

EE3122: Analogue Circuit Fundamentals

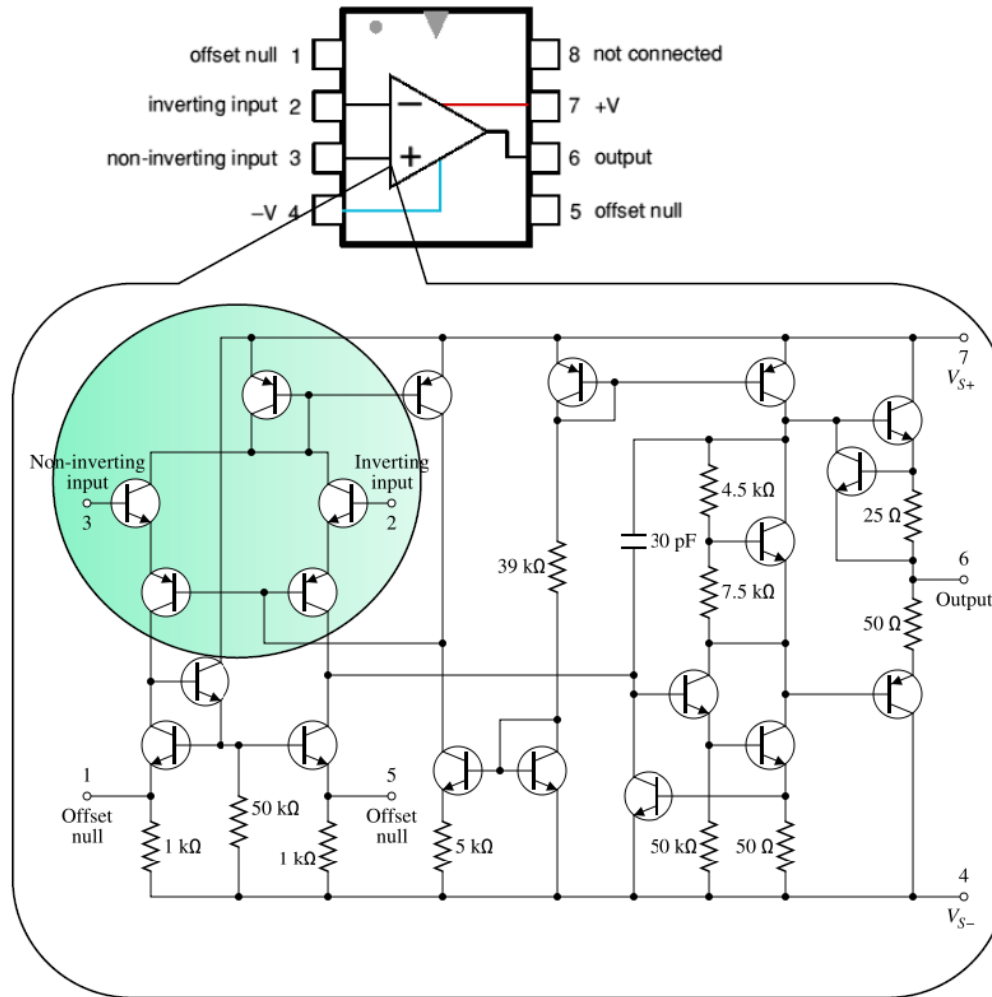
Differential Amplifiers

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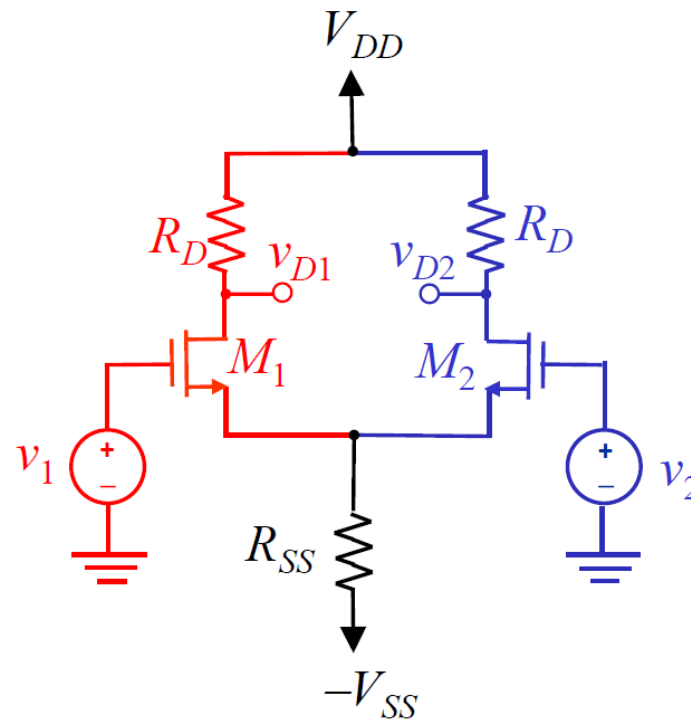
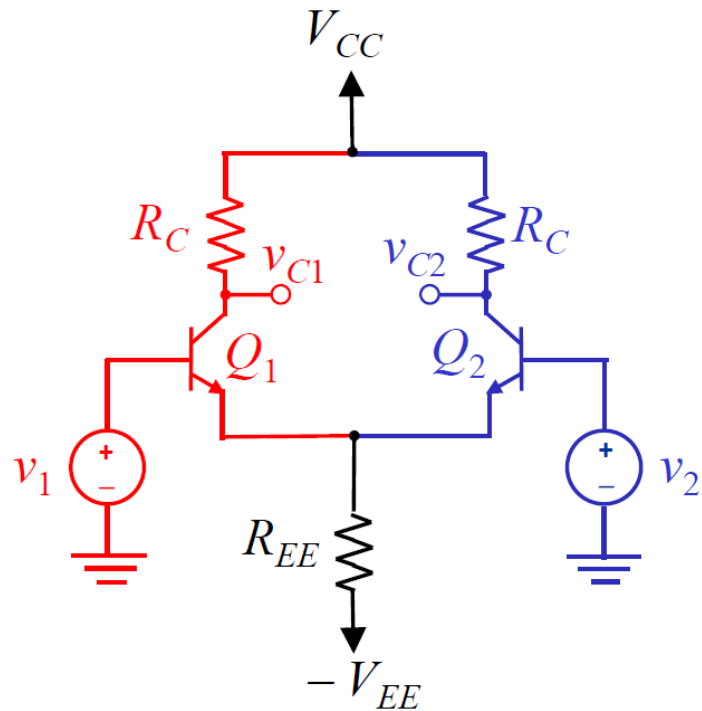
