strange History with the Razor and Blade Strategy.*" September 23, 2010.

Razors & Blades

The analysis conducted earlier suggests: high priced durable, low priced consumable

However, there are also example of firms following the low priced durable, high priced consumable, strategy. (Gillette)

How do we reconcile this?

Until now, it assumed all buyers had the same demand curve.

What if buyers have different demand curves?

Two-Part Tariff with Differing Buyers

Two segments, three buyers.

One Heavy user: $q_H = 9 - p$.

Two Light users: $q_L = 6 - p$.

Constant marginal costs of \$1 a unit.

Two-Part Tariff

Case 1: Set unit price to \$1.

Demand of heavy user will be $9 - 1 = 8$.

Surplus of heavy user will be

$$\int_0^8 (9 - x)dx - 8 \times 1 = 32$$

Demand of light user will be $6 - 1 = 5$.

Surplus of light user will be

$$\int_0^5 (6 - x)dx - 5 \times 1 = \frac{25}{2}$$

If set $F = 32$, only heavy user buys yielding profit of \$32.

If set $F = 12.5$, everyone buys yielding profit of $\$12.5 \times 3 = 37.5$.

Two-Part Tariff with Differing Buyers

Case 2: Set unit price to \$2.

Demand of heavy user will be 7.

Surplus of heavy user will be

$$\int_0^7 (9-x)dx - 7 \times 2 = \frac{49}{2}$$

Demand of light user will be 4.

Surplus of light user will be

$$\int_0^4 (6-x)dx - 4 \times 2 = 8$$

If set $F = \frac{49}{2}$, only heavy user buys yielding profit of $\$24.5 + 7 = 31.5$

If set $F = 8$, all three buy yielding profit of $\$\ 3 \times 8 + 2 \times 4 + 7 = 39$.

Two-Part Tariff with Differing Buyers

How do we find profit maximizing two-part tariff when selling to two segments.

Do we want to sell to both segments?

Do we want to sell to just one of the segments?

Look at each case separately.

Two-Part Tariff with Differing Buyers

Focus on finding the (F, p) combination that maximizes profit *subject* to selling to all buyers.

Let $q_L(p)$ be the quantity bought by light user at price p.

Let $q_H(p)$ be the quantity bought by heavy user at price p.

Compute surplus of light user at price p, call it $S_L(p)$

F must be set to surplus of light users (WHY?).

Profit

$$3 \times S_L(p) + 2 \times (p-1)q_L(p) + (p-1)q_H(p)$$

Two-Part Tariff with Differing Buyers

Choose p.

Light user demand is $q = 6 - p$.

Light user surplus is

$$S_L(p) = \int_0^{6-p} (6 - x)dx - p(6 - p) = (6 - p)[3 - \frac{p}{2}]$$

Heavy user demand is $q = 9 - p$

Profit is

$$3 \times (6 - p)[3 - \frac{p}{2}] + 2(p - 1)(6 - p) + (p - 1)(9 - p)$$

Two-Part Tariff with Differing Buyers

Profit is

$$3 \times (6-p)[3-\frac{p}{2}] + 2(p-1)(6-p) + (p-1)(9-p)$$

$$= 33 + 6p - \frac{3}{2}p^2$$

Differentiate, set to zero, check second order condition, blah, blah, blah

$$p = 2$$

$$S_L(p) = 8$$

So, set $F = 8$ and $p = 2$

Two-Part Tariff with Differing Buyers

Set $F = 8$ and $p = 2$

Profit is 39

Higher than profit from setting p to marginal cost.

Two-Part Tariff with Differing Buyers

Sell to only the heavy user.

Set $p = \$1$ and $F = \$32$

Profit is 32

(See *two-part tariff when all the buyers had the same demand curve*)

Two-Part Tariff with Differing Buyers

Sell to only the light users?

This is not a possibility because any two part tariff attractive to a light user will also attract the heavy user.

Thus, one winds up selling to both segments anyway.

Two-Part Tariff with Differing Buyers

Summary

If we sell to both segments, the profit maximizing two-part tariff is to set $F = 8$ and $p = 2$. Profit is 39.

If we sell to only one of the segments (i.e., the heavy user), the profit maximizing two-part tariff is to set $F = 32$ and $p = 1$. Profit is 32.

If we compare the two cases, it is clear that selling to both segments yields a higher profit.

Pricing Dilemma (Bertrand Model)

100 buyers, each will buy at most one widget, RP of $3 per widget

2 firms

- Production Costs: $1 a unit
- Identical product
- Unlimited Capacity
- Everyone knows everything

The RULES

- Firms announce price simultaneously
- No collusion
- Buyers will buy from lowest priced firm
- If prices are = market splits evenly
- Play Once

Pricing Dilemma

Firms are limited to charge one of two prices: \$3 and \$2.5.

firm 1/firm 2	\$3	\$2.50
\$3	\$100, \$100	\$0, \$150
\$2.50	\$150, \$0	\$75, \$75

Table: Payoff Table

Pricing Dilemma

If I told you (firm 1) that firm 2 intended to choose the high price \$3, what would you do?

If I told you that firm 2 was going to choose the low price \$2.5, what would you do?

No matter which price firm 2 picks, you (firm 1) are always better off pick the low price. (The same argument applies to firm 2.)

So, both firms would end up picking the low price.

Equilibrium

A collection of actions (prices, capacities etc.) one for each firm (not necessarily the same).

No **one** firm has a **unilateral** incentive (measured by profit) to choose a different action.

Holding the actions of all firms but one fixed, ask that one firm if they would prefer to change their action.

Equilibrium (for 2 firms)

GIven, a pair of actions (a_1, a_2) where a_i is the action of firm i.

The pair (a_1, a_2) is an equilibrium if *both* of the following are true

1. Ask firm 1: holding firm 2's action at a_2 would you like to pick something other than a_1? Firm 1 should answer **NO**.
2. Ask firm 2: holding firm 1's action at a_1 would you like to pick something other than a_2? Firm 2 should answer **NO**.

Pricing Dilemma

If one allow firms to choose any price, no firm will choose a price exceeds \$3 (buyer's RP).

No firm will choose a price below \$1 (unit cost).

Is it sensible to predict, say, that firm 1 will price at \$1.5 and firm 2 will price at \$1.6?

If firm 2 predicted that firm 1 would post a \$1.5 price, firm 2 should have priced just below, say \$1.49.

So, firms offering *unequal* prices is not an equilibrium.

Pricing Dilemma

How about both firms offering *equal* prices? For example, \$1.7 each.

At these prices firm 1 makes $\$(1.7 - 1) \times 50 = \$35$.

If firm 1 drops its by a cent to \$1.69, firm 1 can sell to the entire market and make \$69.

So, equal prices above marginal cost is not an equilibrium.

Conclusion: Each firm pricing at marginal cost is the only equilibrium.

Pricing Dilemma

- Play once.
- Unlimited Capacity.
- No Differentiation.
- Full Information
- No Collusion

Pricing Dilemma

- Play once (focused on short term gain)
- Unlimited Capacity (excess capacity)
- No Differentiation (commodity)
- Full Information (transparency)
- No Collusion

Structuring Procurement

- Aggregate so that order represents sizeable chunk of suppliers profit. Get them to focus on winning today.
- Emphasize alternative sources of supply (second hand market, internal production)
- Hold all suppliers to identical requirements terms in request for proposals.
- Let suppliers know about the offers made by others (in so far as promises allow).

Capacity

Firms are limited to charge one of two prices: \$3 and \$2.5.

Each firm has capacity of 20 units.

firm 1/firm 2	\$3	\$2.50
\$3	\$40, \$40	\$40, \$30
\$2.50	\$30, \$40	\$30, \$30

Both firms would pick the high price.

Capacity

Firms are limited to charge one of two prices: \$3 and \$2.5.

Each firm has capacity of 60 units.

firm 1/firm 2	\$3	\$2.50
\$3	\$100, \$100	\$80, \$90
\$2.50	\$90, \$80	\$75, \$75

Both firms would pick the high price.

Capacity

Initially both firms have 20 units of capacity.

Would you like to buy an additional 80 units for $80?

Capacity

Firm 1 has capacity of 100 units.

Firm 2 has capacity of 20 units

100/20	\$3	\$2.50
\$3	\$160, \$40	\$160, \$30
\$2.50	\$150, \$0	\$120, \$30

Firm 1 would pick the high price and, expecting this, firm 2 would also pick the high price.

Capacity

Each firm has capacity of 20 units.

	\$3	\$2.50
\$3	\$40, \$40	\$40, \$30
\$2.50	\$30, \$40	\$30, \$30

Both firms would pick the high price.

Differentiation

One sells RED widgets (production costs \$1 a unit)

One sells BLUE widgets (production costs \$1 a unit)

	RP for RED	RP for BLUE	Number
Red Lover	\$5	\$3	50
Blue Lover	\$3	\$5	50

Differentiation

Pricing at marginal cost is **not** an equilibrium.

Arguments:

- Suppose both firms charge \$1 for their widgets.
- Now ask: could any one firm do better by raising its price?
- Consider the Blue firm, for example.
- At a \$1 price, it makes 0 profit.
- Now suppose it raises its price to \$1.01.
- Blue lovers continue to buy from the Blue firm: $\$(5 - 1.01) > \$(3 - 1)$.
- At this new price, the blue firm makes a positive profit: $\$(1.01 - 1) \times 50$.

Differentiation

Pricing at \$5 is an equilibrium.

Arguments:

- Suppose both firms charge \$5 for their widgets.
- Now ask: could any one firm do better by raising its price?
- Could the Blue firm, for example, make more by dropping its price?
- To get new customers (red lovers), the Blue firm has to drop its price to something slightly below \$3 (i.e., $3 - \epsilon$). (Why?)
- At this new price, its profit will be: $\$(3 - \epsilon - 1) \times 100 < \$(5 - 1) \times 50$.

Differentiation

One sells RED widgets (production costs \$1 a unit)

One sells BLUE widgets (production costs \$1 a unit)

	RP for RED	RP for BLUE	Number
Red Lover	\$5	\$3	1
Blue Lover	\$3	\$5	99

Differentiation

Pricing at \$5 is **not** an equilibrium.

Arguments:

- Suppose both firms charge \$5 for their widgets.
- Now ask: could any one firm do better by raising its price?
- Could the Red firm make more by dropping its price?
- To get new customers (blue lovers), the Red firm has to drop its price to something slightly below \$3 (i.e., $3 - \epsilon$). (Why?)
- At this new price, its profit will be: $\$(3 - \epsilon - 1) \times 100 > \$(5 - 1) \times 1$.

Differentiation

From the point of the Blue firm, the market consists of a **strong market** (of those customers who prefer blue) and a **weak market** (whose who prefer red).

One should differentiate in a way that leads to a sizable strong market.

One should differentiate in a way that leaves a sizable strong market for one's rival.

2.5 Models of Imperfect Competition

Bertrand (Pricing Dilemma): no differentiation, no capacity constraints, choose prices

Hotelling: imperfect substitutes (differentiation), no capacity constraints, choose prices

Cournot: no differentiation, choose capacities/quantities, 'market' determines price

Cournot

Two firms

Each firm chooses a capacity/quantity q_i

Unit cost of capacity for firm i is c_i

If firm i chooses q_i, then market sets unit price for output according to

$$\max\{1 - q_1 - q_2, 0\}$$

Cournot: Finding Equilibrium Quantities

Firm 1 chooses a quantity q_1

Firm 2 chooses a quantity q_2

Market price will be $\max\{1 - q_1 - q_2, 0\}$.

