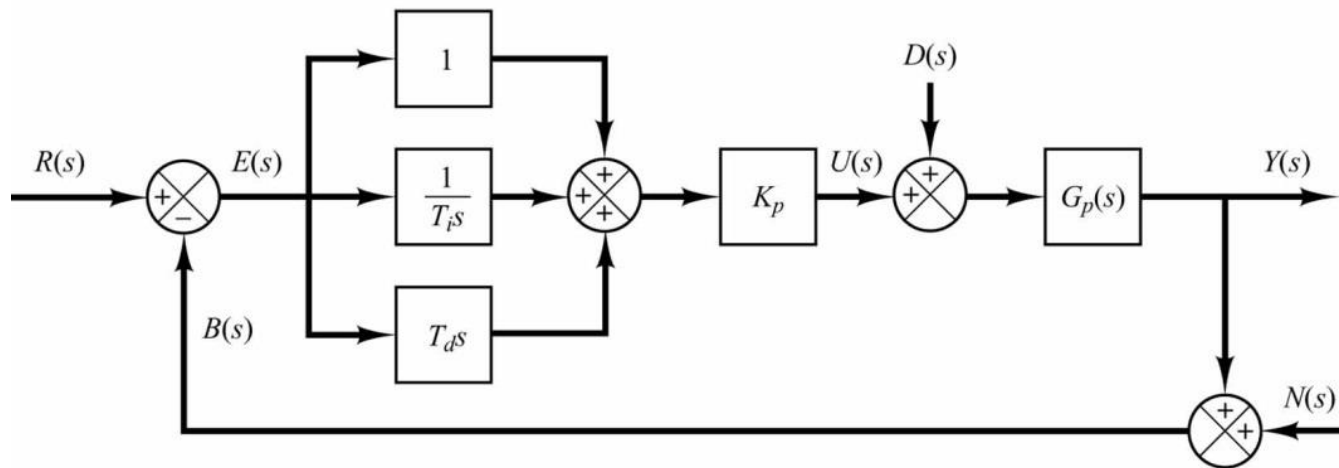


EE3114 Systems and Control

6. PID Controllers

Semester B, 2017-18

PID (Proportional-Integral-Derivative) Controller



More details will follow ...

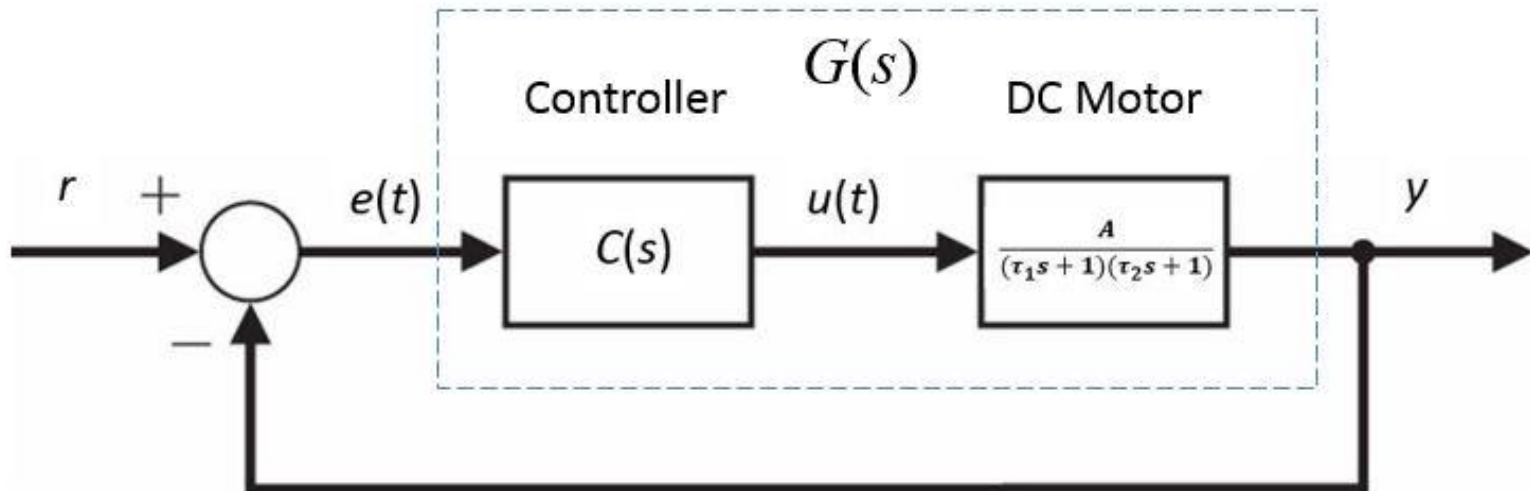
History

- Christian Huygens in 1673: centrifugal governor to regulate the gap between millstones in windmills
- James Watt in 1788: flyball governor for steam engine (proportional P-control)
- James Clerk Maxwell in 1868: mathematical basis for control stability
- Elmer Sperry in 1911: proportional-integral-derivative (PID) controller for ship-steering
- Nicolas Minorsky in 1922: Formal control laws of PID control

Industrial PID control

- Harold Black in late 1920s: Telephone engineering
- Clesson E Mason in 1930: PI-controller, with negative feedback from the controller output
- In the 1950s, when high gain electronic amplifiers available, electronic PID controllers became popular
- Most modern PID controllers in industry are implemented in distributed control systems (DCS), programmable logic controllers (PLCs), or as a panel-mounted digital controller.
- PID controllers are widely applied in industry, e.g., industrial assembly lines, chemical processes, electronic packaging, head positioning of disk drive, power conditioning of power supply, movement-detection circuit of modern seismometer, multicopter drones, ...