Prof. Arindam BASU

Operational Amplifier Core



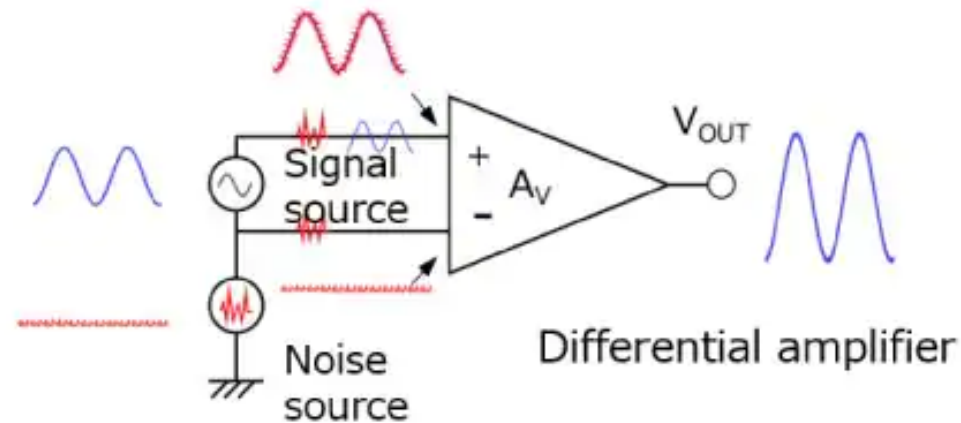
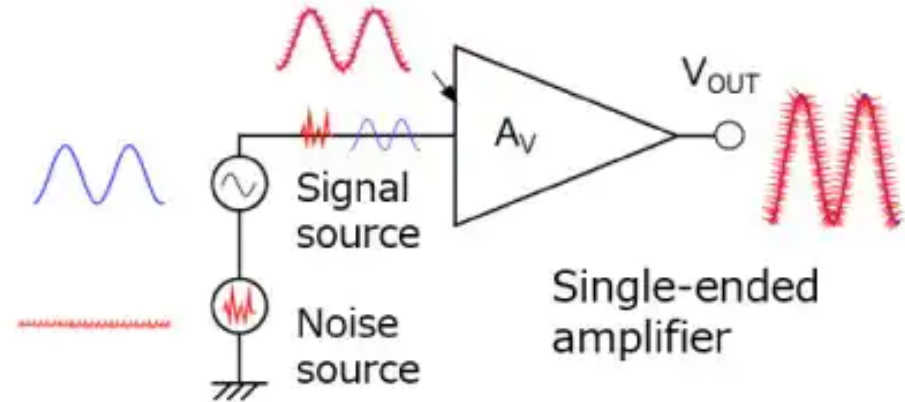
- Previous amplifiers – 1 input, 1 output
- Op-amps – 2 input → Differential amplifier!

