

EE3101 Tutorial 8 Solutions

(1) (a) symbol rate = $\frac{90M}{\log_2 16} = 22.5 \text{MSymbol/s}$

Minimum TX Bandwidth = 22.5MHz.

(b) $\log_2 M \geq \frac{135 \times 10^6}{25 \times 10^6}$, i.e. $\log_2 M \geq 5.4$

$M = 64$ since both $\log_2 M$ and M should be an integer.

→ 64QAM

(2)

(a) The average signal level for signal point i is:

$$\begin{aligned} \text{Average value of } \left\{ \int_0^T E_i \frac{2}{T} \cos^2(\omega_c t) dt + \int_0^T n(t) \sqrt{\frac{2}{T}} \cos(\omega_c t) dt \right\} \\ = \frac{2E_i}{T} \int_0^T \cos^2(\omega_c t) dt \\ = E_i \end{aligned}$$

$$\text{Average noise power } \sigma^2 = \frac{N_0}{2} \int_0^T \left[\sqrt{\frac{2}{T}} \cos(\omega_c t) \right]^2 dt = \frac{N_0}{2} \frac{2}{T} \int_0^T \frac{1 + \cos(2\omega_c t)}{2} dt = \frac{N_0}{2}$$

The distance between the constellation point (i.e. signal point in (a)) and the decision threshold is $2E/2 = E$, i.e. the probability of error for the constellation points $i = 2$ to $M - 1$ is:

$$p_{s_i} = 2Q\left[\frac{E}{\sigma}\right] = 2Q\left[\sqrt{\frac{2E^2}{N_0}}\right]$$

The probability of error for the constellation points $i = 1$ and $i = M$ is:

$$p_{s_i} = Q\left[\sqrt{\frac{2E^2}{N_0}}\right]$$

The total average probability of symbol error is:

$$\begin{aligned} P_S &= \frac{1}{M} \sum_{i=1}^M p_{s_i} \\ P_S &= \left[\frac{1}{M} + \frac{1}{M} \sum_{i=2}^{M-1} 2 + \frac{1}{M} \right] Q\left[\sqrt{\frac{2E^2}{N_0}}\right] \\ &= \frac{2(M-1)}{M} Q\left[\sqrt{\frac{2E^2}{N_0}}\right] \end{aligned}$$

If Gray Code is used, $P_B = \frac{P_S}{\log_2 M}$

→ $P_B = \frac{2(M-1)}{M \log_2 M} Q\left[\sqrt{\frac{2E^2}{N_0}}\right]$

(b) The average symbol energy is:

$$\begin{aligned}
 E_S &= \frac{1}{M} \sum_{i=1}^M E_i^2 = \frac{1}{M} \sum_{i=1}^M [E_1 + 2(i-1)E]^2 \\
 E_S &= \frac{1}{M} \left[\sum_{i=1}^M E_1^2 + 4E_1E(i-1) + 4(i-1)^2 E^2 \right] \\
 &= \frac{1}{M} \left\{ ME_1^2 + 4E_1E \frac{(M-1)[(M-1)+1]}{2} + 4 \frac{(M-1)[(M-1)+1][2(M-1)+1]}{6} E^2 \right\} \\
 &= E_1^2 + 2E_1E(M-1) + \frac{2(M-1)(2M-1)}{3} E^2 \\
 E_b &= \frac{E_S}{\log_2 M} = \frac{1}{\log_2 M} \left[E_1^2 + 2E_1E(M-1) + \frac{2(M-1)(2M-1)}{3} E^2 \right]
 \end{aligned}$$

(c) (i) To minimize the transmission power,

$$\begin{aligned}
 \frac{dE_b}{dE_1} &= 0 \\
 \rightarrow 2E_1 + 2E(M-1) &= 0 \\
 \text{i.e. } E_1 &= -E(M-1)
 \end{aligned}$$

$$\begin{aligned}
 E_b &= \frac{1}{\log_2 M} \left\{ [-E(M-1)]^2 + 2[-E(M-1)]E(M-1) + \frac{2(M-1)(2M-1)}{3} E^2 \right\} \quad (\text{ii}) \\
 &= \frac{E^2}{\log_2 M} \left\{ (M-1)^2 - 2(M-1)^2 + \frac{2(M-1)(2M-1)}{3} \right\} \\
 &= \frac{E^2}{\log_2 M} \frac{(M^2-1)}{3}
 \end{aligned}$$

$$\begin{aligned}
 \rightarrow E^2 &= \frac{3E_b \log_2 M}{M^2-1} \\
 \text{i.e. } P_B &= \frac{2(M-1)}{M \log_2 M} Q \left[\sqrt{\frac{6E_b \log_2 M}{N_0(M^2-1)}} \right]
 \end{aligned}$$

(3) The symbol error prob for M-QAM is: $P_{S-MQAM} \cong 4(1 - \frac{1}{\sqrt{M}})Q[\sqrt{\frac{3E_{b-Q}\log_2 M}{(M-1)N_0}}]$

The symbol error prob for M-PSK is: $P_{S-MPSK} \cong 2Q[\sqrt{\frac{2E_{b-P}\log_2 M}{N_0}} \sin(\frac{\pi}{M})]$