DC Motor as Example for Illustration

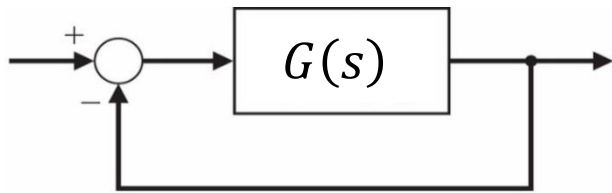


Controller $C(s)$:

Proportional-Integral-Derivative (PID) controller

P (Proportional) Control

- $C(s) = K_p$
- $u(t) = K_p e(t)$ (controller output $\propto e(t)$)
- Closed-loop system: unity feedback with



$$G(s) = \frac{K_p A}{(\tau_1 s + 1)(\tau_2 s + 1)}$$

(controller + plant)

Closed-loop Transfer Function:

$$H(s) = \frac{G(s)}{1 + G(s)} = \frac{K_p A}{\tau_1 \tau_2 s^2 + (\tau_1 + \tau_2)s + (1 + K_p A)}$$

Steady-State Performance

$$G(s) = \frac{K_p A}{(\tau_1 s + 1)(\tau_2 s + 1)}$$

- System type = 0
- Steady-state error for Step Input $r(t)$ is

$$\begin{aligned} e_{ss} &= \lim_{t \rightarrow \infty} (r(t) - y(t)) = \lim_{s \rightarrow 0} s(R(s) - Y(s)) \\ &= \lim_{s \rightarrow 0} sE(s) = \lim_{s \rightarrow 0} s \frac{1}{1+G(s)} R(s) \\ &= \lim_{s \rightarrow 0} s \frac{(\tau_1 s + 1)(\tau_2 s + 1)}{(\tau_1 s + 1)(\tau_2 s + 1) + K_p A} \frac{1}{s} = \frac{1}{1 + K_p A} \end{aligned}$$

- Larger $K_p \rightarrow$ Smaller e_{ss}

Transient Performance

- Closed-loop Transfer Function: $\frac{K_p A}{\tau_1 \tau_2 s^2 + (\tau_1 + \tau_2)s + (1 + K_p A)}$

- Characteristic equation:

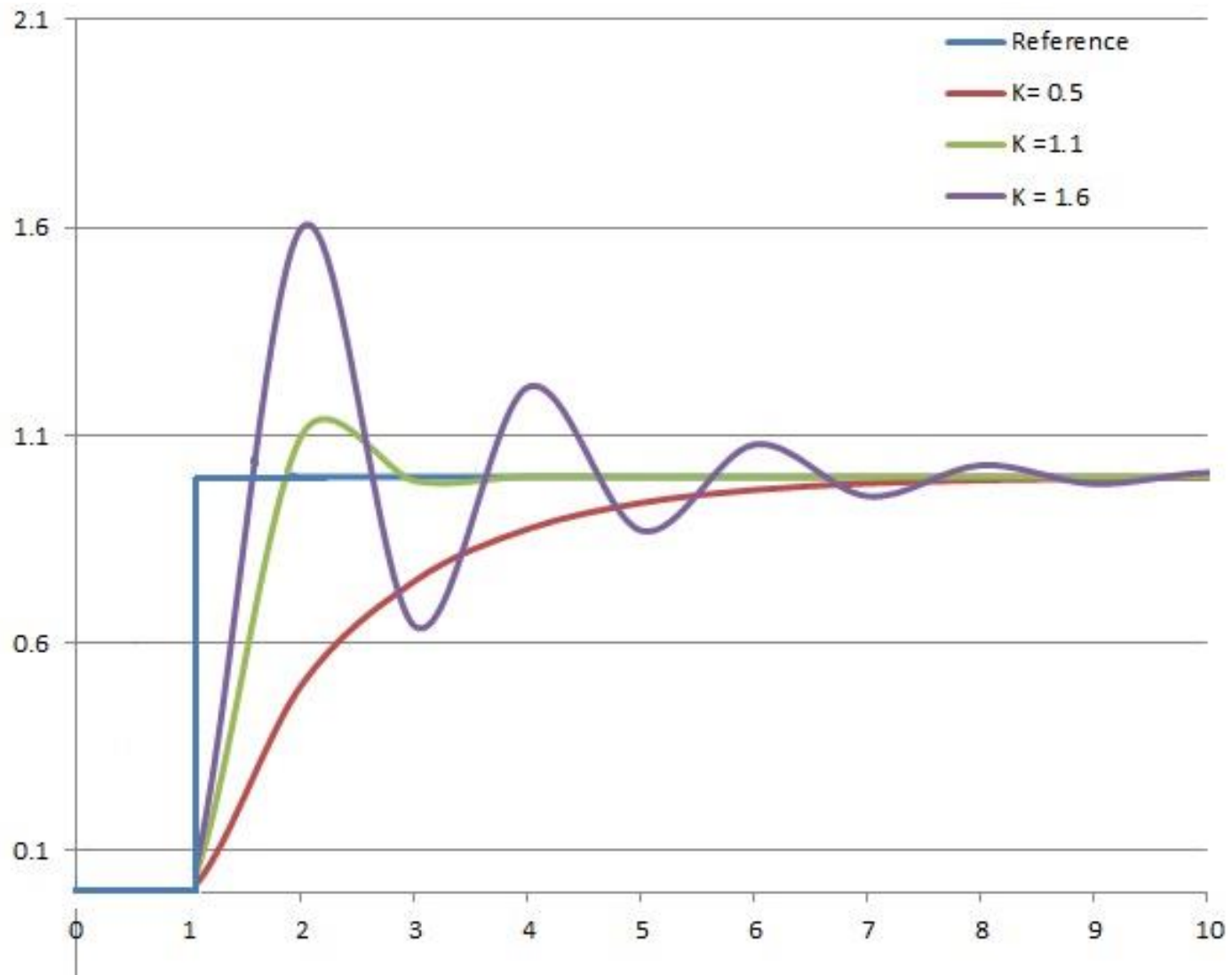
$$\tau_1 \tau_2 s^2 + (\tau_1 + \tau_2)s + (1 + K_p A) = 0$$