Firm 1's profit is

$$\Pi_1(q_1, q_2) = q_1 \max\{1 - q_1 - q_2, 0\} - c_1 q_1$$

Firm 2's profit is

$$\Pi_2(q_1, q_2) = q_2 \max\{1 - q_1 - q_2, 0\} - c_2 q_2.$$

Cournot: Finding Equilibrium Quantities

A pair (q_1^*, q_2^*) form an equilibrium, if no firm unilaterally has an incentive to choose a different quantity:

$$\Pi_1(q_1^*, q_2^*) \geq \Pi_1(q_1, q_2^*) \; \forall q_1 \neq q_1^*$$
$$\Rightarrow \Pi_1(q_1^*, q_2^*) = \max_{q_1} \Pi_1(q_1, q_2^*)$$

$$\Pi_2(q_1^*, q_2^*) \geq \Pi_2(q_1^*, q_2) \; \forall q_2 \neq q_2^*$$
$$\Rightarrow \Pi_2(q_1^*, q_2^*) = \max_{q_2} \Pi_2(q_1^*, q_2)$$

Cournot: Finding Equilibrium Quantities

Find Firm 1's **best-response function**.

Fix firm 2 at some q_2, how should Firm 1 choose q_1 to maximize its profit?

Solve $\max_{q_1} q_1 \max\{1 - q_1 - q_2, 0\} - c_1 q_1$

Will always choose q_1 so that $1 - q_1 - q_2 \geq 0$. So, solve

$$\max_{q_1} q_1(1 - q_1 - q_2) - c_1 q_1$$

Cournot: Finding Equilibrium Quantities

$$\max_{q_1} q_1(1 - q_1 - q_2) - c_1 q_1$$

Differentiate wrt q_1 holding q_2 fixed and set to zero (check second derivative):

$$q_1 = \frac{1 - q_2 - c_1}{2}$$

This is Firm 1's best-response function.

Similar analysis yields Firm 2's best-response function:

$$q_2 = \frac{1 - q_1 - c_2}{2}$$

Cournot: Finding Equilibrium Quantities

Equilibrium is where best-response functions cross. Why?

$$q_1 = \frac{1 - q_2 - c_1}{2}$$

$$q_2 = \frac{1 - q_1 - c_2}{2}$$

Pair of simultaneous equations, solution is

$$q_1^* = \frac{1 - 2c_1 + c_2}{3}$$

$$q_2^* = \frac{1 - 2c_2 + c_1}{3}$$

These are equilibrium quantities.

What can we do with Cournot?

1. Is there a first mover advantage in capacity?
2. Is competition better than monopoly for consumer surplus?
3. Would increasing the number of competitors increase consumer surplus?
4. Suppose the merger of two firms would reduce production costs. Should we allow the merger?
5. I have an innovation that would reduce costs for an industry. Should I sell it to one firm only? Should I license it?

Cournot: Sequential

Firm 1 moves first and chooses q_1.

Firm 1's choice is irrevocable.

Firm 2 observes Firm 1's choice of q_1 and chooses q_2.

How do their profit compare with simultaneous case?

This is also known as the **Stackelberg model**.

Cournot: Sequential

For simplicity only, assume that $c_1 = c_2 = c < 1$.

Given q_1, Firm 2 will choose q_2 such that (WHY?)

$$q_2 = \frac{1 - q_1 - c}{2}.$$

Market price will be

$$1 - q_1 - q_2 = 1 - q_1 - \frac{1 - q_1 - c}{2} = \frac{1 - q_1 + c}{2}.$$

Firm 1's profit will be

$$q_1 \frac{1 - q_1 + c}{2} - cq_1.$$

Value of q_1 that maximizes this profit is $q_1 = (1 - c)/2$.

Cournot: Sequential

Know $q_1 = (1-c)/2$.
From Firm 2's best-response function know

$$q_2 = \frac{1 - q_1 - c}{2} = \frac{1 - \frac{1-c}{2} - c}{2} = (1-c)/4.$$

Firm 1's profits will be

$$\frac{1-c}{2}(1 - \frac{3(1-c)}{4}) - c\frac{(1-c)}{2} = \frac{(1-c)^2}{8}.$$

Firm 1 makes more profit than simultaneous case because

$$\frac{(1-c)^2}{9} < \frac{(1-c)^2}{8}$$

Cournot Merger

Suppose firm 1 and 2 each have a constant marginal cost of c per unit.

If they merge, their marginal costs would fall to αc per unit where $0 < \alpha < 1$.

Would the merger reduce or increase consumer surplus?

Cournot Merger

Pre-merger

$$q_1 = q_2 = \frac{1-c}{3}$$

Market price will be $1 - q_1 - q_2 = 1 - 2(\frac{1-c}{3}) = \frac{1+2c}{3}$

Post-merger

Merged firm will act like a monopolist

Cournot Merger

Merged firm will choose a quantity q to maximize

$$q(1-q) - \alpha c q$$

Differentiate and set to zero (check second derivative)

$$1 - 2q - \alpha c = 0 \;\Rightarrow\; q = \frac{1-\alpha c}{2}$$

Market price will be

$$1 - \frac{1-\alpha c}{2} = \frac{1+\alpha c}{2}$$

Cournot Merger

Market price post-merger

$$\frac{1+\alpha c}{2}$$

Market price pre-merger

$$\frac{1+2c}{3}$$

If

$$\frac{1+\alpha c}{2} < \frac{1+2c}{3} \Rightarrow$$

merger increases consumer surplus.

Hotelling

On a line between 0 and 1 are a 1000 customers, evenly distributed.

Number of customers in any segment between 0 and 1 is proportional to the length of that segment.

For example, the number of customers between 0 and 0.25, will be 250.

One firm, called LEFT (at 0) and another called RIGHT (at 1) both selling Soma.

Hotelling

Each buyer wants at most one unit of Soma.

Travel costs for buyers is \$1 per mile.

Travel cost to buyer at distance d from LEFT of going to LEFT to purchase a unit of Soma is d.

Cost of going to RIGHT is $1 - d$.

Hotelling

Given prices charged by LEFT and RIGHT, buyers choose based on the **delivered** price.

Delivered price is travel cost plus purchase price.

Delivered price of buyer at distance d purchasing from LEFT, is $d + p_L$, where p_L is the price per unit charged by LEFT.

If p_R is unit selling price of RIGHT, this customer will buy from LEFT if and only if

$$d + p_L < 1 - d + p_R.$$

Hotelling

Strategies of each firm is set of possible price for their offering.

Suppose LEFT chooses p_L and RIGHT chooses p_R. What is LEFT's demand curve?

Customers who buy from LEFT must be at most distance d from LEFT such that:

$$p_L + d < 1 - d + p_R.$$

$$\Rightarrow \ 2d < 1 + p_R - p_L.$$

Therefore, total of

$$\frac{(1 + p_R - p_L) \times 1000}{2}$$

customers go LEFT.

Hotelling

LEFT's demand curve:

$$D_L(p_L, p_R) = \frac{(1 + p_R - p_L) \times 1000}{2}$$

Similarly, RIGHT's demand curve is

$$D_R(p_L, p_R) = \frac{(1 + p_L - p_R) \times 1000}{2}$$

Hotelling

Suppose marginal cost of production of each firm is c.

Assume RIGHT has chosen its price, p_R. What is profit maximizing choice of p_L (as a function of p_R) for LEFT?

LEFT's profit is

$$(p_L - c)\frac{(1 + p_R - p_L) \times 1000}{2}$$

Profit maximizing choice of p_L is

$$p_L = \frac{1 + p_R + c}{2}$$

Similarly

$$p_R = \frac{1 + p_L + c}{2}$$

Hotelling

$$p_L = \frac{1 + p_R + c}{2}$$

$$p_R = \frac{1 + p_L + c}{2}$$

Solving pair of equations for p_L and p_R:

$$p_L = p_R = 1 + c$$

Hotelling

With differing costs marginal costs of c_L and c_R:

$$p_L = 1 + \frac{2c_L + c_R}{3}$$

$$p_R = 1 + \frac{2c_R + c_L}{3}.$$

One Side Pays Delivery

Suppose LEFT pays for delivery.

RIGHT does not.

Does this make LEFT better or worse off than before?

One Side Pays Delivery

Suppose LEFT chooses p_L and RIGHT chooses p_R.

Buyer living distance d from LEFT will purchase from LEFT if

$$p_L < p_R + 1 - d \;\Rightarrow\; d < 1 - p_L + p_R$$

Therefore, LEFT's demand will be $1000 \times (1 - p_L + p_R)$.

This customer will purchase from RIGHT if $p_L > p_R + 1 - d$. Therefore, RIGHT's demand will be

$$1000 \times (p_L - p_R)$$

One Side Pays Delivery

To determine RIGHT's best-response function choose p_R to maximize

$$(p_R - c) \times 1000 \times (p_L - p_R)$$

The solution is

$$p_R = \frac{p_L + c}{2}$$

One Side Pays Delivery

For LEFT, the cost of shipping to buyers within any distance d is the average cost of shipping multiplied by total number of customers within distance d.

From uniform distribution, the average cost of shipping per person is $\frac{d}{2}$. So, total cost of shipping is

$$1000 \times d \times \frac{d}{2}.$$

Given prices p_L and p_R, LEFT serves all customers within distance $1 - p_L + p_R$.

One Side Pays Delivery

LEFT's profit is

$$(p_L - c) \times (1 - p_L + p_R) \times 1000 - 1000 \times (1 - p_L + p_R) \times \frac{1 - p_L + p_R}{2}$$

The profit maximizing choice of p_L is

$$\frac{2 + 2p_R + c}{3}$$

One Side Pays Delivery

$$p_R = \frac{p_L + c}{2}$$

$$p_L = \frac{2 + 2p_R + c}{3}$$

$$= \frac{2 + (p_L + c) + c}{3}$$

$$\Rightarrow \frac{2}{3} p_L = \frac{2(1 + c)}{3}$$

$$\Rightarrow p_L = 1 + c$$

$$\Rightarrow p_R = \frac{1}{2} + c$$

One Side Pays Delivery

$$p_L = 1 + c$$

$$p_R = \frac{1}{2} + c$$

LEFT's demand is: $1000 \times (1 - p_L + p_R) = 1000 \times \frac{1}{2} = 500$

RIGHT's demand is: $1000 \times (p_L - p_R) = 1000 \times \frac{1}{2} = 500$

Hotelling

Suppose LEFT and RIGHT each have a constant marginal cost of 0.1 per unit.

If they merge, their marginal costs would fall to 0.05 per unit.

Would the merger reduce or increase consumer surplus?

Hotelling

Pre-merger

$$p_L = p_R = 1 + c = 1 + 0.1 = 1.1$$

Post-merger

Merged firm will act like a monopolist

Hotelling

Merged firm will choose p_L and p_R to maximize combined profit

$$\max(p_L - 0.05)\frac{(1 + p_R - p_L) \times 1000}{2} + (p_R - 0.05)\frac{(1 - p_R + p_L) \times 1000}{2}$$

Given symmetry, we guess that $p_L = p_R = p$

$$\max(p - 0.05)\frac{(1) \times 1000}{2} + (p - 0.05)\frac{(1) \times 1000}{2}$$

Hotelling

$$\max(p - 0.05)\frac{(1) \times 1000}{2} + (p - 0.05)\frac{(1) \times 1000}{2}$$

Optimization problem is unbounded!

Model does not contain upper limit on what a buyer will pay.

Clear from above that merged firm will simply increase price as much as it can.

Merged firm does not pass along cost savings to buyers.

Inferior Entrant

Can an entrant with a higher cost than you, and an inferior product to yours, profitably enter your market?

Inferior Entrant

1. There are 100 buyers.
2. Entrant chooses a capacity N.
3. Entrant chooses a per unit price p.
4. Incumbent offers one price to all buyers. Incumbent has sufficient capacity to serve all buyers.
5. Buyers choose offer that yields largest surplus. In case of tie, they choose incumbent.
6. Game ends.

Inferior Entrant

	Incumbent	Entrant
Reservation Price	\$200	\$160
Unit Cost	\$100	\$120

Inferior Entrant

To succeed, entrant must choose p and N so as to encourage Incumbent to let it be.