Differential Amplifiers



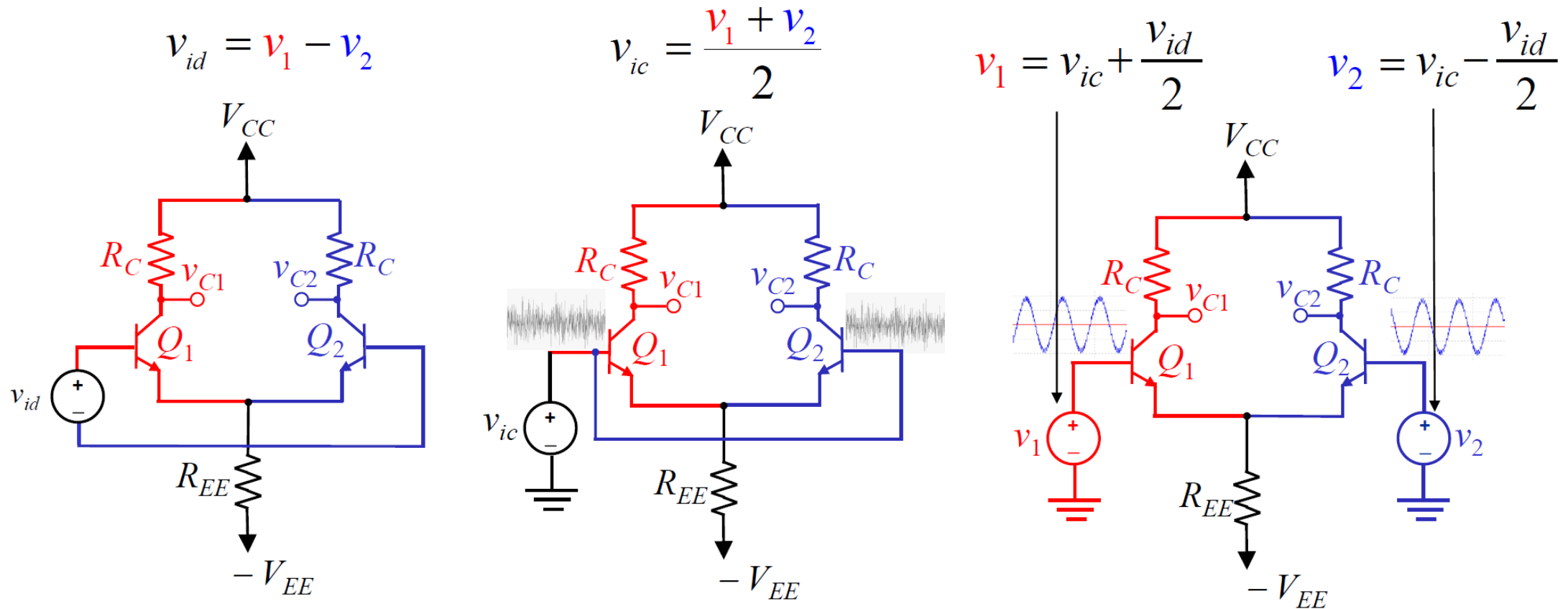
- Differential amplifier forms the heart of operational amplifier.
- It contains two transistors in symmetrical configuration.
- It has two inputs and two outputs.
- Ideal differential amplifier uses perfectly matched transistors.

Why Differential Amplifiers



Differential and Common Mode operations

The input signals, v_1 and v_2 can be decomposed into a difference signal, v_{id} and a common signal, v_{ic} .