$\uparrow K_p \rightarrow \uparrow \omega_n \rightarrow \downarrow$ Rise Time (faster response)

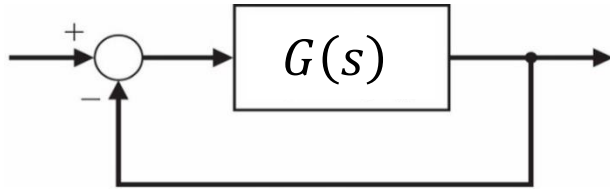
Damping ratio $\zeta \propto \frac{1}{\sqrt{1 + K_p A}}$

$\uparrow K_p \rightarrow \downarrow \zeta \rightarrow \uparrow$ Overshoot

Typical Unit Step Response of P -Control



Example:



$G(s) = (P\text{-controller} + \text{plant})$

$$\text{Type-0: } G(s) = K_p G_0(s) = \frac{K_p}{\left(\frac{s}{2}+1\right)\left(\frac{s}{50}+1\right)}$$

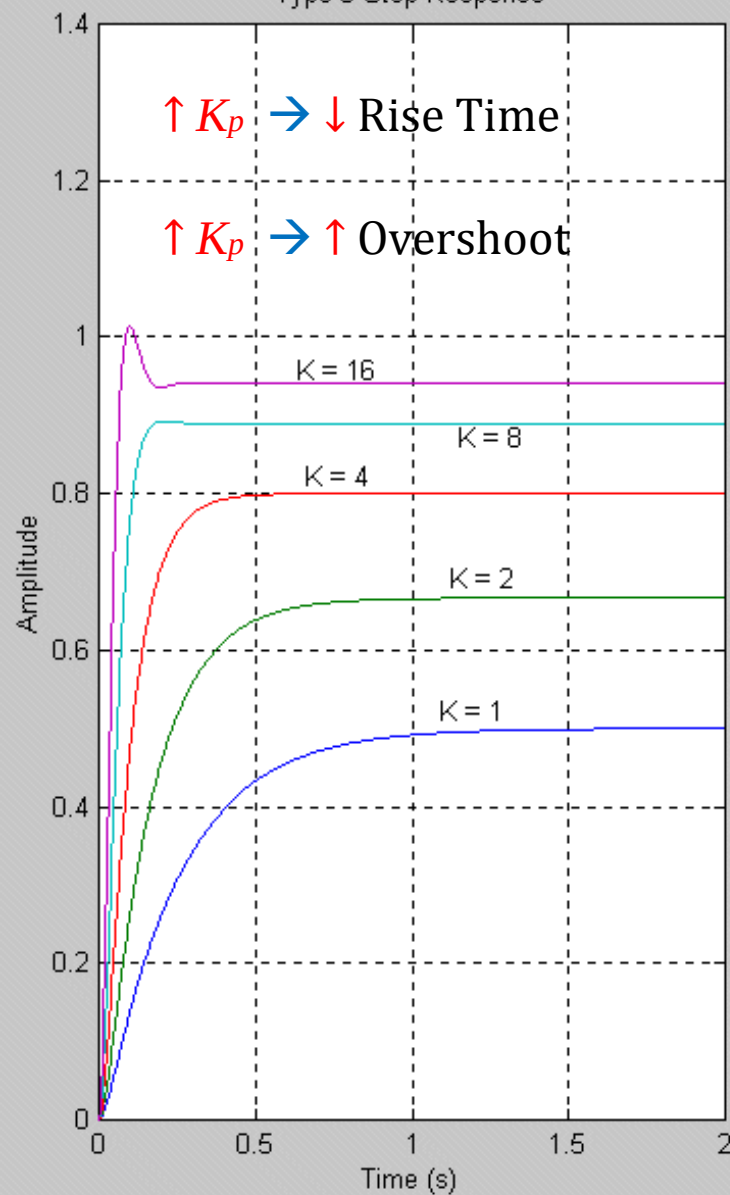
$$\text{Type-1: } G(s) = K_p G_1(s) = \frac{K_p}{s\left(\frac{s}{2}+1\right)\left(\frac{s}{50}+1\right)}$$

$$\text{Type-2: } G(s) = K_p G_2(s) = \frac{K_p(s+1)}{s^2\left(\frac{s}{2}+1\right)\left(\frac{s}{50}+1\right)}$$