If Incumbent is 'passive' profit is

$$(200 - 100) \times (100 - N)$$

If Incumbent 'fights', it chooses price of $p + 40$ (why?) yielding profit of

$$(p + 40 - 100) \times 100$$

Entrant needs

$$(p + 40 - 100) \times 100 \leq (200 - 100) \times (100 - N)$$

Inferior Entrant

Entrant must choose p and N so that

$$(p + 40 - 100) \times 100 \leq (200 - 100) \times (100 - N)$$

$$\Rightarrow \ N \leq 160 - p$$

Entrant's problem is

$$\max (p - 120) \times N$$

$$\text{s.t. } N \leq 160 - p$$

Inferior Entrant

Entrant's problem is

$$\max(p - 120) \times N$$

$$\text{s.t. } N \leq 160 - p$$

Because Entrant's profit is strictly increasing in N, it chooses $N = 160 - p$.

Entrant's problem becomes

$$\max(p - 120)(160 - p)$$

The profit maximizing $p = 140$. The optimal capacity is $N = 160 - 140 = 20$.

Preferences

A bundle of goods will be represented by an n dimensional non-negative vector x.

The ith component of x, denoted x_i will denote the quantity of good i in the bundle x.

Suppose our goods are sugar, salt and butter (i.e. $n = 3$). Then the vector $(2, 3, 2.7)$ means a bundle that contains 2 units of sugar, 3 units of salt and 2.7 units of butter.

Do mean 'goods', so there are no 'bads'.

Preferences

Consumers have preferences over bundles, i.e., non-negative vectors.

For any *any* two bundles of goods, x and y, a consumer will say exactly one of the following

1. She (strictly) prefers x to y, written $x \succ y$.
2. She (strictly) prefers y to x, written $y \succ x$.
3. She is indifferent between x and y, written $x \sim y$.

The consumers preferences are said to be **complete** in this case.

If consumer prefers x at least as much as y this is written as $x \succeq y$. We also say that x is *weakly preferred* to y.

Preferences

Given a set of bundles to choose from, a consumer will choose their most preferred bundle from the set.

Preferences

Preferences must satisfy the following consistency conditions:

1. **Monotonicity** More of a good thing is better (and certainly no worse) than less of it other things held fixed.
2. **Irreflexivity** Given two identical bundles, you should never prefer one to the other.
3. **Transitivity** If x is preferred to y and y is preferred to z *then* x is preferred to z.
   If I prefer apples to oranges and oranges to grapefruit, then I prefer apples to grapefruit.

Utility Function

A utility function is a **numerical representation** of a preference ordering over the set of bundles.

A function U represents a preference ordering $\succ$ satisfying conditions 1 to 3 above if whenever $x \succeq y$ then, $U(x) \geq U(y)$ and vice-versa.

A utility function will be monotone: $x \geq y \Rightarrow U(x) \geq U(y)$.

Note $x \geq y$ means $x_i \geq y_i$ for all components $i = 1, \ldots, n$.

A utility function is strictly monotone: $x \geq y$ and $x \neq y \Rightarrow U(x) > U(y)$.

Utility Function

1. x is strictly preferred to y if $U(x) > U(y)$.
2. x is weakly preferred to y if $U(x) \geq U(y)$.
3. one is indifferent between x and y if $U(x) = U(y)$.

Utility Function

Two *different* utility functions can represent the *same* preferences.

$$U_1(x_1, x_2) = x_1^{1/3} x_2^{2/3}, U_2(x_1, x_2) = \ln x_1 + 2 \ln x_2 - 5$$

To show they represent the same preferences must verify

1. $U_1(x_1, x_2) \geq U_1(y_1, y_2) \Rightarrow U_2(x_1, x_2) \geq U_2(y_1, y_2)$
2. $U_2(x_1, x_2) \geq U_2(y_1, y_2) \Rightarrow U_1(x_1, x_2) \geq U_1(y_1, y_2)$

Utility Function

Verify that $U_1(x_1, x_2) \geq U_1(y_1, y_2) \Rightarrow U_2(x_1, x_2) \geq U_2(y_1, y_2)$

Suppose that $U_1(x_1, x_2) \geq U_1(y_1, y_2)$. Then

$$
\begin{aligned}
& x_1^{1/3} x_2^{2/3} \geq y_1^{1/3} y_2^{2/3} \\
\Rightarrow & \ln(x_1^{1/3} x_2^{2/3}) \geq \ln(y_1^{1/3} y_2^{2/3}) \\
\Rightarrow & \frac{1}{3} \ln x_1 + \frac{2}{3} \ln x_2 \geq \frac{1}{3} \ln y_1 + \frac{2}{3} \ln y_2 \\
\Rightarrow & \ln x_1 + 2 \ln x_2 - 5 \geq \ln y_1 + 2 \ln y_2 - 5 \\
\Rightarrow & U_2(x_1, x_2) \geq U_2(y_1, y_2).
\end{aligned}
$$

The other direction is left to you as exercise.

Consumer Choice Problem

Consumer has income I and p_i is the per unit price of good $i = 1, \ldots, n$.

Consumer chooses a bundle $(x_1^*, x_2^*, \ldots, x_n^*)$ that solves:

$$\max U(x_1, x_2, \ldots, x_n)$$

$$\text{s.t.} \sum_{i=1}^{n} p_i x_i \leq I$$

$$x_i \geq 0 \ \forall i = 1, 2, \ldots, n$$

Consumer Choice Problem

Consumer chooses a bundle x^* that solves:

$$\max U(x_1, x_2, \ldots, x_n)$$

$$\text{s.t.} \sum_{i=1}^{n} p_i x_i \leq I$$

$$x_i \geq 0 \ \forall i = 1, 2, \ldots, n$$

For consumer choice problem to have a solution need u to be continuous.

To solve consumer choice problem using Lagrangean, need U to be **concave** or **quasi-concave**.

Convex Set

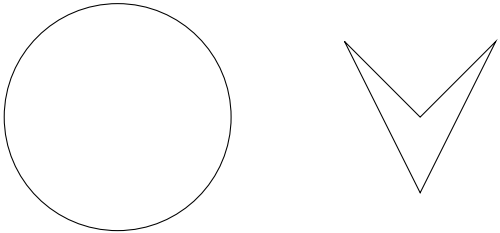
A set C of vectors is convex if for every $\lambda \in [0,1]$ and pair of $x, y \in C$ the following is true:

$$\lambda x + (1-\lambda)y \in C$$

The straight line joining any two points in C lies entirely in C.

Convex Set

The one on the left is convex while the one on the right is not.



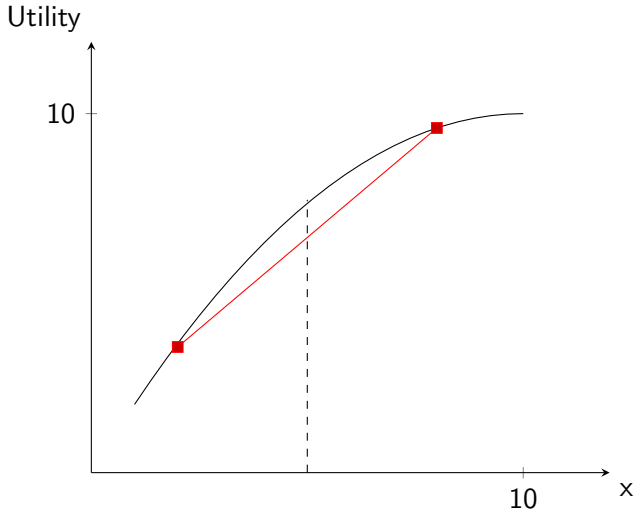
Concave Utility

A utility function (indeed any function) U whose domain is a convex set is concave if for each $\lambda \in [0, 1]$ and for any two vectors x and y in the domain of U

$$U(\lambda x + (1 - \lambda)y) \geq \lambda U(x) + (1 - \lambda)U(y)$$

The utility of the 'average' is larger than the the 'average' of the utility.

Concave Utility



Concave Utility

If x is a number and $U(x)$ is twice differentiable, then concavity of U is equivalent to

$$\frac{d^2U(x)}{dx^2} \leq 0 \ \forall x$$

If $x = (x_1, x_2)$ and the second order partials of $U(x)$ exist, concavity of u can be determined by a determinant test involving the Hessian:

$$\frac{\partial^2 U}{\partial x_1^2}, \frac{\partial^2 U}{\partial x_2^2} \leq 0 \ \forall x_1, x_2$$

$$(\frac{\partial^2 U}{\partial x_1^2})(\frac{\partial^2 U}{\partial x_2^2}) - (\frac{\partial^2 U}{\partial x_1 x_2})(\frac{\partial^2 U}{\partial x_2 x_1}) \geq 0 \ \forall x_1, x_2$$

Concave Utility

If a consumer's utility function is continuous and concave, then, the optimal solution to her consumer choice problem is given by the first order conditions of the Lagrangian.

No need to check the second order condition.

The first order conditions need not have a solution so one is still expected to check the corners.

Indifference Curve

The **indifference curve** of function U at threshold q is the set of vectors:

$$I_q(U) = \{x : U(x) = q\}.$$

All bundles that deliver utility *exactly* q.

Indifference Curve

Important properties:

1. Indifference curves farther from the origin represent higher utilities.
2. Indifference curves slope downward.
3. Indifference curves cannot cross.

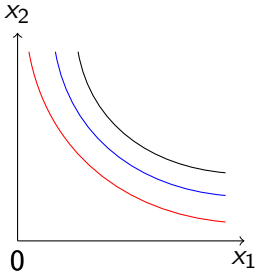


Figure: Indifference curves at various thresholds

Upper Contour Set

The **upper countour** set of function U at threshold q is the set of vectors:

$$C_q(U) = \{x : U(x) \geq q\}.$$

All bundles that deliver a utility of *at least* q.

Upper Contour Set

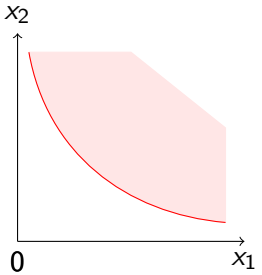


Figure: Upper Contour Set

Quasi-Concavity

Utility function U is **quasi-concave** if $C_q(U)$ is a convex set for each value of q

If U is quasi-concave, then FOC for optimality suffice.

If U is concave, $C_q(U)$ is a convex set for each value of q.

U is concave implies quasi-concavity but not the reverse.

Why Quasi-concavity of Utility?

Consumer would buy only *one* good.

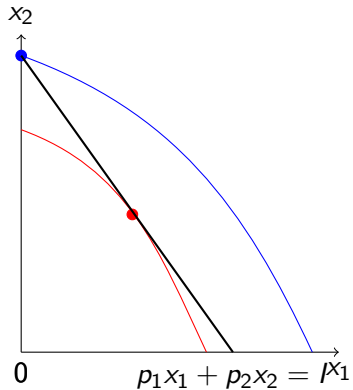


Figure: Strictly Concave Indifference Curves

Why Quasi-concavity of Utility?

FOC for optimality suffice.

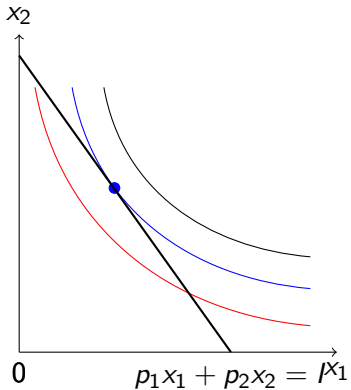


Figure: Convex Upper Contour Set

Consumer Problem

Two goods denoted 1 and 2.

x_i = quantity of good i selected by agent for $i = 1, 2$

p_i = price per unit of good i

Utility of agent for bundle (x_1, x_2) is $U(x_1, x_2) = x_1 x_2$.

I is income of agent

$$\max x_1 x_2$$
$$\text{s.t. } p_1 x_1 + p_2 x_2 \leq I$$
$$x_1, x_2 \geq 0$$

Concavity vs. Quasi-Concavity

$U(x_1, x_2) = x_1 x_2$ is quasi-concave because (*visually*) upper contour sets are convex.