Differential and Common Mode operations

Therefore, the AC analysis can then be divided into two parts:

- (1) Differential mode analysis
- (2) Common mode analysis

$$v_{out} = A_{dm} v_{id} + A_{cm} v_{ic}$$

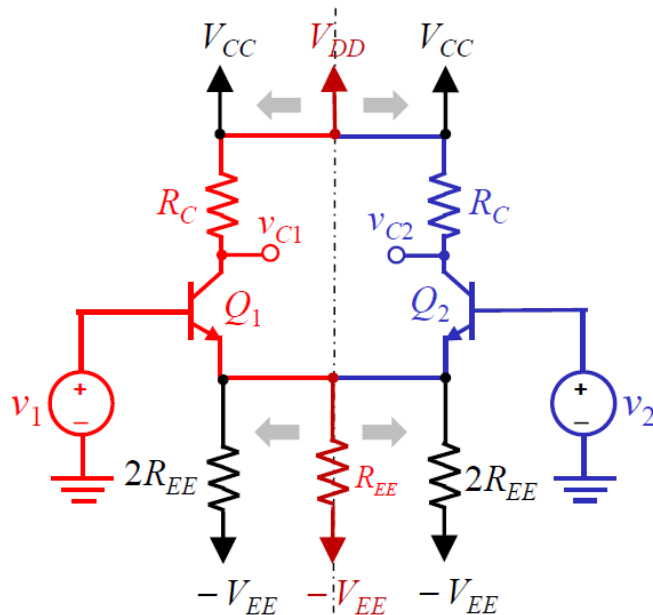
The o/p signal can be taken differentially between the two o/p terminals, v_{C1} and v_{C2} .

- For BJT differential amplifier, $v_{od} = v_{C1} - v_{C2}$.
- For MOSFET differential amplifier, $v_{od} = v_{D1} - v_{D2}$.

Single ended outputs are ground referenced outputs taken between the collector and ground for BJT differential amplifier or between the drain and ground for MOSFET differential amplifier.

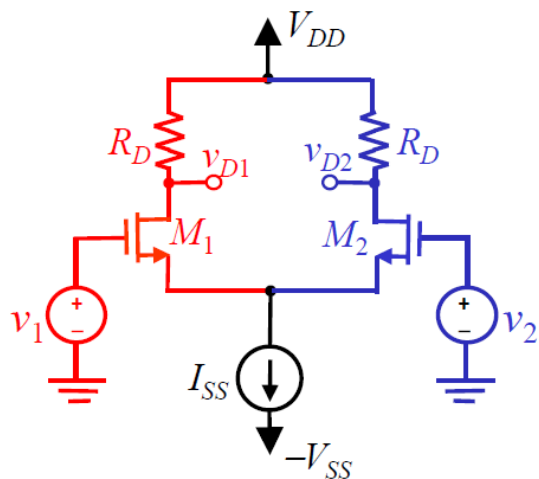
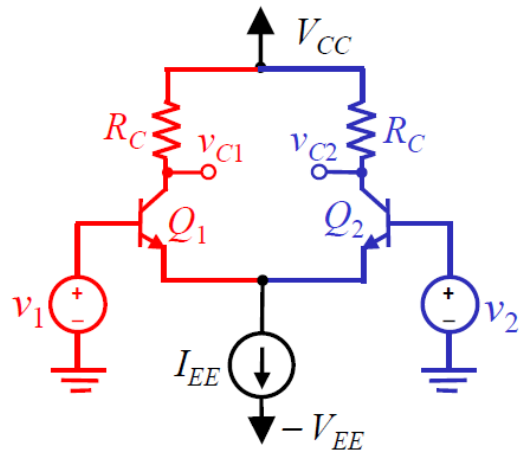
Half Circuit Analysis