As shown in (2), the symbol error prob for the M-ASK is:

$$P_{S-MASK} = \frac{2(M-1)}{M} Q \sqrt{\frac{6E_{b-A}\log_2 M}{N_0(M^2-1)}}$$

As the bit error mainly depends on x value in $Q(x)$, a 10% decrease in x will at least double P_S value (for $M \geq 16$ and $P_S \leq 10^{-2}$). For example, $Q(3.719) = 10^{-4}$, $Q(3.540) = 2 \times 10^{-4}$, $Q(3.719 \times 0.9) = 4.1 \times 10^{-4}$. Thus, for a given P_S for all three systems,

$$\begin{aligned} \sqrt{\frac{3E_{b-Q}\log_2 M}{(M-1)N_0}} &\approx \sqrt{\frac{2E_{b-P}\log_2 M}{N_0}} \sin(\frac{\pi}{M}) \approx \sqrt{\frac{6E_{b-A}\log_2 M}{N_0(M^2-1)}} \\ \frac{P_{MQAM}}{P_{MPSK}} &= \frac{E_{b-Qx}(\text{bit rate})}{E_{b-Px}(\text{bit rate})} \approx \frac{2\sin^2(\frac{\pi}{M})}{\frac{3}{(M-1)}} \sim \frac{2\pi^2}{3M} \\ P_{MPSK} &\sim \frac{3M}{2\pi^2} P_{MQAM} \\ \text{i.e. } P_{MPSK}(\text{dB}) &\sim P_{MQAM}(\text{dB}) + 10\log(\frac{3M}{2\pi^2}) \text{ dB} \end{aligned}$$

This is the reason that at 16 level or above, M-QAM is preferable than M-PSK.

$$\frac{P_{MASK}}{P_{MPSK}} = \frac{E_{b-Ax}(\text{bit rate})}{E_{b-Px}(\text{bit rate})} = \frac{2\sin^2(\frac{\pi}{M})}{\frac{6}{(M^2-1)}} \sim \pi$$

$$\text{i.e. } P_{MASK}(\text{dB}) \sim P_{MPSK}(\text{dB}) + 5\text{dB}$$

(4) Consider the autocorrelation for impulses at I-channel:

$$R_0 = (-2.8A)^2 0.25 + (-A)^2 0.25 + (+A)^2 0.25 + (+2.8A)^2 0.25 = 4.42 A^2$$

for $r \neq 0$,

$$\begin{aligned} R_r &= (-2.8A)(2.8A)(0.25)^2 + (-2.8A)(A)(0.25)^2 + (-2.8A)(-A)(0.25)^2 \\ &+ (-2.8A)(-2.8A)(0.25)^2 + (-A)(2.8A)(0.25)^2 + (-A)(A)(0.25)^2 + (-A)(-A)(0.25)^2 \\ &+ (-A)(-2.8A)(0.25)^2 + \dots \\ &= 0 \end{aligned}$$

Or, since the symbols are uncorrelated (the positive terms will be balanced out by the negative terms) $R_r = 0$

$$\text{i.e. } P(\omega) = \frac{1}{T} \sum_{r=-\infty}^{\infty} R_r \exp(-jr\omega T) = \frac{4.42A^2}{T}$$

$$H(\omega) = \frac{T \sin(\omega T / 2)}{\omega T / 2}$$

$$T = T_b \log_2 16 = 4T_b$$

$$S(\omega) = |H(\omega)|^2 P(\omega)$$

$$\rightarrow S_I(\omega) = 17.68A^2T_b \sin^2(2\omega T_b) / (2\omega T_b)^2$$

$$\text{Similar for the Q-channel, } S_Q(\omega) = 17.68A^2T_b \sin^2(2\omega T_b) / (2\omega T_b)^2$$

Since the effect of the carrier is to provide a frequency translation, and the two carriers $[\sin(\omega t)$ and $\cos(\omega t)]$ are orthogonal and the data are random. The overall psd at modulator output is:

$$S(\omega) = 2 \times \frac{1}{4} \times 17.68A^2T_b \left\{ \frac{\sin^2[2(\omega - \omega_c)T_b]}{[2(\omega - \omega_c)T_b]^2} + \frac{\sin^2[2(\omega + \omega_c)T_b]}{[2(\omega + \omega_c)T_b]^2} \right\}$$

$$S(\omega) = 8.84A^2T_b \left\{ \frac{\sin^2[2(\omega - \omega_c)T_b]}{[2(\omega - \omega_c)T_b]^2} + \frac{\sin^2[2(\omega + \omega_c)T_b]}{[2(\omega + \omega_c)T_b]^2} \right\}$$

(5)

(a) 1.3us

(b) $-3.4ABT_s$, $-ABT_s$, ABT_s , $3.4ABT_s$

(c) $\sigma^2 = \frac{N_0}{2} \int_0^{T_s} B^2 dt = \frac{N_0 B^2 T_s}{2}$

(d) The 3 threshold levels are $-2.2ABT_s$, 0, and $+2.2ABT_s$

$$P_s = \frac{1}{4}Q\left(\frac{1.2ABT_s}{\sigma}\right) + \frac{1}{4}Q\left(\frac{1.2ABT_s}{\sigma}\right) + \frac{1}{4}Q\left(\frac{ABT_s}{\sigma}\right) + \frac{1}{4}Q\left(\frac{ABT_s}{\sigma}\right) + \frac{1}{4}Q\left(\frac{1.2ABT_s}{\sigma}\right) + \frac{1}{4}Q\left(\frac{1.2ABT_s}{\sigma}\right)$$
$$= Q\left(\frac{1.2ABT_s}{\sigma}\right) + \frac{1}{2}Q\left(\frac{ABT_s}{\sigma}\right)$$

$$\frac{ABT_s}{\sigma} = \sqrt{\frac{(ABT_s)^2}{N_0 B^2 T_s / 2}} = \sqrt{\frac{2A^2 T_s}{N_0}}$$

i.e. $P_s = Q\left(\sqrt{\frac{2.88A^2 T_s}{N_0}}\right) + \frac{1}{2}Q\left(\sqrt{\frac{2A^2 T_s}{N_0}}\right)$