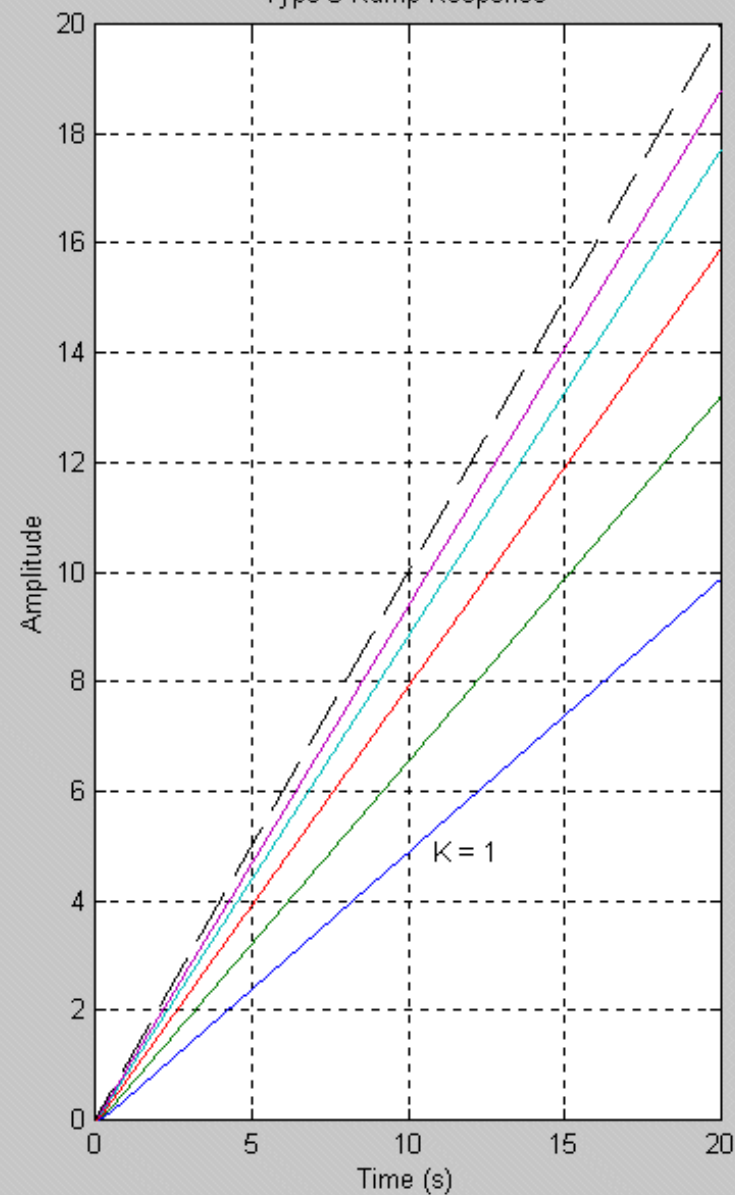
$$\text{Input: } R(s) = \frac{A}{s^2} \quad (\text{ramp input}) \quad A - \text{constant}$$

$$K_p G_0(s) = \frac{K_p}{\left(\frac{s}{2} + 1\right) \left(\frac{s}{50} + 1\right)}$$