Line of symmetry



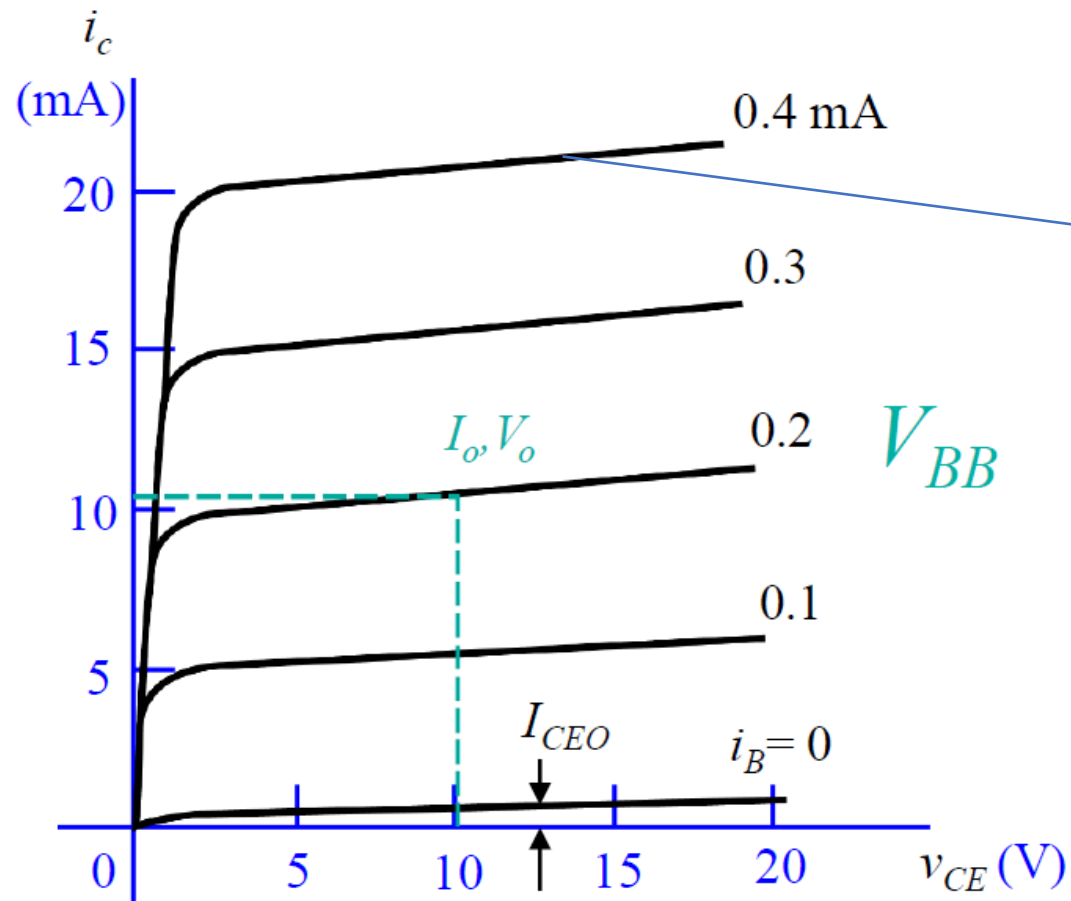
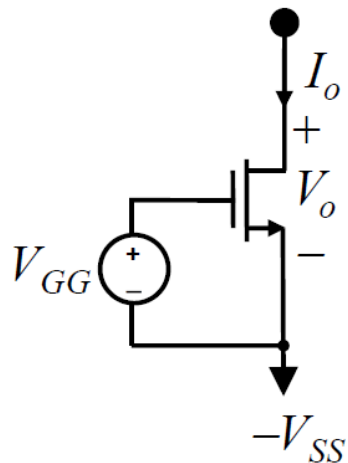
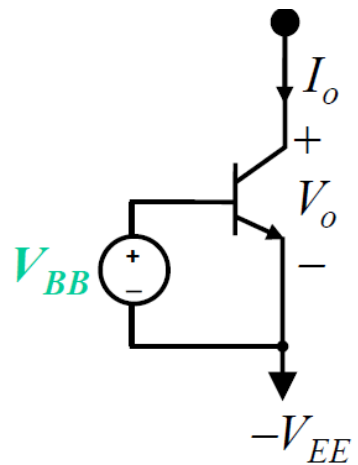
- Half-circuits are constructed by first drawing the differential amplifier in a fully symmetrical form.
 - Power supplies are split into two equal halves in parallel.
 - Emitter resistor is separated into two equal resistors in parallel.
- None of the currents or voltages in the circuit are changed.
- For differential mode signals, points on the line of symmetry are virtual grounds connected to ground for AC analysis.
- For common-mode signals, points on line of symmetry are replaced by open circuits.

Biasing With Electronic Current Source



- Differential amplifiers are biased using electronic current sources to stabilise the operating point and increase effective value of R_{EE} to improve $CMRR$.
- Electronic current source has a DC current equal to I_{EE} (or I_{SS}) and an output resistance equal to R_{EE} (or R_{SS}).
- DC model of the electronic current source is a DC current source, I_{EE} (or I_{SS}) while AC model is a resistance R_{EE} (or R_{SS}).

