

EE1002 Principles of Electrical Engineering Tutorial 3 Solution

1. Evaluate the following integrations.

(a) $\int_0^2 x e^x dx$

(b) $\int_0^2 x^2 e^x dx$

(c) $\int \frac{2e^{\tan x}}{\cos^2 x} dx$.

Solution

(a) We let

$$u=x \text{ and } \frac{du}{dx} = 1 \text{ Then } \frac{du}{dx} = 1 \text{ and } v = e^x$$

Using the integration by parts formula for definite integrals we have

$$\int_0^2 x e^x dx = [x e^x]_0^2 - \int_0^2 e^x dx = 2e^2 - [e^x]_0^2 = 2e^2 - [e^2 - 1] = e^2 + 1$$

(b) We let $u = x^2$ and $\frac{dv}{dx} = e^x$

Then $\frac{du}{dx} = 2x$ and $v = e^x$

Using the integration by parts formula we have

$$\int_0^2 x^2 e^x dx = [x^2 e^x]_0^2 - \int_0^2 2x e^x dx = 4e^2 - 2 \int_0^2 x e^x dx$$

Now $\int_0^2 x^2 e^x dx = 4e^2 - 2[e^2 + 1] = 12.78$

(c) Put $z = \tan x$. Then $\frac{dz}{dx} = \sec^2 x$, $dz = \frac{dx}{\cos^2 x}$. Hence,

$$\int \frac{2e^{\tan x}}{\cos^2 x} dx = \int 2e^z dz = 2(e^z + c) = 2e^{\tan x} + 2c$$

2. Solve the following differential equations for y .

(a) $\frac{dy}{dx} = 3 \cos 2x$

(b) $\frac{dy}{dx} = 3x^2 e^{-y}$ with the initial condition of $y(0) = 1$

(c) $\frac{dy}{dx} = \frac{y^2}{x^2} + \frac{y}{x} + 1$ (Hint: You may let $z = \frac{y}{x}$)

Solution:

(a) Given that $\frac{dy}{dx} = 3 \cos 2x$, then $y = \int 3 \cos 2x dx = \frac{3}{2} \sin 2x + C$. This is the required general solution.

(b) Here $g(y) = e^{-y}$ and $f(x) = 3x^2$. Separating the variable and integrating we find

$$\int e^y dy = \int 3x^2 dx$$

So that $e^y = x^3 + C$. Imposing the initial condition $y(0) = 1$ we find $e^1 = (0)^3 + C$

so that $C = e$. Therefore,

$$e^y = x^3 + e$$

Note that since the exponential function is always positive, the solution will be valid only for $x^3 + e > 0$. Taking natural logarithms gives the particular solution explicitly: $y = \ln(x^3 + e)$

(c) If $z = \frac{y}{x}$ then $y = zx$. Because the solution, y , is a function of x the variable z depends upon

$$x \text{ also. The product rule gives } \frac{dy}{dx} = z + x \frac{dz}{dx}, \text{ so that Equation becomes } z + x \frac{dz}{dx} = z^2 + z + 1$$

That is,

$$x \frac{dz}{dx} = z^2 + 1$$

This new equation has independent variable x and dependent variable z , and is separable. So

$$\int \frac{dz}{z^2 + 1} = \int \frac{dx}{x}$$

So that $\tan^{-1} z = \ln|x| + C$. Writing $C = \ln|D|$ we have

$$\tan^{-1} z = \ln|x| + \ln|D| = \ln |Dx|$$

So that $z = \tan(\ln|Dx|)$. Returning to be original variable we see that the general solution is

$$y = zx = x \tan(\ln |Dx|)$$

3.

(a) Reduce each of the following expressions to a single sinusoidal function:

(i) $(\sin x - \cos x)^2 - 1$

(ii) $\sin x \cos x \cos 2x \cos 4x$

(b) Reduce the following expression to a single sinusoidal function and state the amplitude, frequency and phase of the single sinusoidal function:

$$2 \sin(3t + \pi/6) + 4 \cos 3t$$

Solutions:

(a)

(i) Considering that

$$(A - B)^2 = A^2 + B^2 - 2AB$$

$$\text{So, } (\sin x - \cos x)^2 - 1 = \sin^2 x + \cos^2 x - 2 \sin x \cos x - 1$$

$$= 1 - 2 \sin x \cos x - 1$$

$$= -2 \sin x \cos x = -\sin 2x$$

(ii) Considering that

$$\sin A \cos A = \frac{1}{2} \sin 2A$$

$$\text{So, } \sin x \cos x \cos 2x \cos 4x = \frac{1}{8} (2 (2 (\sin x \cos x) \cos 2x) \cos 4x)$$

$$= \frac{1}{8} \sin 8x$$

(b)

$$2 \sin(3t + \pi/6) + 4 \cos 3t = \sqrt{3} \sin 3t + (1 + 4) \cos 3t$$

$$= \sqrt{3 + 25} \sin [3t + \tan^{-1} (5/\sqrt{3})]$$

Since $\tan^{-1} (5/\sqrt{3}) = 1.2373$,

we obtain

$$2 \sin (3t + \pi/3) + 4 \cos 3t = 2\sqrt{7} \sin (3t + 1.2373)$$

In this single wave, the amplitude is $2\sqrt{7}$, the frequency is $\frac{3}{2\pi}$ Hz, and the phase is 1.2373 radians.