Type 0 Step Response

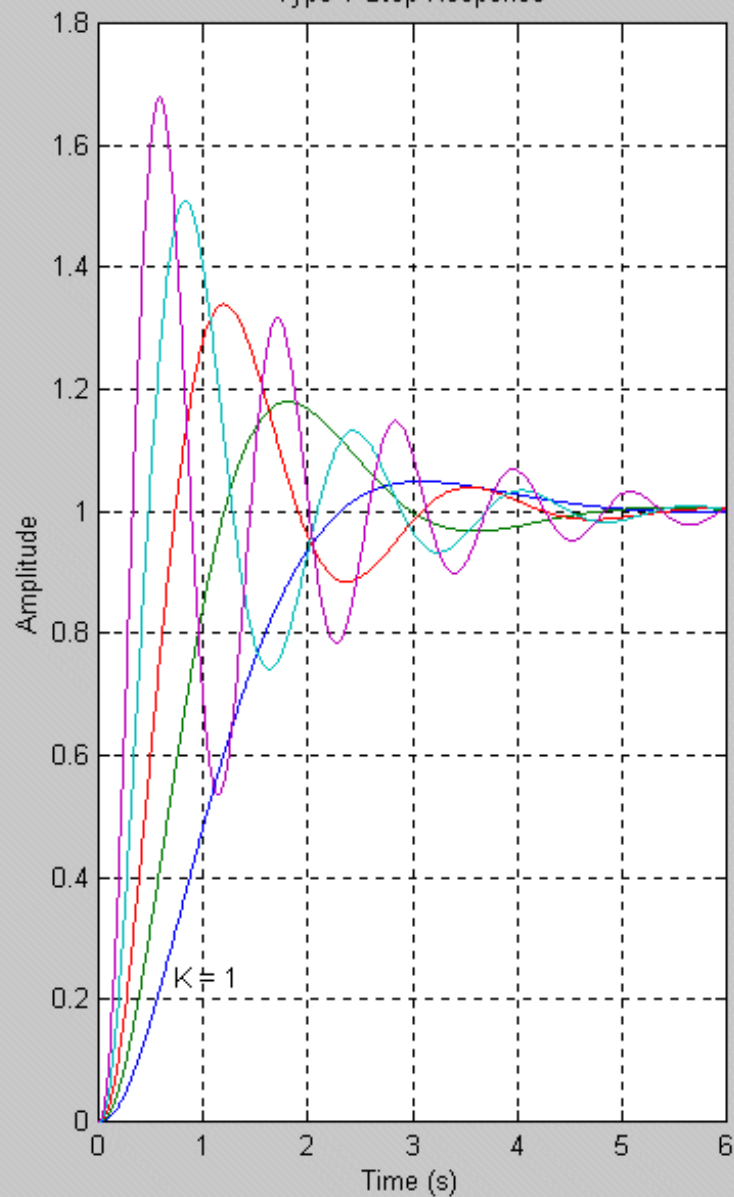


Type 0 Ramp Response

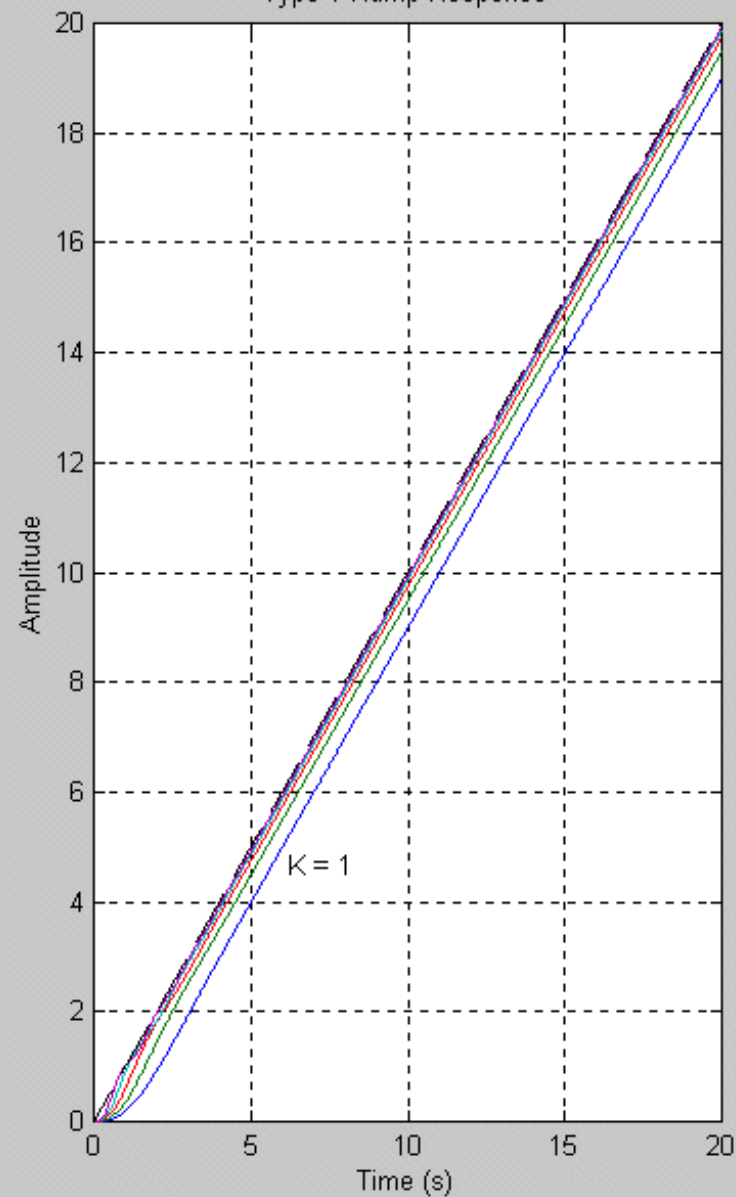


$$K_p G_1(s) = \frac{K_p}{s \left(\frac{s}{2} + 1 \right) \left(\frac{s}{50} + 1 \right)}$$