Example of Electronic Current Source

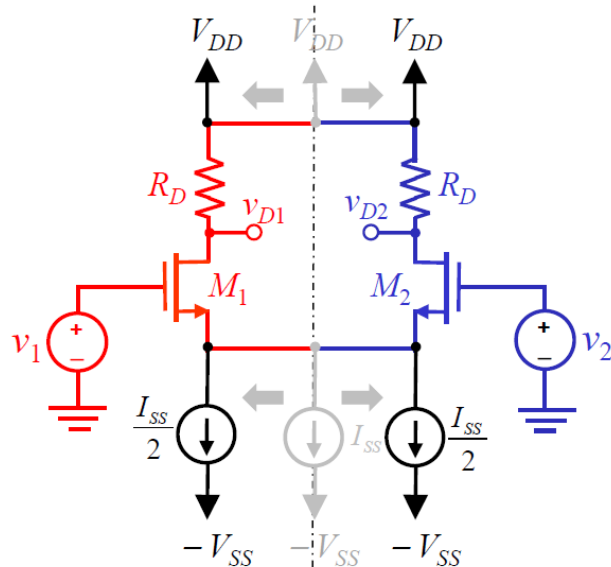


DC current: I_{ss}

AC resistance: r_o

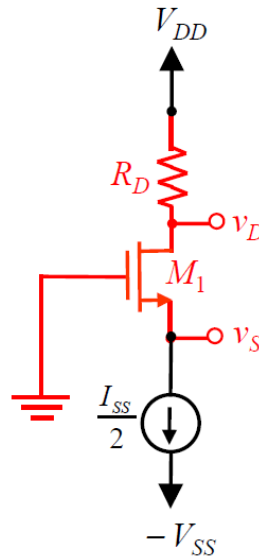
MOS Differential Amplifier: DC Analysis

Line of symmetry



**DC Half Circuit =
Common-Mode**

DC half-circuit



$$I_D = I_S = \frac{I_{SS}}{2}$$

$$I_D = \frac{K_n}{2} (V_{GS} - V_{TN})^2$$

$$V_{GS} = V_{TN} + \sqrt{\frac{2I_D}{K_n}}$$

$$= V_{TN} + \sqrt{\frac{I_{SS}}{K_n}}$$

$$V_{D1} = V_{D2} = V_{DD} - I_D R_D$$

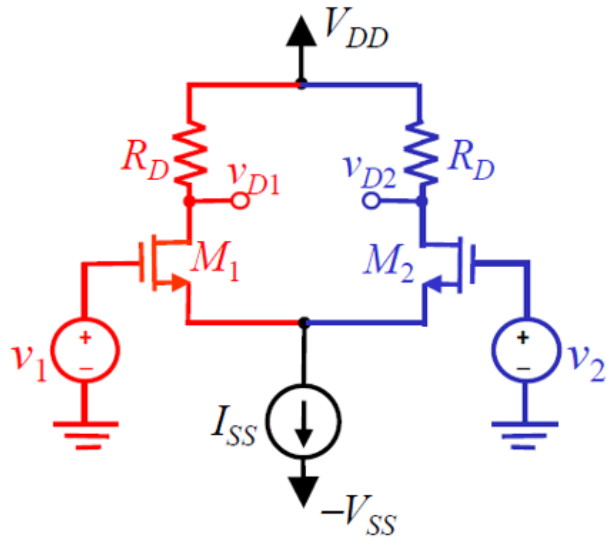
$$V_{OD} = V_{D1} - V_{D2} = 0$$

$$V_{DS} = V_D - V_S$$

$$= V_{DD} - I_D R_D - (-V_{GS})$$

$$= V_{DD} - I_D R_D + V_{GS}$$

MOS Differential Amplifier: AC Analysis

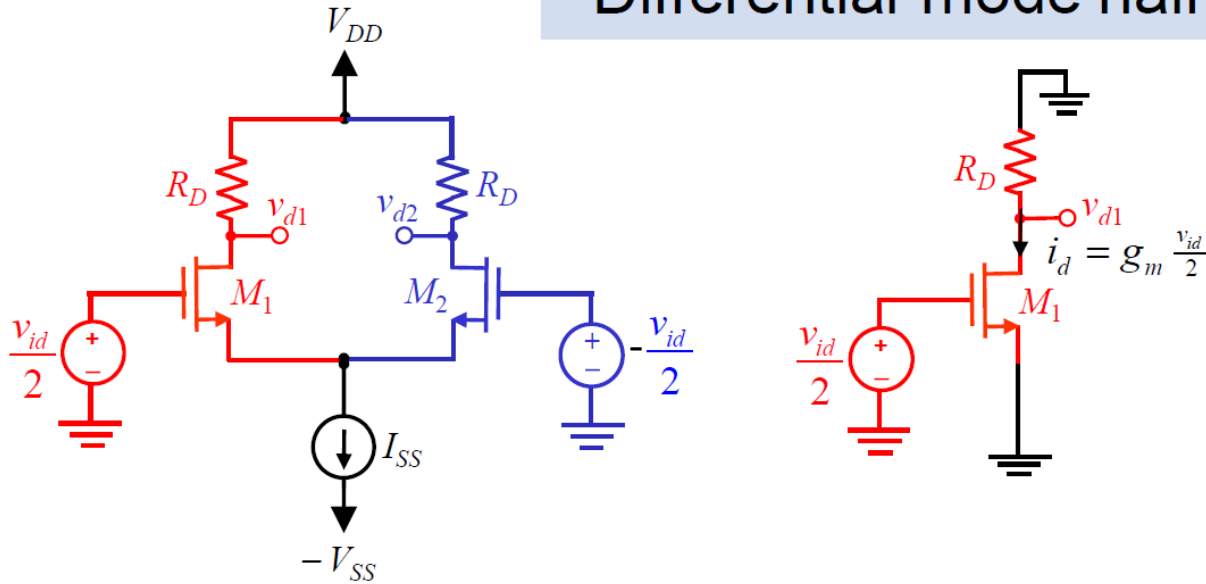


- $v_{od} = v_{D1} - v_{D2}$ (output differential)
- $v_{id} = v_1 - v_2$ (input differential)
- Input of 2 types: dm (differential mode) OR cm (common mode)
- Output of 2 types: diff ($v_{D1} - v_{D2}$) OR se (single ended: v_{D1})
- 4 possible gain definition

Input Output	dm	cm
diff	$A_{dm-diff}$	$A_{cm-diff}$
