

EE1001 Fundamentals of Digital Techniques

Test 1 Solution

Semester B, 2022-23

The test consists of 2 parts.

PART A

The part A consists of 17 questions and each question carry 3 marks.

QA1. 3 is the smallest prime number.

- A. True
- B. False

Solution: B. The smallest prime number should be 2.

QA2. Difference of two distinct prime number is?

- A. Odd and prime
- B. Even and composite
- C. None of the mentioned
- D. All of the mentioned

Solution: C. $3 - 2 = 1$ is neither prime nor composite.

QA3. In 2's complement, what do all the positive numbers have in common?

Solution: The most significant bit (MSB) is 0.

QA4. In a 4-bit 1's complement, how can the value 0 be represented?

Solution: 0000 and 1111.

QA5. Suppose we are using 8-bit floating-point representation where there is 1 sign-bit (that is before the radix point), 4 mantissa bits, and 3 exponent (in 2's complement) bits. What is $(3.25)_{10}$ in this floating-point representation?

Write your answer in the follow format: **0.0000 [000]**.

Solution:

$$(3)_{10} = (11)_2$$

$$(0.25)_{10} = (0.01)_2$$

$$(3.25)_{10} = (11.01)_2$$

Sign bit is 0, mantissa bits are $(1101)_2$,

Shift radix point left for 2 places, exponent bits are $(010)_2$.

So, the answer is 0.1101 [010].

QA6. Convert a decimal number $(-31)_{10}$ to binary using a 6-bit 2's complement representation.

Solution: Since $(31)_{10} = (01\ 1111)_2$, $(-31)_{10}$ in 6 bit 2's complement is 10 0001.

QA7. Write the hexadecimal $(-3E)_{16}$ in an 8-bit 1's complement representation.

Solution: Since $(3E)_{16} = (0011\ 1110)_2$, $(-3E)_{16}$ in 8-bit 1's complement is 1100 0001.

QA8. Perform unsigned binary numbers addition, write down the result in binary.

$$(1010\ 0101)_2 + (0100\ 0100)_2$$

Solution:

$$\begin{array}{r} 1010\ 0101 \\ +\ 0100\ 0100 \\ \hline 1110\ 1001 \end{array}$$

QA9. Suppose $A = \{0, 1, 8, \{1, 2\}, \{1, 2, 3\}, \{1, 3, 6, 7\}, \{1, 7, 8, 9\}\}$. What is the cardinality of the power set $P(A)$?

Solution: $|P(A)| = 2^7 = 128$.

QA10. Which of the following is the correct set builder notation for the given set below?

$$\{1.1, 2.2, 3.3, 4.4, 5.5, 6.6, 7.7, 8.8, 9.9\}$$

- A. $\{x \in \mathbf{R} \mid 1.1 \leq x \leq 9.9\}$
- B. $\{(1.1x) \mid x \in \mathbf{R}, 1 \leq x \leq 9\}$
- C. $\{(1.1x) \mid x \in \mathbf{Z}, 1 < x \leq 9\}$
- D. $\{(1.1x) \mid x \in \mathbf{Z}, 1 < x < 9\}$
- E. None of the above.

Solution: E.

QA11. $A = \left\{ \frac{p}{q} \mid p \in \mathbf{Z}, q \in \mathbf{Z}, \text{ and } q \neq 0 \right\}$. What is A ?

Solution: Set of rational numbers ($\mathbb{Q}$).

QA12. Between any two numbers, there are

- A. Two rational numbers
- B. No rational number
- C. Infinite rational numbers
- D. None of the above

Solution: C.

QA13. $B = \{-9, -7, -5, -3, -1, 1, 3, 5\}$: which of the following is the correct set builder form to describe B ?