Type 1 Step Response

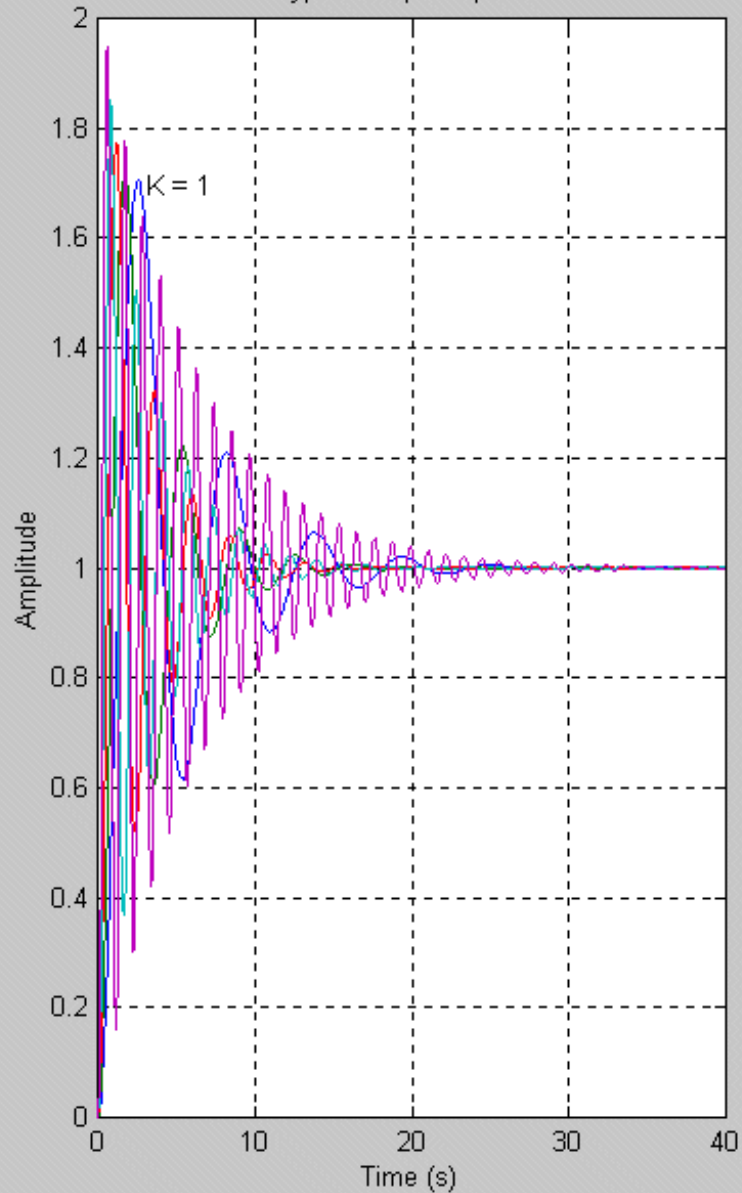


Type 1 Ramp Response

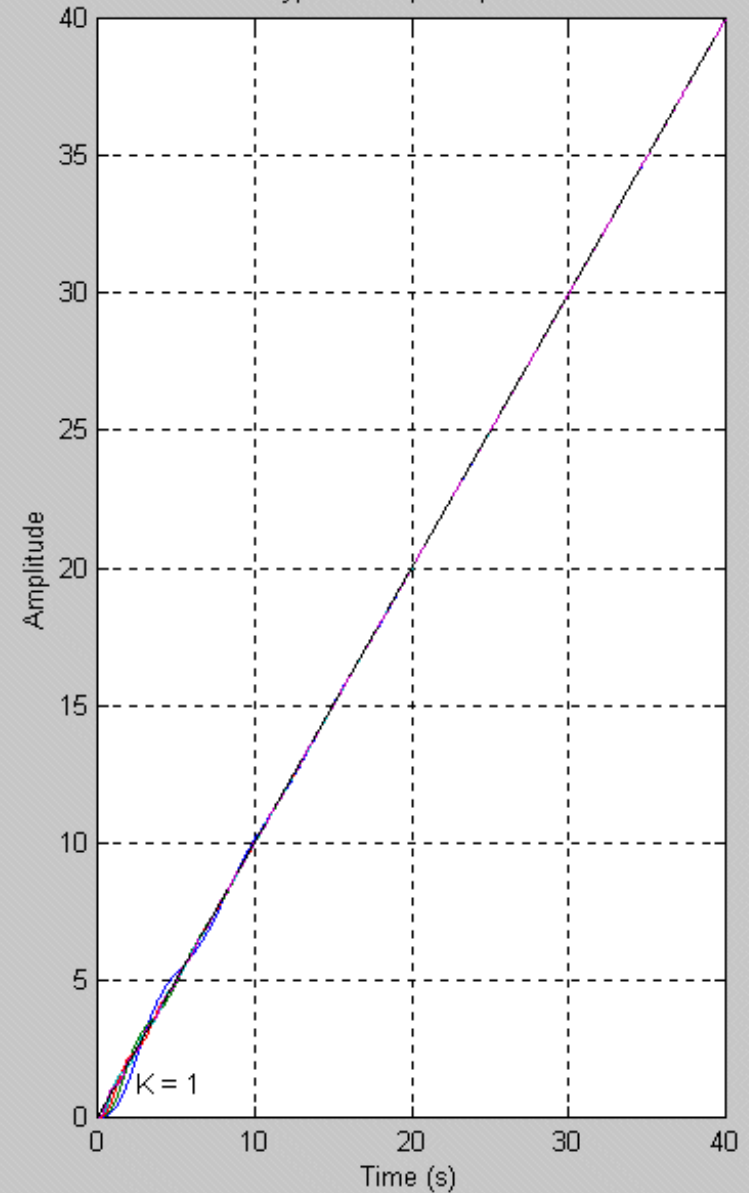


$$K_p G_2(s) = \frac{K_p(s+1)}{s^2 \left(\frac{s}{2} + 1\right) \left(\frac{s}{50} + 1\right)}$$

Type 2 Step Response



Type 2 Ramp Response

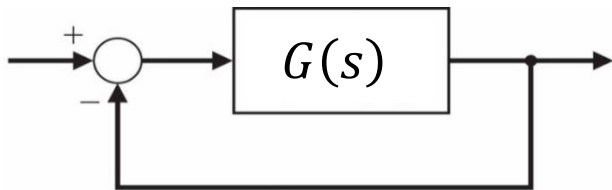


I (Integral) Control

- $C(s) = \frac{K_i}{T_I s}$ (Integral time T_I is a scaling constant)
- $u(t) = \frac{K_i}{T_I} \int_0^t e(\tau) d\tau$ (controller output $\propto$ integral of $e(t)$)

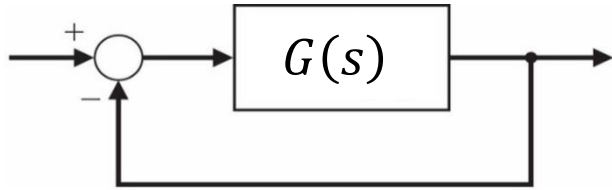
$$U(s) = \frac{K_i}{sT_I} \quad (\text{Integral control})$$