$U(x_1, x_2) = x_1 x_2$ is *not* concave.

Concavity test:

$$\frac{\partial^2 U}{\partial x_1^2}, \frac{\partial^2 U}{\partial x_2^2} \leq 0 \ \forall x_1, x_2$$

$$(\frac{\partial^2 U}{\partial x_1^2})(\frac{\partial^2 U}{\partial x_2^2}) - (\frac{\partial^2 U}{\partial x_1 x_2})(\frac{\partial^2 U}{\partial x_2 x_1}) \geq 0 \ \forall x_1, x_2$$

Consumer Problem: Example

Two goods, 1 and 2 with $U(x_1, x_2) = 16x_1^4x_2^8$.

$$\begin{aligned} &\max 16x_1^4x_2^8 \\ \text{s.t. } &p_1x_1 + p_2x_2 \leq I \\ &x_1, x_2 \geq 0 \end{aligned}$$

Consumer Problem: Example

$$\max 16x_1^4x_2^8$$
$$\text{s.t. } p_1x_1 + p_2x_2 = I$$
$$x_1, x_2 \geq 0$$

How do we know budget constraint holds with equality?

$$L(x_1, x_2, \lambda) = 16x_1^4x_2^8 + \lambda(I - p_1x_1 - p_2x_2)$$
$$\frac{\partial L}{\partial x_1} = 64x_1^3x_2^8 - \lambda p_1 = 0$$
$$\frac{\partial L}{\partial x_2} = 128x_1^4x_2^7 - \lambda p_2 = 0$$
$$\frac{\partial L}{\partial \lambda} = I - p_1x_1 - p_2x_2 = 0$$

Consumer Problem: Example

$$\frac{\partial L}{\partial x_1} = 64x_1^3x_2^8 - \lambda p_1 = 0$$

$$\frac{\partial L}{\partial x_2} = 128x_1^4x_2^7 - \lambda p_2 = 0$$

$$\frac{\partial L}{\partial \lambda} = I - p_1x_1 - p_2x_2 = 0$$

$$x_1 = \frac{I}{3p_1},\ x_2 = \frac{2I}{3p_2}$$

Consumer Problem: Example

$$V(p_1, p_2, I) = \max 16x_1^4 x_2^8$$
$$\text{s.t. } p_1x_1 + p_2x_2 \leq I$$
$$x_1, x_2 \geq 0$$

$V(p_1, p_2, I)$ is called the **indirect** utility function.

Denote by $x_1(p_1, p_2, I)$ and $x_2(p_1, p_2, I)$ the optimal solution to the above problem.

$$V(p_1, p_2, I) = U(x_1(p_1, p_2, I), x_2(p_1, p_2, I))$$

Consumer Problem: Example

$$V(p_1, p_2, I) = 16x_1(p_1, p_2, I)^4 x_2(p_1, p_2, I)^8$$
$$= 16(\frac{I}{3p_1})^4(\frac{2I}{3p_2})^8$$

Indirect utility function tells us how utility varies with price and income.

Marginal This & That

$\frac{\partial U}{\partial x_i}$ is marginal utility wrt good i.

$\frac{\partial V}{\partial I}$ is marginal utility of income.

$\frac{\frac{\partial U}{\partial x_i}}{\frac{\partial U}{\partial x_j}}$ is the marginal rate of substitution (MRS) between good i and j.

MRS is the negative of the slope of the indifference curve.

Equi-marginal Rule

To maximize utility, consumer will allocate income between two goods in such a way as to equalize the ratio of the marginal utilities to the ratio of the prices.

$$\frac{\frac{\partial U(x_1(p_1,p_2,I))}{\partial x_1}}{\frac{\partial U(x_2(p_1,p_2,I))}{\partial x_2}} = \frac{p_1}{p_2}$$

Only true if $(x_1(p_1, p_2, I), x_2(p_1, p_2, I))$ is interior to budget line and not otherwise.

Equi-marginal Rule

$$\max U(x_1, x_2)$$
$$p_1 x_1 + p_2 x_2 \leq I$$

Write down the Lagrangean:

$$\mathcal{L} = U(x_1, x_2) + \lambda(I - p_1 x_1 - p_2 x_2)$$

First order conditions:

$$\frac{\partial U}{\partial x_1} - \lambda p_1 = 0$$
$$\frac{\partial U}{\partial x_2} - \lambda p_2 = 0$$

Divide one by the other to get the equi-marginal rule.

Equi-marginal Rule

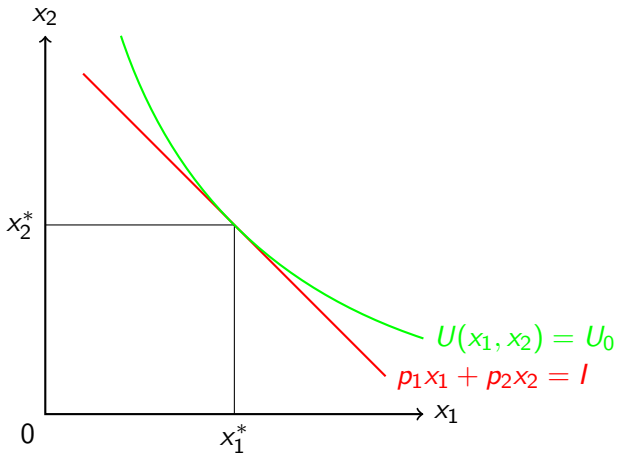


Figure: Utility maximization

Failure of Equi-marginal Rule

$$\max U(x_1, x_2) = 2x_1 + 3x_2$$
$$\text{s.t. } p_1 x_1 + p_2 x_2 \leq I$$
$$x_1, x_2 \geq 0$$

$$\frac{\partial U}{\partial x_1} - \lambda p_1 = 2 - \lambda p_1 = 0$$
$$\frac{\partial U}{\partial x_2} - \lambda p_2 = 3 - \lambda p_2 = 0$$

Equi-Marginal rule forces $\frac{p_1}{p_2} = \frac{2}{3}$ which need not be true.

Failure of Equi-marginal Rule

$$\max U(x_1, x_2) = 2x_1 + 3x_2$$

$$\text{s.t. } p_1 x_1 + p_2 x_2 \leq I$$

$$x_1, x_2 \geq 0$$

If $\frac{2}{p_1} > \frac{3}{p_2}$ optimal solution is $x_1 = \frac{I}{p_1}, x_2 = 0$.

If $\frac{2}{p_1} < \frac{3}{p_2}$ optimal solution is $x_1 = 0, x_2 = \frac{I}{p_2}$.

If $\frac{2}{p_1} = \frac{3}{p_2}$ optimal solution is any point on budget line.

Failure of Equi-marginal Rule

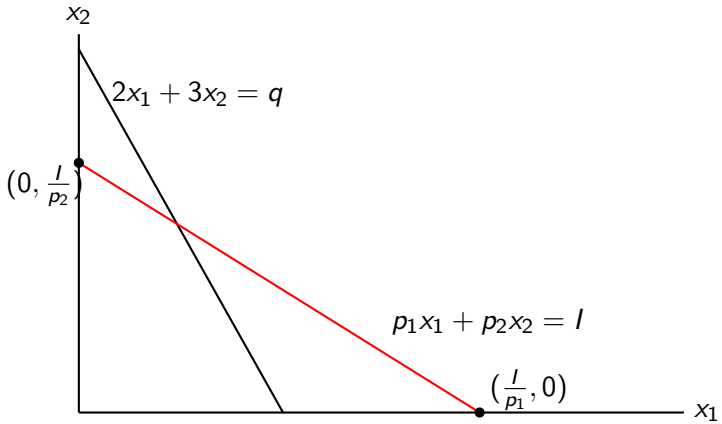


Figure: A non-interior optimal

Demand vs. Price

Suppose utility function of $U(x_1, x_2) = x_1 x_2$.

Consumer choice problem:

$$\max x_1 x_2$$

$$\text{s.t. } p_1 x_1 + p_2 x_2 \leq I$$

$$x_1, x_2 \geq 0$$

FOC from Lagrangean:

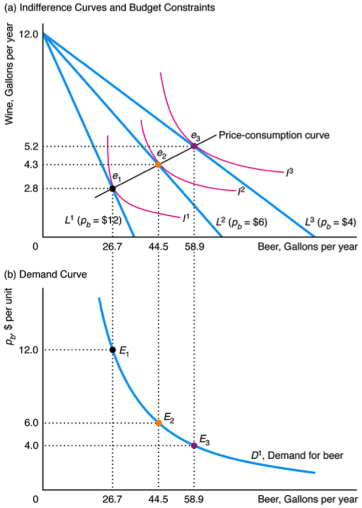
$$x_2 - \lambda p_1 = 0$$

$$x_1 - \lambda p_2 = 0$$

$$I - p_1 x_1 - p_2 x_2 = 0$$

$$x_1 = \frac{I}{2p_1}, \; x_2 = \frac{I}{2p_2}$$

Demand vs. Price



Substitutes vs. Complements

Good 1 is a substitute for good 2 if $\frac{dx_1}{dp_2} > 0$.

Good 1 and 2 are perfect substitutes if MRS is constant, i.e., indifference curves are straight lines. $U(x_1, x_2) = 2x_1 + 4x_2$

Good 1 is a complement for good 2 if $\frac{dx_1}{dp_2} < 0$.

Good 1 and 2 are perfect complements if indifference curve is 'L' shaped.
$U(x_1, x_2) = \min\{x_1, x_2\}$

Marginal Utility of Income

$$\max U(x_1, x_2)$$

$$p_1 x_1 + p_2 x_2 \leq I$$

If λ^* is optimal Lagrange multiplier, then,

$$\frac{\partial V(p_1, p_2, I)}{\partial I} = \lambda^*$$

Marginal Utility of Income

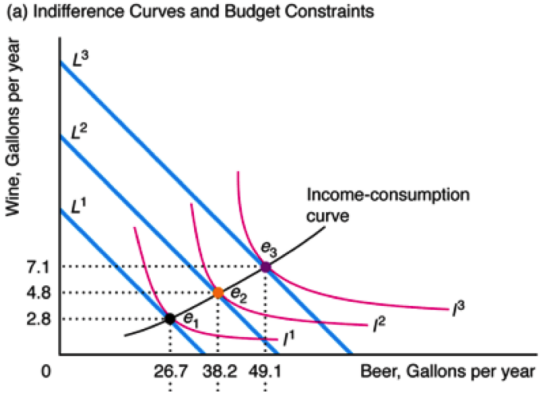
$$\begin{aligned}\frac{\partial V}{\partial I} =&(\frac{\partial U}{\partial x_1})(\frac{\partial x_1}{\partial I}) + (\frac{\partial U}{\partial x_2})(\frac{\partial x_2}{\partial I}) \\ =&\lambda^* p_1 \frac{\partial x_1}{\partial I} + \lambda^* p_2 \frac{\partial x_2}{\partial I} \\ =&\lambda^*(p_1 \frac{\partial x_1}{\partial I} + p_2 \frac{\partial x_2}{\partial I}) = \lambda^*\end{aligned}$$

The last equality follows from differentiating both sides of the budget line with respect to I:

$$\begin{aligned}&p_1 x_1(p_1, p_2, I) + p_2 x_2(p_1, p_2, I) = I \\ \Rightarrow &p_1 \frac{\partial x_1}{\partial I} + p_2 \frac{\partial x_2}{\partial I} = 1\end{aligned}$$

Demand vs. Income

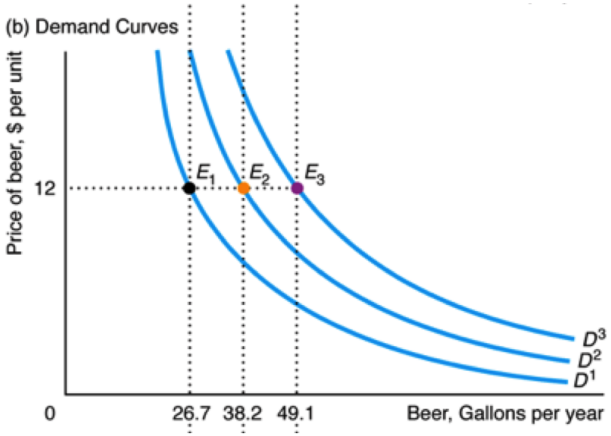
The **income-consumption curve:** consumer chooses *new* optimal bundle when income changes.