se	A_{dm-se}	A_{cm-se}

MOS Differential Amplifier: AC Analysis

Differential-mode half-circuit



$$v_{od} = v_{d1} - v_{d2} = -g_m v_{id} R_D$$

$$A_{dm-diff} = \frac{v_{od}}{v_{id}} = -g_m R_D$$

$$R_{id} = \infty$$

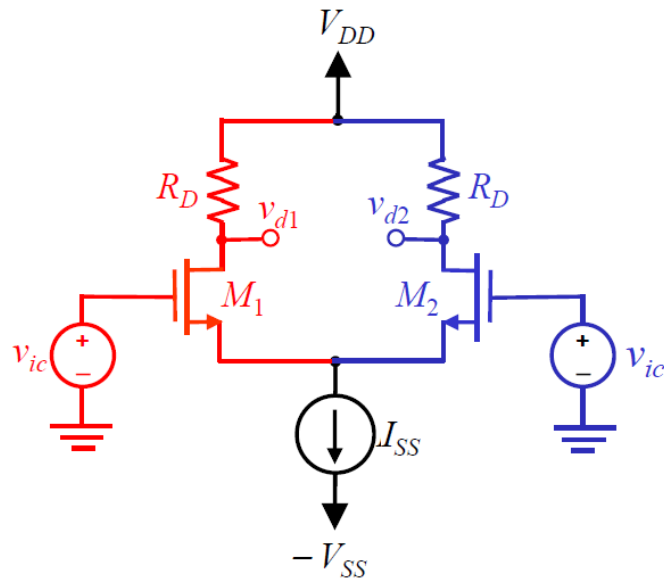
$$R_{o1} = R_{o2} = r_o \parallel R_D \approx R_D$$

$$R_{od} = R_{o1} + R_{o2} \approx 2R_D$$

$$v_{d1} = -g_m \left(\frac{v_{id}}{2} \right) R_D \Rightarrow A_{dm-se1} = \frac{v_{d1}}{v_{id}} = -\frac{g_m R_D}{2}$$

$$v_{d2} = -v_{d1} = g_m \left(\frac{v_{id}}{2} \right) R_D \Rightarrow A_{dm-se2} = \frac{v_{d2}}{v_{id}} = \frac{g_m R_D}{2}$$

MOS Differential Amplifier: AC Analysis

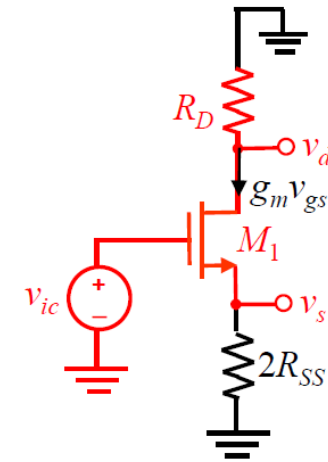


$$v_{d1} = v_{d2} = -g_m v_{gs} R_D$$

$$v_{ic} = v_{gs} + g_m v_{gs} (2R_{SS})$$

$$v_{od} = v_{d1} - v_{d2} = 0$$

Common-mode half-circuit



$$A_{cm-se} = \frac{v_d}{v_{ic}} = -\frac{g_m R_D}{1 + 2g_m R_{SS}} \approx -\frac{R_D}{2R_{SS}}$$

$$A_{cm-diff} = \frac{v_{od}}{v_{ic}} = 0$$

$$R_{ic} = \infty$$

Common Mode Rejection Ratio (CMRR)