- Closed-loop system: with unity feedback



$$G(s) = \frac{K_i A / T_I}{s(\tau_1 s + 1)(\tau_2 s + 1)} \quad (\text{Type-1})$$

Steady-State Performance



$$G(s) = \frac{K_i A / T_I}{s(\tau_1 s + 1)(\tau_2 s + 1)}$$

- Steady-state error for ramp input of $r(t)$ is

$$e_{ss} = \lim_{t \rightarrow \infty} (r(t) - y(t)) = \lim_{s \rightarrow 0} s(R(s) - Y(s))$$

$$= \lim_{s \rightarrow 0} sE(s) = \lim_{s \rightarrow 0} s \frac{s(\tau_1 s + 1)(\tau_2 s + 1)}{s(\tau_1 s + 1)(\tau_2 s + 1) + K_i A / T_I} \frac{1}{s^2} = \frac{1}{K_i A / T_I}$$

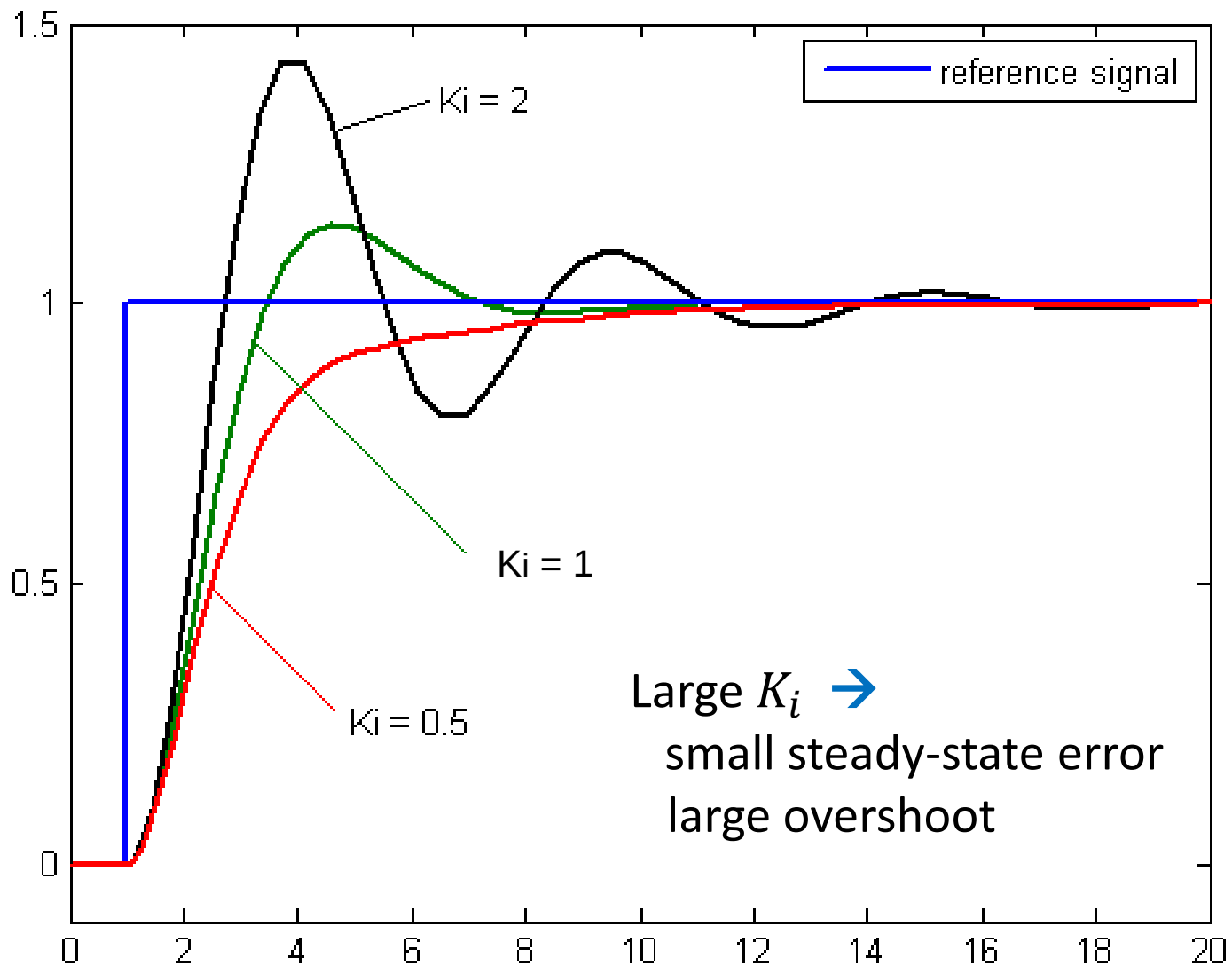
- Larger $K_i \rightarrow$ Smaller e_{ss}
 - $\rightarrow$ Reduce steady-state error for ramp input
(Performance better than P -control)