Income changes *shift* the budget constraint in or out.

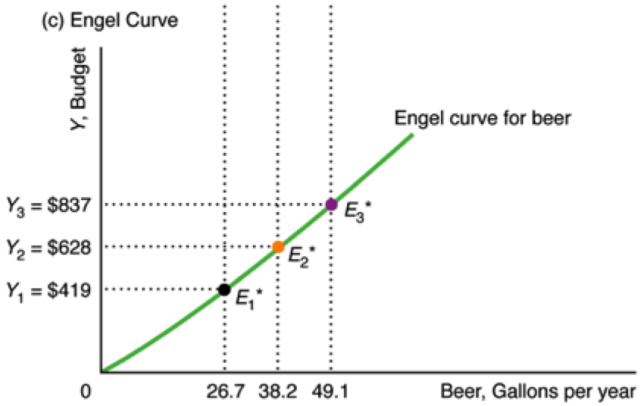
Demand vs. Income

An *increase* in income, holding prices and preference constant, causes a *shift* of the **demand curve**.



Demand vs. Income

Engel curve shows the direct relationship between demand and income.



Income Elasticitiy

How does percentage quantity demanded *change* with percentage change in income?

$$\begin{aligned}\xi &= \frac{\%\text{ change in quantity demanded}}{\%\text{ change in income}} \\ &= \frac{\partial x_i(p_1, p_2, I)}{\partial I}\frac{I}{x_i(p_1, p_2, I)}\end{aligned}$$

1. Normal good: demand *more* as income increases
   - Luxury good: demand rises more than income, $\xi > 1$
   - Necessity good: demand rises less than income, $0 \leq \xi \leq 1$
2. Inferior good: demand *less* as income increases, $\xi < 0$.

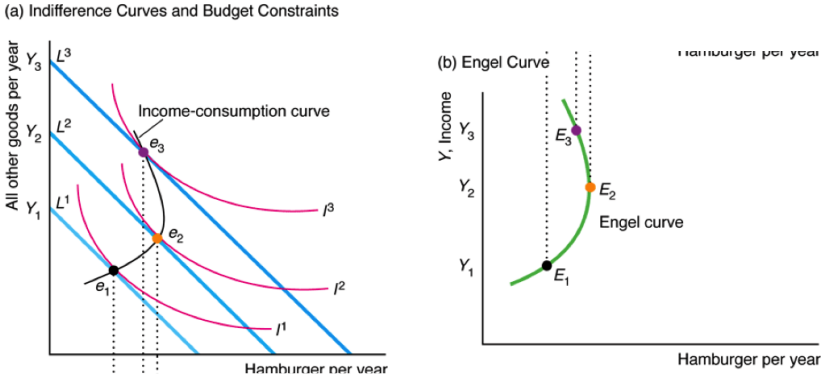
Example

Suppose $U(x_1, x_2) = x_1^{0.4} x_2^{0.6}$, yielding demand functions

$$x_1(p_1, p_2, I) = \frac{0.4I}{p_1} \text{ and } x_2(p_1, p_2, I) = \frac{0.6I}{p_2}$$

Derive the income elasticity and Engel curve for x_1.

A good which is both normal and inferior



Income Tax vs Sales Tax

Which is better?

To ensure apples with apples comparison, fix sales tax, say. Then compare with an income tax that would raise an equivalent amount of money.

How do we measure 'better'?

By determining which gives consumer more utility.

Income Tax, $U(x_1, x_2) = x_1 x_2$

Suppose income tax of fraction t. Consumer problem is

$$\max x_1 x_2$$

$$p_1 x_1 + p_2 x_2 \leq (1 - t) I$$

$$x_1, x_2 \geq 0$$

Note, $U(x_1, x_2) = x_1 x_2$ is quasi-concave.

FOC from Lagrangean:

$$x_2 - \lambda p_1 = 0$$

$$x_1 - \lambda p_2 = 0$$

$$(1 - t) I - p_1 x_1 - p_2 x_2 = 0$$

Income Tax, $U(x_1, x_2) = x_1 x_2$

FOC from Lagrangean:

$$x_2 - \lambda p_1 = 0$$

$$x_1 - \lambda p_2 = 0$$

$$(1-t)I - p_1 x_1 - p_2 x_2 = 0$$

$$x_1^t = \frac{(1-t)I}{2p_1}, \; x_2^t = \frac{(1-t)I}{2p_2}$$

$$V^t(p_1, p_2, I) = \frac{(1-t)^2 I^2}{4p_1 p_2}$$

Sales Tax, $U(x_1, x_2) = x_1 x_2$

Suppose sales tax of $1 + r$ on good 2. Consumer problem is

$$\max x_1 x_2$$

$$p_1 x_1 + (1 + r) p_2 x_2 \leq I$$

$$x_1, x_2 \geq 0$$

FOC from Lagrangean:

$$x_2 - \lambda p_1 = 0$$

$$x_1 - \lambda (1 + r) p_2 = 0$$

$$I - p_1 x_1 - (1 + r) p_2 x_2 = 0$$

Sales Tax, $U(x_1, x_2) = x_1 x_2$

FOC from Lagrangean:

$$x_2 - \lambda p_1 = 0$$

$$x_1 - \lambda(1 + r)p_2 = 0$$

$$I - p_1 x_1 - (1 + r)p_2 x_2 = 0$$

$$x_1^r = \frac{I}{2p_1}, \; x_2^r = \frac{I}{2(1 + r)p_2}$$

$$V^r(p_1, p_2, I) = \frac{I^2}{4(1 + r)p_1 p_2}$$

Income vs. Sales Tax

Fix r. How can we choose t so that income and sales tax raise same \$ amount?

$$tI = rp_2x_2^r = \frac{rI}{2(1+r)}$$

$$\Rightarrow \quad t = \frac{r}{2(1+r)}$$

Income vs. Sales Tax

$$t = \frac{r}{2(1+r)}$$

Which is bigger, $V^t(p_1, p_2, I)$ or $V^r(p_1, p_2, I)$?

$$V^t(p_1, p_2, I) = \frac{(1-t)^2 I^2}{4p_1 p_2} > \frac{I^2}{4(1+r)p_1 p_2} = V^r(p_1, p_2, I)$$

$$(1-t)^2 > \frac{1}{1+r}$$

$$(1 - \frac{r}{2(1+r)})^2 > \frac{1}{1+r}$$

$$\frac{(2+r)^2}{4} > 1 + r$$

Income vs. Sales Tax

Under sales tax $1 + r$, consumer solves

$$\max U(x_1, x_2)$$

$$\text{s.t. } p_1 x_1 + (1 + r) p_2 x_2 \leq I$$

$$x_1, x_2 \geq 0$$

Let (x_1^r, x_2^r) be an optimal solution. Amount R raised in taxes is

$$R = r p_2 x_2^r$$

Choose an income tax rate t so that $tI = R$.

Income vs. Sales Tax

Under income tax, consumer solves

$$\max U(x_1, x_2)$$

$$\text{s.t. } p_1 x_1 + p_2 x_2 \leq (1-t)I = I - tI = I - R$$

$$x_1, x_2 \geq 0$$

Let (x_1^t, x_2^t) be an optimal solution.

Observe (x_1^r, x_2^r) is a feasible solution to the optimization problem above because

$$p_1 x_1^r + p_2 x_2^r = p_1 x_1^r + (1+r) p_2 x_2^r - r p_2 x_2^r = I - R$$

Therefore $U(x_1^t, x_2^t) \geq U(x_1^r, x_2^r)$.

Substitution & Income Effect

A price increase has both

- a *substitution effect*: the relative price has changed, and
- an *income effect*: fewer bundles affordable

holding constant other prices, income and preferences.

The *total* price effect is the sum of the substitution and income effects.

Substitution & Income Effect

Substitution Effect: Change in demand resulting from a change in the price ratio that leaves utility unchanged.

Income Effect: Change in demand resulting from a change in income (movement from the initial indifference curve to the final indifference curve), leaving the price ratio unchanged.

total effect = substitution effect + income effect

Pure Income Effect

One good, consumer choice problem:

$$\max U(x_1)$$

$$\text{s.t. } p_1 x_1 \leq I$$

$$x_1 \geq 0$$

As utility is monotone, optimal quantity $x_1 = \frac{I}{p_1}$.

Now raise price to $p_1 + \Delta$. Utility maximizing bundle is now

$$x_1 = \frac{I}{p_1 + \Delta} = \frac{\frac{p_1 I}{p_1 + \Delta}}{p_1}$$

Effect of price increase equivalent to a reduction in income by a factor of $\frac{p_1}{p_1 + \Delta}$.

Pure Substitution Effect

$$\max x_1 + x_2$$

$$\text{s.t. } p_1 x_1 + p_2 x_2 \leq I$$

$$x_1, x_2 \geq 0$$

If $p_1 < p_2$ optimal solution is $x_1 = \frac{I}{p_1}, x_2 = 0$.

If $p_1 > p_2$ optimal solution is $x_1 = 0, x_2 = \frac{I}{p_2}$.

If $p_1 = p_2$ optimal solution is any point on budget line.

Suppose the initial price $p_1 < p_2$ and initial consumption of good 1 is $\frac{I}{p_1}$. Increase price of good 1 to $p_1 + \Delta > p_2$, and new consumption of good 1 is 0. This change in good 1's consumption cannot be mimicked by a change in income.

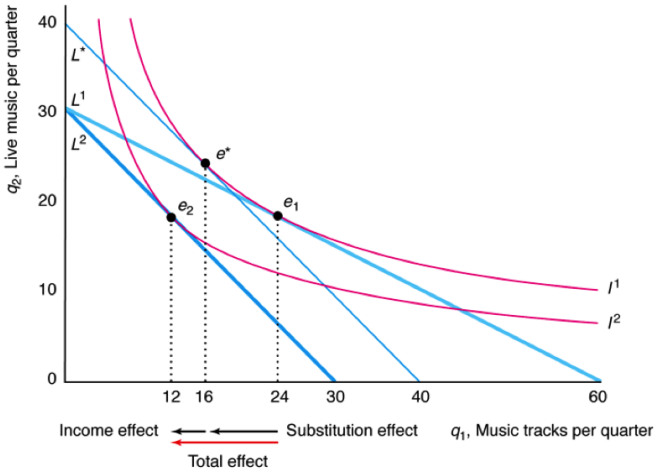
Income & Substitution Effect

Initially prices of (p_1, p_2), initial budget line is L^1 and initial consumption of $e_1 = (x_1, x_2)$.

Increase price of good 1 to $p_1 + \Delta$ shifts budget line to L^2 and new consumption is $e_2 = (y_1, y_2)$ with $y_1 < x_1$.

L^* is a fictitious budget line parallel to L^2 that allows consumer to maintain initial utility at price $(p_1 + \Delta, p_2)$.

The picture: normal good



Example

Suppose $u(x_1, x_2) = x_1^{0.4} x_2^{0.6}$, yielding demand functions

$$x_1(p_1, p_2, I) = 0.4I/p_1 \text{ and } x_2(p_1, p_2, I) = 0.6I/p_2$$

Let $p_1 = 0.5$, $p_2 = 1$ and $I = 30$, and consider a price increase to $p_1' = 1$.

Find the substitution, income and total price effect.

