

EE3114 - Homework 9 - Solutions

9-1 Design a negative state-feedback controller $u = -Kx$ such that the controlled system

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -2 & -2 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

has two desired stable poles $-3 \pm j3$.

Answer: (4 marks)

$$u = -Kx = -[k_1 \quad k_2]x$$

$$\det(sI - (A - BK)) = s^2 + (2 + k_2)s + 2 + k_1$$

$$\prod_{i=1}^2 (s - p_i) = (s + 3 + j3)(s + 3 - j3) = s^2 + 6s + 18$$

Thus,

$$K = [k_1 \quad k_2] = [16 \quad 4].$$

9-2 Given a state-space system as follows:

$$\dot{x} = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

- a) Is this system observable?
- b) Design a state-feedback observer such that it has desired poles to be the roots of the following characteristic equation:

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

Namely find the observer gain $L = \begin{bmatrix} L_1 \\ L_2 \end{bmatrix}$ in terms of ζ and ω_n .

- c) Find the numerical observer gain L if $\zeta = 1$ and $\omega_n = 6$.

Answer: (6 marks)

- a) (2 marks) The system is observable if $\begin{bmatrix} C \\ CA \end{bmatrix}$ has a full rank. We have

$$C = [1 \quad 0]$$

and

$$\begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix}$$

which has a full rank (equal to 2). Hence the system is observable.

b) (2 marks) We have:

$$A - LC = \begin{bmatrix} 2 - L_1 & 3 \\ -1 - L_2 & 4 \end{bmatrix}$$

and

$$\det(A - LC) = s^2 + s(L_1 - 6) + (11 - 4L_1 + 3L_2)$$

The condition for the poles to be the roots of the equation

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

is

$$s^2 + s(L_1 - 6) + (11 - 4L_1 + 3L_2) = s^2 + 2\zeta\omega_n s + \omega_n^2$$

Or, equivalently

$$L_1 - 6 = 2\zeta\omega_n$$

$$11 - 4L_1 + 3L_2 = \omega_n^2$$

which yields

$$L_1 = 2\zeta\omega_n + 6$$

$$L_2 = \frac{\omega_n^2 + 8\zeta\omega_n + 13}{3}.$$

c) (2 marks) For if $\zeta = 1$ and $\omega_n = 6$, we have

$$L_1 = 2\zeta\omega_n + 6 = 18$$

$$L_2 = \frac{\omega_n^2 + 8\zeta\omega_n + 13}{3} = \frac{97}{3} \simeq 32.34.$$