- A. $B = \{2x + 1 | x \in \mathbb{Z}, -5 \leq x \leq 2\}$
- B. $B = \{2x - 1 | x \in \mathbb{Z}, -5 \leq x \leq 2\}$
- C. $B = \{2x - 1 | x \in \mathbb{Z}, -4 \leq x < 3\}$
- D. $B = \{2x + 1 | x \in \mathbb{Z}, -5 < x < 2\}$
- E. None of the above

Solution: A.

QA14. Power set of empty set has exactly ____ subset.

- A. Zero
- B. One
- C. Undefined
- D. Two
- E. None of the above

Solution: B.

QA15. Suppose R, Q, P are 3 different ordered pairs, A and B are 2 sets such that $A = \{a, b, c\}$ and $B = \{1, 2, 3\}$. Consider the following statements:

- i. $R = \{(a, 1), (b, 2), (c, 2)\}$ is a relation from A to B .
 - ii. $Q = \{(1, a), (2, b)\}$ is a relation from A to B .
 - iii. $P = \{(a, a), (b, c), (b, a)\}$ is a relation from A to A .
- A. i. True, ii. True, iii. True.
 - B. i. True, ii. True, iii. False.
 - C. i. False, ii. False, iii. False.
 - D. i. True, ii. False, iii. True.
 - E. None of the above.

Solution: D.

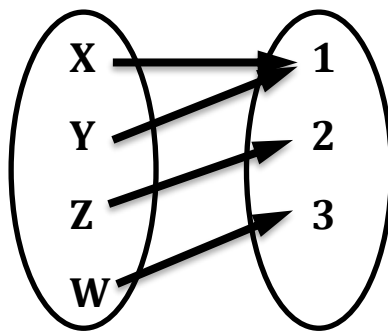
QA16. Which of the below is the Cartesian product of $A = \{1, 2\}$ and $B = \{a, b\}$?

- A. $\{(1, a), (1, b), (2, a), (b, b)\}$
- B. $\{(1, 1), (2, 2), (a, a), (b, b)\}$
- C. $\{(1, a), (2, a), (1, b), (2, b)\}$
- D. $\{(1, 1), (a, a), (2, a), (1, b)\}$
- E. $\{(1, 1), (1, a), (1, b), (2, b)\}$

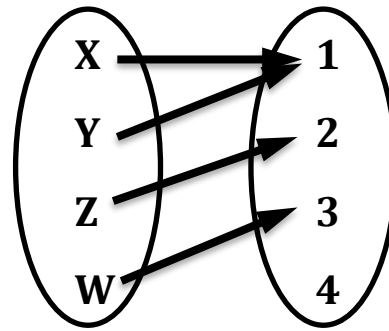
Solution: C.

QA17. Consider the following mappings A and B , which statement is true?

- A. A is injective, B is surjective.
- B. A is injective, B is not surjective.
- C. A is surjective, B is injective.
- D. A is surjective, B is not surjective.
- E. None of the above.



A



B

Solution: D.

PART B

The part B consists of 5 questions and each question carry 10 marks.

QB1. Using the properties of sets to prove

$$A - (A - B) = A \cap B$$

Start the proof from the Left-hand side. Use algebraic rules to prove it. State the name of the algebraic rule you used in each step. Do **not** skip any steps.

List of algebraic rules:

$A \cap B = B \cap A$ $A \cup B = B \cup A$	Commutative laws
$(A \cap B) \cap C = A \cap (B \cap C)$ $(A \cup B) \cup C = A \cup (B \cup C)$	Associative laws
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$	Distributive laws
$A - B = A \cap B^c$	Difference law
$A \cup \emptyset = A = A \cap U$	Identity laws
$A \cup A^c = U$ $A \cap A^c = \emptyset$	Complement laws
$A \cup A = A \cap A = A$	Idempotent laws
$A \cup U = U$ $A \cap \emptyset = \emptyset$	Domination laws

$A \cup (A \cap B) = A$ $A \cap (A \cup B) = A$	Absorption laws
$(A \cup B)^c = A^c \cap B^c$ $(A \cap B)^c = A^c \cup B^c$	De Morgan's laws
$(A^c)^c = A$	Double complement law
$U^c = \emptyset$ $\emptyset^c = U$	Complement laws of U and $\emptyset$