Example

To compute point e_1 (utility maximizing bundle at initial prices)

we use

$$x_1(p_1, p_2, I) = 0.4I/p_1 \text{ and } x_2(p_1, p_2, I) = 0.6I/p_2$$

Plug in $p_1 = 0.5$, $p_2 = 1$ and $I = 30$.

$$x_1 = \frac{0.4 \times 30}{0.5} = 24, \; x_2 = \frac{0.6 \times 30}{1} = 18$$

Example

To compute point e_2 (utility maximizing bundle at new prices)

we use

$$x_1(p_1, p_2, I) = 0.4I/p_1 \text{ and } x_2(p_1, p_2, I) = 0.6I/p_2$$

Plug in $p_1 = 1$, $p_2 = 1$ and $I = 30$.

$$x_1 = \frac{0.4 \times 30}{1} = 12, \; x_2 = \frac{0.6 \times 30}{1} = 18$$

Example

To compute point e^*, bundle that gives same utility as e_1 but wrt new prices, we need to find 'new' income, Y.

We use

$$x_1(p_1, p_2, I) = 0.4I/p_1 \text{ and } x_2(p_1, p_2, I) = 0.6I/p_2$$

Plug in $p_1 = 1$, $p_2 = 1$ and $I = Y$.

$$x_1 = \frac{0.4 \times Y}{1} = 0.4Y \; x_2 = \frac{0.6 \times Y}{1} = 0.6Y$$

Example

Utility is $(0.4)^{0.4}(0.6)^{0.6}Y$ and this must be set equal to utility of bundle e_1:

$$(0.4)^{0.4}(0.6)^{0.6}Y = (24)^{0.4}(18)^{0.6} \Rightarrow Y = 10 \times 6^{0.4} \times 3^{0.6}$$

Therefore, $x_1 = 0.4 \times Y = 0.4 \times 10 \times 6^{0.4} \times 3^{0.6} = 15.83$

(in picture this rounded to 16)

$x_2 = 0.6Y = 0.6 \times 10 \times 6^{0.4} \times 3^{0.6} = 23.75$

How Should One Measure Benefit of a Price Change?

Compute maximum utility subject to budget constraint at current prices.

Change price of good 1, say.

Compute maximum utility subject to new budget constraint.

Compute change in utility.

What is wrong with this?

How Should One Measure Benefit of a Price Change?

Scaling one's utility function by 10 or 100 does not change the underlying preferences only the magnitude of the utils.

If a change reduces one person's utility but raises another, how can we decide if the change is, on balance, a good thing?

The trick is to convert this question into one about money.

Expenditure Problem

Find a minimum cost bundle that achieves utility of U.

$$E(p_1, p_2, U) = \min p_1 x_1 + p_2 x_2$$
$$\text{s.t. } u(x_1, x_2) = U$$
$$x_1, x_2 \geq 0$$

$E(p_1, p_2, U)$ is called the expenditure function.

Optimal solution is denoted $(x_1^c(p_1, p_2, U), x_2^c(p_1, p_2, U))$ (called compensated demand).

$$E(p_1, p_2, U) = \min p_1 x_1 + p_2 x_2$$
$$\text{s.t. } u(x_1, x_2) = U$$
$$x_1, x_2 \geq 0$$

Write down the Lagrangean:

$$\mathcal{L} = p_1 x_1 + p_2 x_2 + \lambda(U - u(x_1, x_2))$$

First order conditions:

$$p_i = \lambda \frac{\partial u}{\partial x_i} \text{ for } i = 1, 2$$

Compensating Variation

Initial prices are (p_1, p_2)

New prices are (p_1', p_2) where $p_1' > p_1$.

$$CV = E(p_1', p_2, U) - E(p_1, p_2, U)$$

How much extra does one have to pay consumer to leave them with same utility.

Compensating Variation

$$E(p_1, p_2, U) = p_1 x_1^c(p_1, p_2, U) + p_2 x_2^c(p_1, p_2, U)$$

$$\frac{\partial E(p_1, p_2, U)}{\partial p_1} = x_1^c(p_1, p_2, U) + p_1 \frac{\partial x_1^c(p_1, p_2, U)}{\partial p_1} + p_2 \frac{\partial x_2^c(p_1, p_2, U)}{\partial p_1}$$

Use FOC: $p_i = \lambda \frac{\partial u}{\partial x_i}$

$$= x_1^c + \lambda \left[\frac{\partial u}{\partial x_1} \frac{\partial x_1^c}{\partial p_1} + \frac{\partial u}{\partial x_2} \frac{\partial x_2^c}{\partial p_1} \right] = x_1^c(p_1, p_2, U)$$

Compensating Variation

Why

$$\frac{\partial u}{\partial x_1}\frac{\partial x_1^c}{\partial p_1} + \frac{\partial u}{\partial x_2}\frac{\partial x_2^c}{\partial p_1} = 0$$

Recall $u(x_1^c(p_1, p_2, U), x_2^c(p_1, p_2, U)) = U$

Differentiate both sides wrt p_1:

$$\frac{\partial u}{\partial x_1}\frac{\partial x_1^c}{\partial p_1} + \frac{\partial u}{\partial x_2}\frac{\partial x_2^c}{\partial p_1} = 0$$

Compensating Variation

$$\frac{\partial E(p_1, p_2, U)}{\partial p_1} = x_1^c(p_1, p_2, U)$$

$$CV = E(p_1', p_2, U) - E(p_1, p_2, U) = \int_{p_1}^{p_1'} x_1^c(t, p_2, U)dt$$

Problem: we never observe $x_1^c(t, p_2, U)$.

So, use $\int_{p_1}^{p_1'} x_1(t, p_2, I)dt$ instead (Consumer Surplus).

If income effect is small, CS is good approximation of CV.

Central vs Decentralized

The immediate 'common sense' answer to the question 'What will an economy motivated by individual greed and controlled by a very large number of different agents look like?' is probably: There will be chaos.

Exchange Economy

Stylized model of a sharing economy.

$A =$ set of agents

n goods

Each agent $i \in A$ has a quasi-concave utility function $u_i(\cdot)$.

Each agent $i \in A$ has an endowment w^i, which is an $n-$ vector. The jth component of w^i denoted w^i_j is the quantity of good j that agent i owns.

Exchange Economy: Example

Two agents: 1 and 2.

Two goods: sugar and salt.

Endowment of agent 1: $w^1 = (7, 3)$

Endowment of agent 2: $w^2 = (0, 4)$

Utility of agent 1: $u_1(x_1, x_2) = x_1^2 x_2$

Utility of agent 2: $u_2(y_1, y_2) = 3 \ln y_1 + 2y_2$

Exchange Economy: Example

$u_1(x_1, x_2) = x_1^2 x_2,\ w^1 = (7, 3)$

$u_2(y_1, y_2) = 3 \ln y_1 + 2y_2,\ w^2 = (0, 4)$

If no trade, agent 1 enjoys utility: $7^2 \times 3$

If no trade, agent 2 enjoys utility: $3 \ln 0 + 2 \times 4$

Exchange Economy: Example

$u_1(x_1, x_2) = x_1^2 x_2,\ w^1 = (7, 3)$

$u_2(y_1, y_2) = 3 \ln y_1 + 2y_2,\ w^2 = (0, 4)$

Is there a trade that would make them mutually better off?

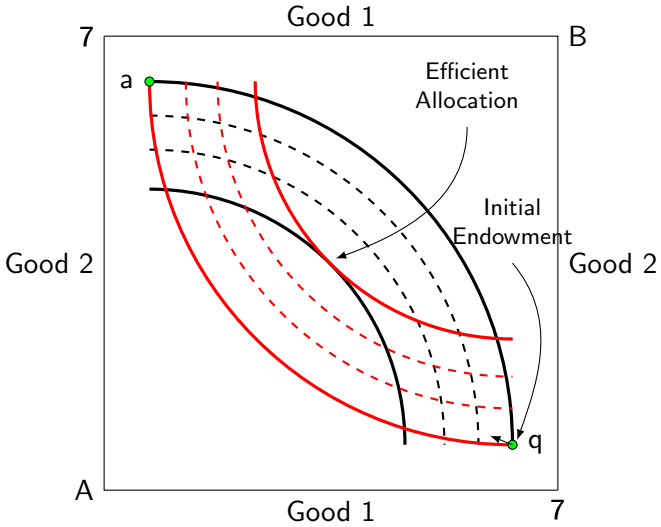
Agent 1 gives up one unit of good 1 to agent 2.

Agent 2 gives up 4 units of good 2 to agent 1.

New utility: $u_1(6, 7) = 6^2 \times 7 > 7^2 \times 3 = u_1(7, 3)$

New utility $u_2(1, 0) = 3 \ln 1 > 3 \ln 0 + 2 \times 4 = u_2(0, 4)$

Edgeworth Box



Exchange Economy: Individually Rational

x_i = quantity of good i consumed by agent 1

y_i = quantity of good i consumed by agent 2

An allocation X^* of the goods is **feasible** if:

$$x_1 + y_1 = 7 + 0$$

$$x_2 + y_2 = 3 + 4$$

$$x_1, x_2, y_1, y_2 \geq 0$$

An allocation X^* is **individually rational**:

$$u_1(x_1, x_2) \geq u_1(7, 3)$$

$$u_2(y_1, y_2) \geq u_2(0, 4)$$

Exchange Economy: Pareto Optimality

A feasible allocation X^* of the goods is **Pareto optimal** if there is no other **feasible** allocation that gives each agent at least as much utility as they enjoy under X^* with at least one agent enjoying strictly more.

The allocation that gives agent 1 the bundle $(7, 3)$ and agent 2 the bundle $(0, 4)$ is NOT Pareto optimal because

$u_1(6, 7) = 6^2 \times 7 > 7^2 \times 3 = u_1(7, 3)$

$u_2(1, 0) = 3 \ln 1 > 3 \ln 0 + 2 \times 4 = u_2(0, 4)$

Exchange Economy: Pareto Optimality

A pareto optimal allocation can be found by solving (for any choice of nonegative w_1, w_2):

$$\max w_1 u_1(x, x_2) + w_2 u_2(y_1, y_2)$$

subject to

$$x_1 + y_1 = 7$$

$$x_2 + y_2 = 7$$

$$x_1, x_2, y_1, y_2 \geq 0$$

Exchange Economy: Pareto Optimality

Substitution method:

$$\max w_1 u_1(x_1, x_2) + w_2 u_2(7 - x_1, 7 - x_2)$$

First order conditions:

$$w_1 \frac{\partial u_1(x_1, x_2)}{\partial x_1} - w_2 \frac{\partial u_2(7 - x_1, 7 - x_2)}{\partial y_1} = 0$$
$$w_1 \frac{\partial u_1(x_1, x_2)}{\partial x_2} - w_2 \frac{\partial u_2(7 - x_1, 7 - x_2)}{\partial y_2} = 0$$

Divide one by the other to get

$$\frac{\frac{\partial u_1(x_1,x_2)}{\partial x_1}}{\frac{\partial u_1(x_1,x_2)}{\partial x_2}} = \frac{\frac{\partial u_2(y_1,y_2)}{\partial y_1}}{\frac{\partial u_2(y_1,y_2)}{\partial y_2}}$$

Perfectly Competitive Market

1. A finite number of consumers specified by utilities and endowments.
2. A finite number of commodities.
3. Each consumer is aware of the price of every good.
4. The transaction costs of a sale, purchase etc are zero.
5. Consumers can buy and sell as much or as little as they want at the going price. Each assumes their individual transactions do not affect the going price. This is called the **price-taking** assumption.

Exchange Economy: Example

Unit prices: p_1, p_2 set by benevolent middleman.

1. Middleman announces (p_1, p_2).
2. Each agent reports their utility maximizing bundle at announced prices.
   maximize utility subject to expenditure = dollar value of endowment
3. Middleman chooses (p_1^*, p_2^*) to 'clear' the market, i.e., the demand for good 1 is equal to the total supply of good 1 and the demand for good 2 equals the total supply of good 2.

The pair of market clearing prices (p_1^*, p_2^*) are called Walrasian equilibrium prices.

The utility maximizing bundles consumed by the agents at prices (p_1^*, p_2^*) are called a Walrasian allocation.