- For single-ended output, $CMRR = \left| \frac{A_{dm-se}}{A_{cm-se}} \right| = \frac{\frac{g_m R_D}{2}}{R_{SS}} = g_m R_{SS}$
- $R_{SS} \gg R_{EE}$ (thus provides more Q-point stability) should be maximised.
- If the MOSFET amplifier is biased by a resistor, then $R_{SS} = \frac{V_{SS} - V_{GS}}{I_{SS}}$

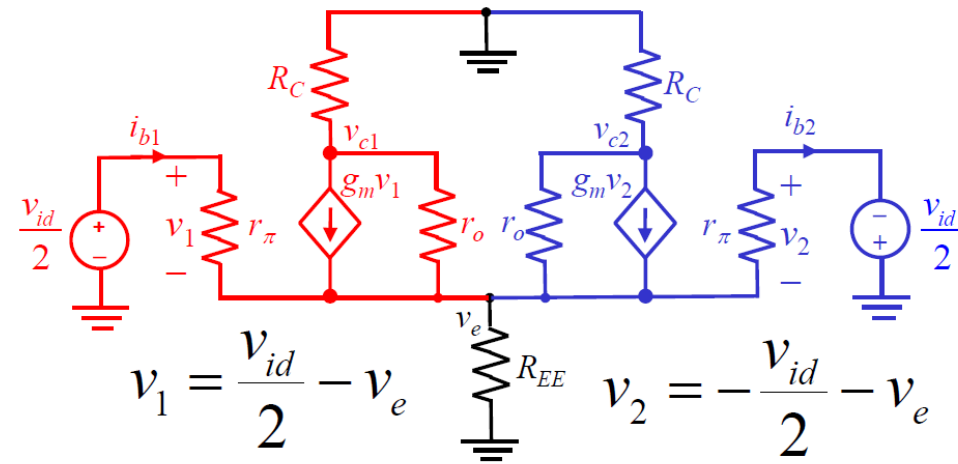
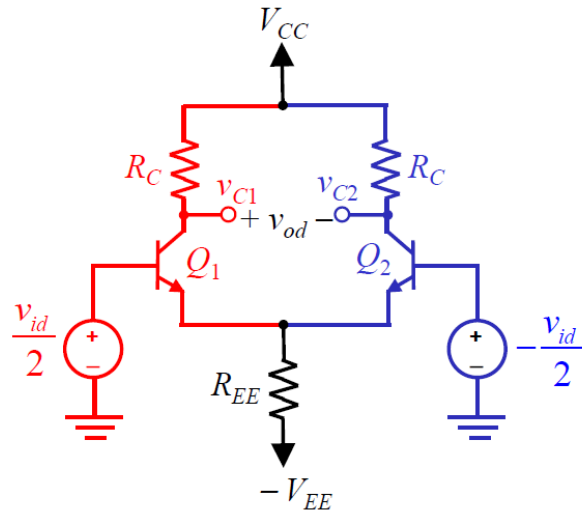
$$CMRR = g_m R_{SS} = \frac{2I_D R_{SS}}{V_{GS} - V_{TN}} = \frac{I_{SS} R_{SS}}{V_{GS} - V_{TN}} = \frac{V_{SS} - V_{GS}}{V_{GS} - V_{TN}}$$

For high $CMRR$, V_{GS} should be small, but $V_{GS} > V_{TN}$. Therefore, $CMRR$ is limited by V_{SS} .

- To increase $CMRR$ in BJT and MOSFET amplifiers, current sources with higher R_{SS} or R_{EE} are used.

Appendix

Differential Amplifier: Proof of Virtual GND



Ignore current through r_o since $i_{ro} \ll g_m v_{be}$

$$\frac{v_1}{r_\pi} + g_m v_1 + \frac{v_2}{r_\pi} + g_m v_2 = \frac{v_e}{R_{EE}}$$

$$\left(g_m + \frac{1}{r_\pi} \right) (v_1 + v_2) = \frac{v_e}{R_{EE}}$$

Since $v_1 + v_2 = -2v_e$,

$$v_e \left(\frac{1}{R_{EE}} + \frac{2}{r_\pi} + 2g_m \right) = 0 \Rightarrow v_e = 0$$

Emitter node represents virtual ground for differential mode i/p signal