Proof:

$$\begin{aligned}
 A - (A - B) &= A \cap (A \cap B^c)^c && \text{By the set difference law} \\
 &= A \cap (A^c \cup B) && \text{By De Morgan's law} \\
 &= A \cap (A^c \cup B) && \text{By the double complement law} \\
 &= (A \cap A^c) \cup (A \cap B) && \text{By the distributive law} \\
 &= \emptyset \cup (A \cap B) && \text{By the complement law for } \cap \\
 &= A \cap B && \text{By the identity law for } \cup
 \end{aligned}$$

QED

QB2. Use IEEE 754 32-bit format to represent

$$(-88.75)_{10}.$$

The IEEE 754 32-bit floating-point format is defined as:

Sign (1 bit)	Exponent (8 bits)	Mantissa (23 bits)
0	0000 0000	0000 0000 0000 0000 0000 000

Solution:

Step 1: Convert to binary number:

$$\begin{aligned}
 (-88.75)_{10} &= -(64 + 16 + 8 + 0.5 + 0.25) \\
 &= -(2^6 + 2^4 + 2^3 + 2^{-1} + 2^{-2}) \\
 &= -(101\ 1000.11)_2
 \end{aligned}$$

Step 2: Move the radix point just behind the leading bit, count how many shifts (**6 shifts**) and that is the exponent. Then obtain the scientific representation.

$$-(101\ 1000.11)_2 = -(1.0110\ 0011)_2 \times 2^{133-127}$$

Step 3: Write in IEEE 754 32-bit format:

- Sign bit: 1(Negative)
- Mantissa: $1.0110\ 0011 = 1 + \text{fraction}$ (Add trailing zeros)
- Actual Exponent: $(133)_{10} = (1000\ 0101)_2$

Sign (1 bit)	Exponent (8 bits)	Mantissa (23 bits)
1	1000 0101	0110 0011 0000 0000 0000 000

QB3. Suppose

$$\begin{aligned}g &: \{a, b, c\} \rightarrow \{a, b, c\}, g(a) = b, g(b) = c, g(c) = a. \\f &: \{a, b, c\} \rightarrow \{1, 2, 3\}, f(a) = 3, f(b) = 2, f(c) = 1.\end{aligned}$$

Determine if the following composite function exists:

- 1) $f \circ g$.
- 2) $g \circ f$.

If it exists, write down the output when each element in the domain is used as input. If it does not exist, explain the reasons.

Solution:

1)

$$(f \circ g)(a) = f(g(a)) = f(b) = 2.$$

$$(f \circ g)(b) = f(g(b)) = f(c) = 1.$$

$$(f \circ g)(c) = f(g(c)) = f(a) = 3.$$

2) $g \circ f$ is **undefined** as the range of f is not a subset of the domain of g .

QB4. Use contrapositive proof to prove:

If $7y + 9$ is even, then y is odd.

Proof:

We prove by contrapositive.

Suppose y is even, then $y = 2p$ where p is an integer.

Therefore, $7y + 9 = 14p + 9 = 2(7p + 4) + 1$.

Since p is an integer, $2(7p + 4) + 1$ **must be odd**.

Hence, by contrapositive, the given statement is true.

QED

QB5. Given the following functions:

$$\begin{aligned}f &: R \rightarrow R, f(x) = x^2, \\g &: R \rightarrow R, g(x) = 4x - 1,\end{aligned}$$

h : A mapping from bank account numbers to bank account customers. The bank account numbers are domain, and the bank account customers are codomain.

- 1) Determine whether they are injective, surjective and bijective.
- 2) Determine which of the above functions is/are invertible.

Solution:

1)

f is not injective, not surjective, not bijective.

g is injective, surjective, bijective.

h is not injective, surjective, not bijective.

2)

g is invertible.