Exchange Economy: Example

$u_1(x_1, x_2) = x_1^2 x_2,\ w^1 = (7, 3)$

Agent 1 solves:

$$\max x_1^2 x_2$$

$$p_1 x_1 + p_2 x_2 \leq 7p_1 + 3p_2$$

$$x_1, x_2 \geq 0$$

Exchange Economy: Example

$u_2(y_1, y_2) = 3 \ln y_1 + 2y_2$, $w^2 = (0, 4)$

Agent 2 solves:

$$\max 3 \ln y_1 + 2y_2$$

$$p_1 y_1 + p_2 y_2 \leq 0 \times p_1 + 4p_2$$

$$y_1, y_2 \geq 0$$

Exchange Economy: Example

We can always choose the price of one product to be 1.

WHY?

Product chosen to have a price of 1 is called the numeraire.

Exchange Economy: Example

Agent 1 solves:

$$\max x_1^2 x_2$$

$$x_1 + p_2 x_2 \leq 7 + 3p_2$$

$$x_1, x_2 \geq 0$$

Solution

$$x_1 = \frac{2(7+3p_2)}{3},\ x_2 = \frac{7+3p_2}{3p_2}$$

Exchange Economy: Example

Agent 2 solves:

$$\max 3 \ln y_1 + 2y_2$$

$$y_1 + p_2 y_2 \leq 0 \times 1 + 4p_2$$

$$y_1, y_2 \geq 0$$

Solution

$$y_1 = \frac{3p_2}{2}, \; y_2 = 2.5$$

Exchange Economy: Example

Agent 1 demand:

$$x_1 = \frac{2(7+3p_2)}{3},\ x_2 = \frac{7+3p_2}{3p_2}$$

Agent 2 demand:

$$y_1 = \frac{3p_2}{2},\ y_2 = 2.5$$

Supply =Demand

$$\frac{2(7+3p_2)}{3} + \frac{3p_2}{2} = 7 \ \Rightarrow p_2 = \frac{2}{3}$$

$$\frac{7+3p_2}{3p_2} + 2.5 = 7 \ \Rightarrow p_2 = \frac{2}{3}$$

Exchange Economy: Example

Equilibrium prices: $p_1 = 1$, $p_2 = \frac{2}{3}$

Equilibrium allocation

Agent 1: $(6, 4.5)$

Agent 2: $(1, 2.5)$

So what?

Exchange Economy: Example

Agent 1: $(6, 4.5)$, $u_1(6, 4.5) = 6^2 \times 4.5 > 7^2 \times 3 = u_1(7, 3)$

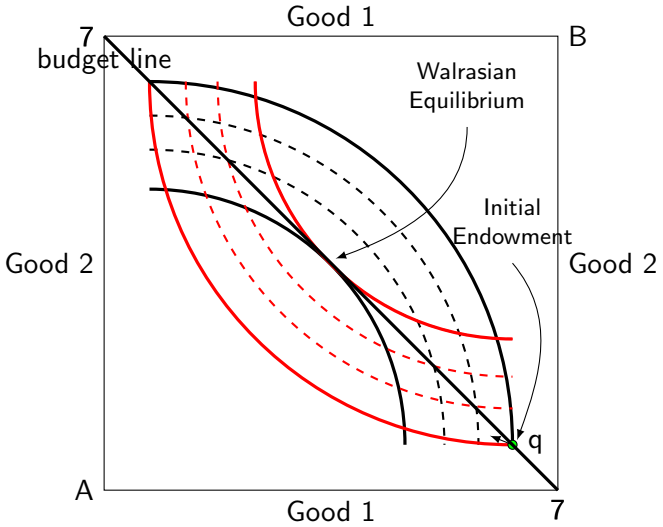
Agent 2: $(1, 2.5)$, $u_2(1, 2.5) = 3 \ln 1 + 2 \times 2.5 > u_2(0, 4)$

Is there a feasible allocation that can do better? In what sense?

Exchange Economy: Walrasian Equilibrium

Walrasian Equilibrium Allocation is individually rational *and* pareto optimal.

Edgeworth Box



Perfectly Competitive Market with Production

1. A finite number of agents specified by utilities, endowments and shares in the profit of each firm.
2. A finite number of commodities.
3. A finite number of firms specified by their production functions.
4. All firms have technologies that exhibit decreasing returns to scale.

Perfectly Competitive Market with Production

5. Each agent (consumers and firms) is aware of the price of every good.
6. The transaction costs of a sale, purchase etc are zero.
7. Agents can buy and sell as much or as little as they want at the going price. Each assumes their individual transactions do not affect the going price. This is called the **price-taking** assumption.

Production

2 agents: Bunter and Quelch.

Each is endowed with a unit of a good called ‘input’.

A factory that uses ‘input’ to produce two outputs called good 1 and 2.

The utility of Bunter and Quelch depends on the quantity of goods 1 and 2 each consumes.

Bunter and Quelch each own half a share of the factory profits.

Production

Middleman announces unit prices for each of the goods (input, good 1 & good 2).

Factory chooses quantity of input (demand) and mix of goods 1 & 2 to produce (supply) to maximize profit.

Bunter & Quelch maximize utility subject to expenditure = dollar value of endowment plus share in profit of factory

Prices selected to clear market.

Production

p_1 unit price of good 1, p_2 unit price of good 2, p unit price of input

Bunter's utility: $u_B(x_1, x_2) = x_1 x_2$

Quelch's utility: $u_Q(y_1, y_2) = y_1 y_2$

Bunter and Quelch each own half a share of the factory and are entitled to half the profits each.

z_i = quantity of input used in production of good $i = 1, 2$

Production function of good 1 is $\sqrt{z_1}$ and the production function of good 2 is $\frac{\sqrt{z_2}}{2}$

Production: Factory Problem

$$\Pi = \max p_1\sqrt{z_1} + p_2\frac{\sqrt{z_2}}{2} - p(z_1 + z_2)$$
$$\text{s.t. } z_1, z_2 \geq 0$$

$$z_1 = (\frac{p_1}{2p})^2,\ z_2 = (\frac{p_2}{4p})^2$$

$$\Pi = \frac{4p_1^2 + p_2^2}{16p}$$

These are the factory's supply curves and profit.

Production: Bunter's Problem

$$\max x_1 x_2$$

$$\text{s.t. } p_1 x_1 + p_2 x_2 \leq p \times 1 + \frac{\Pi}{2}$$

$$x_1, x_2 \geq 0$$

$$x_1 = \frac{p + \frac{\Pi}{2}}{2p_1}, \; x_2 = \frac{p + \frac{\Pi}{2}}{2p_2}$$

Bunter's demand curves (same for Quelch).

Production: Supply = Demand

demand for good 1 = supply of good 1

$$2 \times \frac{p + \frac{\Pi}{2}}{2p_1} = \frac{p_1}{2p}$$

demand for good 2 = supply of good 2

$$2 \times \frac{p + \frac{\Pi}{2}}{2p_2} = \frac{p_2}{8p}$$

demand for input = supply of input

$$(\frac{p_1}{2p})^2 + (\frac{p_2}{4p})^2 = 2$$

Efficiency of Walrasian Equilibrium

Price is selected to match supply with demand.

Goods are allocated in a way that is pareto optimal for the agents.

The mix and quantity of goods generated is produced at minimum total cost.

Efficiency of Walrasian Equilibrium

Imagine two factories with cost functions $C_1(q)$ and $C_2(q)$ where q is the quantity produced.

Suppose Walrasian equilibrium requires us to produce Q units in total to meet demand. Will this amount be produced at minimum cost?

Efficiency of Walrasian Equilibrium

What would central planner do?

Choose how much each factory should make to meet demand at minimum cost.

$$\min C_1(q_1) + C_2(q_2)$$
$$\text{s.t. } q_1 + q_2 = Q$$
$$q_1, q_2 \geq 0$$

Efficiency of Walrasian Equilibrium

Central planner solves:

$$\min C_1(q_1) + C_2(q_2)$$

$$\text{s.t. } q_1 + q_2 = Q$$

$$q_1, q_2 \geq 0$$

$$L = C_1(q_1) + C_2(q_2) + \lambda(Q - q_1 - q_2)$$

$$\frac{\partial L}{\partial q_i} = \frac{dC_i(q_i)}{dq_i} - \lambda = 0$$

Let (q_1^*, q_2^*) be optimal quantities and λ^* be optimal multiplier value.

Efficiency of Walrasian Equilibrium

What would Walrasian middleman do?

Announce a price p per unit for output.

Each factory i chooses q_i to solve:

$$\max pq_i - C_i(q_i)$$
$$\text{s.t. } q_i \geq 0$$

Efficiency of Walrasian Equilibrium

Each factory i chooses q_i to solve:

$$\max pq_i - C_i(q_i)$$
$$\text{s.t. } q_i \geq 0$$

FOC tells us that optimal q_i must solve $p = \frac{dC_i(q_i)}{dq_i}$.

Choose q_i so that *marginal* cost = unit price.

Choose p so that total output is equal to Q.

Efficiency of Walrasian Equilibrium

If Walrasian middleman chooses $p = \lambda^*$, supply meets demand and does so at minimum total cost.

Properties of Walrasian Equilibrium

- Walrasian equilibrium exists (under conditions specified).
- Walrasian allocation is feasible and gives to each agent at least as much utility as her endowment.
- Walrasian allocation is pareto optimal.
- Walrasian allocation is produced at minimum total cost.
- Every pareto optimal allocation can be achieved as a Walrasian equilibrium by redistributing the endowment.
- Only information each agent needs is the price.

Perfectly Competitive Market

A market which produces the Walrasian prices and allocations.

Do they exist?

What is required for their existence?

- Preferences that can be represented by quasi-concave utility functions.
- Production functions exhibit decreasing returns to scale.
- Agents do not believe their actions (consumption and production levels) affect the price (price-taking).

Automation Redux

Two agents, one factory.

The factory takes labor (measured in units of time) as input and outputs Soylent Green.

Each agent is endowed with one unit of time which they can use for either leisure or labor.

Agent's derive utility from the consumption of leisure and Soylent.

If x is the amount of leisure consumed and z the amount of Soylent Green, utility is xz.

Agent 1 owns a share μ of factory profits, while agent 2 owns a share $1 - \mu$.

Automation Redux

Factory production function is $f(L) = 2\sqrt{L}$.

w is unit price of labor, at equilibrium $w = 1$

p is unit price of Soylent Green at equilibirum is $\sqrt{\frac{2}{3}}$

Equilibrium allocation: $x_1 = \frac{1+\mu(\frac{2}{3})}{2}$, $z_1 = \frac{1+\mu(\frac{2}{3})}{2\sqrt{\frac{2}{3}}}$, $x_2 = \frac{1+(1-\mu)(\frac{2}{3})}{2}$ and $z_2 = \frac{1+(1-\mu)(\frac{2}{3})}{2\sqrt{\frac{2}{3}}}$.

The utility of agent 1 is $[\frac{1+\frac{2}{3}\mu}{2}]^2(\sqrt{\frac{3}{2}})$.

Automation Redux

Automation allows more Soylent to be produced for same amount of labor.

$f(L) = 4\sqrt{L}$

The equilibrium prices will be $w = 1$, $p = \sqrt{\frac{1}{2}}$.

The equilibrium quantities are $x_1 = \frac{1+2\mu}{2}$, $z_1 = \frac{1+2\mu}{\sqrt{2}}$, $x_2 = \frac{1+2(1-\mu)}{2}$ and $z_2 = \frac{1+2(1-\mu)}{\sqrt{2}}$.

Utility of agent 1 is $[\frac{1+2\mu}{2}]^2(\frac{2}{\sqrt{2}})$.

Automation Redux

The utility of agent 1 in has increased! The same is true for agent 2 as well.

How can that be?

The new technology requires *less* labor to produce the same level of output. In fact, the total amount of labor supplied goes down. This **hurts** the agents.

Automation lowered the factory's production costs which allowed it to produce more Soylent using less labor. This **benefits** the agents.

