

Due date: Tuesday, 26 July 2022 before 11:59 p.m. HK time

Contribution of this assignment to the final mark: 5%

- Answer **all** questions. Show **all** steps.
- Please submit the softcopy of your work (as a **single pdf file**) to Canvas on or before the due date.

Questions

1. Solve the following inequality. Give your answers in set/interval notations.

$$|3x - 2| + 3 \geq 3x + |4 - x|$$

2. Consider the following system of linear equations

$$\begin{cases} x + 2y + z = 1 \\ x + (k+1)y - 3z = k-6 \\ 3x + 5y + (k+2)z = 5 \end{cases},$$

where k is a real number.

Using Gaussian elimination, determine the value(s) of k such that the system has

- (a) no solution;
- (b) a unique solution;
- (c) infinitely many solutions.

Then solve the system in part (c).

3. (a) Let $x, y > 0$. Use the AM-GM inequality to find the minimum value of $S = \frac{1}{x^2} + \frac{1}{y} + 2xy$ and determine the values of x and y at which this minimum value of S is attained.

- (b) If α, β, γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, where p, q and r are real numbers, express $\alpha^2 + \beta^2 + \gamma^2$ and $\alpha^4 + \beta^4 + \gamma^4$ in terms of p, q and r .

4. (a) Find the smallest possible positive integer N such that $N! > N^3$. Then prove by mathematical induction that $n! > n^3$ for all positive integers $n \geq N$.

- (b) A sequence $\{a_n\}$ is defined by $a_1 = 3$, $a_2 = 11$ and $a_{n+2} = 6a_{n+1} - 7a_n$ for $n = 1, 2, 3, \dots$. Prove by mathematical induction that

$$a_n = \frac{(3 + \sqrt{2})^n + (3 - \sqrt{2})^n}{2} \quad \text{for } n = 1, 2, 3, \dots$$

5. (a) How many different arrangements can be obtained from the letters of the word PROPORTION?
- (b) How many different arrangements can be obtained from the letters of the word TEMPERATURE such that the first two and the last three letters are vowels?
- (c) How many different words can be formed by taking 5 letters at a time from the letters of the word SEMESTER? Assume that the letters are taken without replacement.
- (d) Find the coefficient of $\frac{1}{x^5}$ in the binomial expansion of $\left(4x^2 - \frac{1}{2x^3}\right)^{10}$.

- End -