1. The equilibrium price of Soylent declines.
2. The profit of the factory goes up and each agent owns some share of the profits.

The unanswered question is the effect on those who are endowed only with labor.

Externality

Consumption or production by one agent imposes costs (negative externalities) or benefits (positive externalities) on another.

Formally, my utility or costs depend not just on my consumption but yours as well.

Externality

1. My ownership of a Caravaggio prevents your ownership of the same.
2. Michael Jordan's presence increases regular-season away-game tickets.
3. My decision to enter a congested highway, increases the delay experienced by those who follow after me.
4. Increase in light track SUVs on road leads to additional deaths on the part of non-SUV drivers (each 1 mill SUVs leads to 30-100 additional non-SUV deaths).
5. Two competing firms that sell complementary goods.
6. Pollution.
7. Being polite/rude.

Externality

Externalities are not a problem for markets as long as they are 'priced in.'

Market clearing price in ownership example does exactly that.

One painting, two potential buyers with RPs of $900 and $1000 exactly.

Any price above $900 and below $1000 clears the market.

The market price is large enough that a potential owner must pay a price that covers the externality imposed on the buyer who is deprived of the ownership opportunity.

Externality

Externalities are a problem if they are not 'priced in.'

By not pricing them in, the producer of the externality has an incentive to either under or over produce the externality.

Externality: Example

Two soyabean farms

Farm 1 produces quantity x_1.

The production of soyabeans on farm 1 uses Dicamba a herbicide. Some of this herbicide spreads to farm 2 which does not grow Dicamba resistant soyabeans.

Farm 1 can control the amount, z, of Dicamba used, but it has the effect of raising Farm 2's cost of production.

Externality: Example

Farm 2 produces x_2 units of soyabeans.

Presence of Dicamba pushes up farm 2's production costs (by reducing yield).

Externality: Example

Farm 1's cost function: $C_1(x_1, z)$

$$\frac{\partial C_1}{\partial x_1} > 0, \ \frac{\partial^2 C_1}{\partial x_1^2} > 0, \ \ \frac{\partial^2 C_1}{\partial z^2} > 0, \ \ \frac{\partial^2 C_1}{\partial x_1 \partial z} = 0$$

$$\frac{\partial C_1}{\partial z} < 0 \text{ for small } z, \text{ and } \frac{\partial C_1}{\partial z} > 0 \text{ for large } z$$

Farm 2's cost function: $C_2(x_2, z)$

$$\frac{\partial C_2}{\partial x_2} > 0, \ \frac{\partial^2 C_2}{\partial x_2^2} > 0, \ \frac{\partial C_2}{\partial z} > 0$$

Externality: Example

Suppose p_1 and p_2 are the unit prices for outputs of farm 1 and 2 to generate Q_1 and Q_2 units of outputs 1 and 2 respectively.

Farm 1 will choose x_1, z to maximize $p_1 x_1 - C_1(x_1, z)$, i.e.,

$$\frac{\partial C_1}{\partial x_1} = p_1, \ \frac{\partial C_1}{\partial z} = 0$$

Let (Q_1, z^*) be the profit maximizing choice for farm 1.

Farm 2 chooses x_2 to maximize $p_2 x_2 - C_2(x_2, z^*)$, i.e.,

$$\frac{\partial C_2}{\partial x_2} = p_2$$

Externality: Example

Central planner chooses x_1, x_2 and z to solve

$$\min C_1(x_1, z) + C_2(x_2, z)$$

$$\text{s.t. } x_1 = Q_1$$

$$x_2 = Q_2$$

$$x_1, x_2, z \geq 0$$

Using Lagrangean

$$\frac{\partial C_1}{\partial x_1} = \lambda_1, \ \frac{\partial C_2}{\partial x_2} = \lambda_2$$

$$\frac{\partial C_1}{\partial z} + \frac{\partial C_2}{\partial z} = 0$$

Central Planner $\neq$ Walrasian equilibrium ($\frac{\partial C_1}{\partial z} = 0$)

Externality: Example

$$\frac{\partial C_1}{\partial z} + \frac{\partial C_2}{\partial z} = 0$$

Recall, $\frac{\partial C_1}{\partial z}$ and $\frac{\partial C_2}{\partial z}$ have opposite sign.

Increasing z pushes down farm 1's costs *but* pushes up farm 2's costs.

Central planner balances cost savings enjoyed by Farm 1 against cost increases suffered by Farm 2.

Externality: Internalizing an Externality

1. Regulation.
2. Taxation.
3. Merger.

Externality: Regulation

Solve the central planner's problem:

$$\min C_1(x_1, z) + C_2(x_2, z)$$

$$\text{s.t. } x_1 = Q_1$$

$$x_2 = Q_2$$

$$x_1, x_2, z \geq 0$$

Let (x_1^*, x_2^*, z^*) be the solution and require that Farm 1 not use more than z^* units of Dicamba.

Externality: Taxation

Tax Farm 1 t units per unit of Dicamba used.

Farm 1 now solves:

$$\max p_1 x_1 - C_1(x_1, z) - tz$$
$$\text{s.t. } x_1, z \geq 0$$

FOCs are

$$\frac{\partial C_1}{\partial x_1} = p_1$$
$$\frac{\partial C_1}{\partial z} + t = 0$$

Choose $t = \frac{\partial C_2}{\partial z}$. Then,

$$\frac{\partial C_1}{\partial z} + \frac{\partial C_2}{\partial z} = 0$$

Mimics central planner's solution.

Externality: Merger

Merged farm will solve:

$$\max_{x_1,x_2,z} p_1 x_1 - C_1(x_1, z) + p_2 x_2 - C_2(x_2, z)$$

FOC:

$$\frac{\partial C_1}{\partial x_1} = p_1, \ \frac{\partial C_2}{\partial x_2} = p_2$$

$$\frac{\partial C_1}{\partial z} + \frac{\partial C_2}{\partial z} = 0$$

Externality: Fire

Railway track through a stretch of land owned by a farmer. Upon the land, near the tracks are crops.

Coal burning locomotive emits sparks that set fire to farmer's crops from time to time.

Who should be liable for the damage caused by the sparks?

Externality: Fire

Running the train yields $1000 worth of profit but causes $2000 worth of crop damage.

Damage exceeds profit generated.

Externality: Fire

Running the train yields $1000 worth of profit but causes $2000 worth of crop damage.

It costs the farmer $100 to ensure the safety of their crop.

Externality: Fire

Let W be total economic value *before* train runs (status quo).

Farmer liable: train runs, farmer pays \$100 to safeguard crop. Total economic value is now

$$W + 1000 - 100 = W + 900 > W$$

Make farmer liable?

Externality: Fire

Make railroad liable?

Railroad pays Farmer at least \$100 to move crops.

Total economic value :

$$W + 1000 - 100 = W + 900 > W$$

Does *not* matter who we make liable.

Coase 'Theorem'

The initial allocation of legal entitlements does not matter from an efficiency standpoint so long as the transaction costs of their exchange are zero.

Coase 'Theorem'

Suppose Railway is made liable.

Cost to farmer of moving crops is c, known only to farmer.

If $c \leq 1000$, Railway should 'buy out' farmer.

If $c > 1000$, Railway should shut down.

How can Railway figure out what c is?

Coase 'Theorem'

In the absence of transaction costs it does not matter how legal entitlements are allocated. An inefficient misallocation of legal entitlements will be cured in the marketplace.

- Legal entitlements need to be precisely defined and assigned.
- Contracts for the exchange of legal entitlements need to be enforceable.
- The transaction costs associated with the exchange of such entitlements should be minimized.
- Competitive market for the exchange of such entitlements.

Externality: Internalizing an Externality (Recap)

1. Regulation.
   Requires regulator to know each firm's cost function.
   If can optimally regulate Soma emissions, why not outputs of goods 1 &2?
2. Taxation.
   Same informational requirements as regulating output of Soma.
   What do we do with tax?
3. Merger.
   Merged firm internalizes externality and acts 'like' central planner. However, merger may have other effects in output market.
4. Define a property right over externality and allow trade.
   - A right to pollute?
   - A right to clean water?

   Coase: irrelevant as long as there is a perfectly competitive market to trade them we get efficient outcome.

Is it possible to set up Perfectly Competitive Markets?

Coase says we should define property rights and let market trade in them.

Can this always be done?

Public Good

A good that is non-excludable and non-rivalrous.

Non-excludable: no one can be barred from consumption of the good.

Non-rivalrous: consumption by one does not materially reduce availability of the good to others.

Paying for public goods sets up a free rider problem.

Public Good: Examples

A recipie

Air

National defense

Education

Public Good: Vaccination

n families.

c per family cost for a vaccination.

Vaccination does not provide 100 % protection against infection, but the more who are vaccinated, the better off each family is (vaccinated or not).

$B(k)$ = the dollar benefit enjoyed by each family (vaccinated or not) when exactly k families receive the vaccination.

Public Good: Vaccination

Diminishing benefits:

$$B(n) - B(n-1) \leq B(n-1) - B(n-2) \leq \ldots \leq B(1) - B(0).$$

$$B(n) - B(n-1) < 2B(n) - 2B(n-1) < c$$

$$nB(n) - nB(n-1) > c$$

Public Good: Vaccination

Voluntary vaccination.

Each family simultaneously and independently decides whether to get vaccinated.

Each family has two strategies: vaccinate (V) or not (N).

Is there a Nash equilibrium of this game where everyone chooses V?

Public Good: Vaccination

If every family chooses V, then, family #1, say, enjoys a surplus of $B(n) - c$.

Holding the choices of the other families fixed, we ask what would family #1's benefit from choosing N instead?

It is $B(n-1)$.

As $B(n-1) > B(n) - c$, family #1 prefers N.

Everyone choosing V is NOT an equilibrium.

No family takes into account the benefit they confer on the other families when they choose to get vaccinated.

Public Good: Vaccination

Will trade in vaccine permits 'solve the problem'?

Suppose the price to buy or sell a permit is set at p.

If a family buys one permit but does not sell its permit, it must pay p but does not have to vaccinate.

A family that sells its permit must vaccinate.

Is there a price p where we achieve the surplus maximizing outcome?

Public Good: Vaccination

If there is such a price, everyone must sell their permit. In order to clear the market the permits sold must all be bought.

Consider family #1. Suppose it sold its permit and bought some number d from other families. Its benefit is

$$B(n) + p - c - dp.$$

Among all the purchase decisions it could make this should maximize its benefit. If sells its permit but buys $(d-1)$ permits from others, its benefit is

$$B(n-1) + p - c - (d-1)p.$$

In order to sustain p as a market clearing price we need

$$B(n) + p - c - dp \geq B(n-1) + p - c - (d-1)p \;\Rightarrow B(n) - B(n-1) \geq p.$$

Public Good: Vaccination

If family #1 does not sell its permit but buys d permits from others, its benefit is

$$B(n-1) - dp.$$

In order for everyone to sell their permit we need

$$B(n) + p - c - dp \geq B(n-1) - dp \;\Rightarrow B(n) - B(n-1) \geq c - p.$$

Therefore,

$$2B(n) - 2B(n-1) \geq B(n) - B(n-1) + p \geq c.$$

Contradiction.

Summary

1. Under the price taking assumption, competitive markets deliver outcomes that in a sense cannot be improved upon (Pareto Optimal and cost minimizing).
2. If externalities are present, they can be incorporated by 'pricing' them in by defining a property right and setting up a competitive market in the property right.
3. Breaks down with a public good. No competitive market can deliver the efficient outcome with a public good.
4. If market cannot provide the public good, can Government?

Government Provision

C = cost of the public good

b_i = benefit of citizen i from public good, $i = 1, 2$

Government should only provide the good if $b_1 + b_2 \geq C$ and not otherwise.

How to determine b_i?

How to pay for the good?

Government Provision

Desireable Features

Only build if $b_1 + b_2 \geq C$ (efficiency)

If build, raise exactly enough money to cover the costs (budget balance).

If build, never charge anyone more than their benefit (individually rational)

CAN'T BE DONE!