Transient Performance

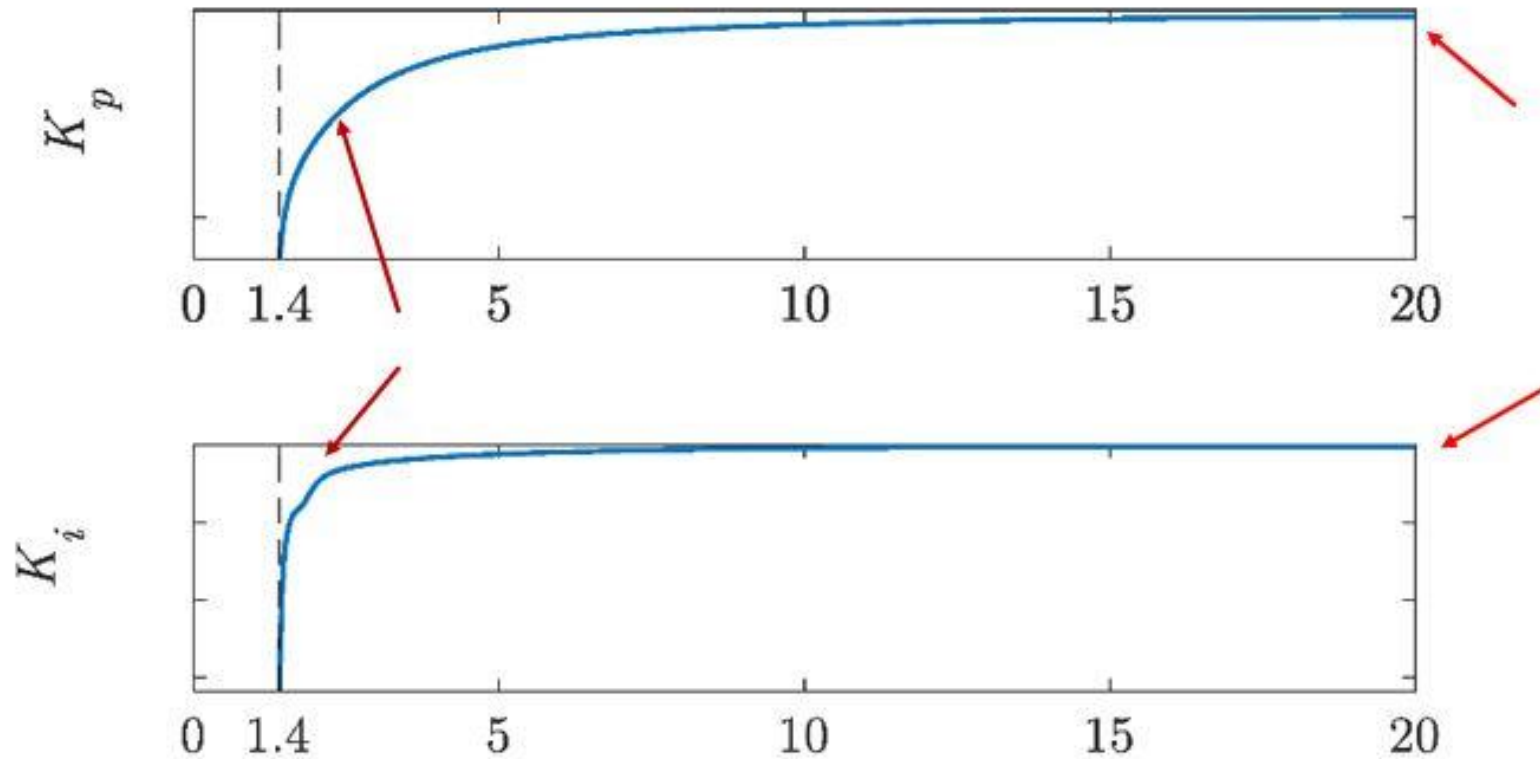
- One more pole than P -control
- Poles closer to imaginary axis
→ Lower damping ratio ζ
- Instability for large K_i

(Worse performance than P -control)

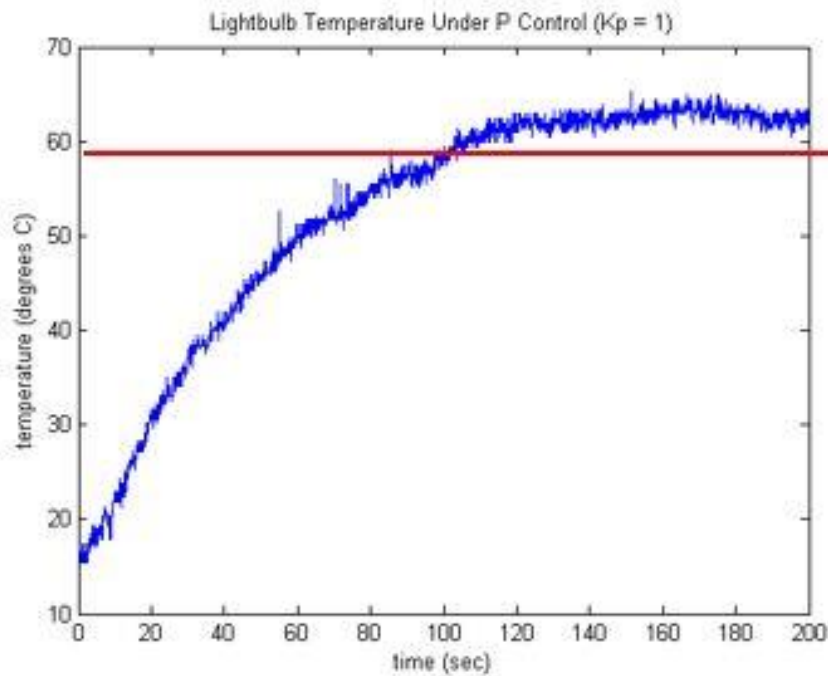
Typical Unit Step Response of I-Control



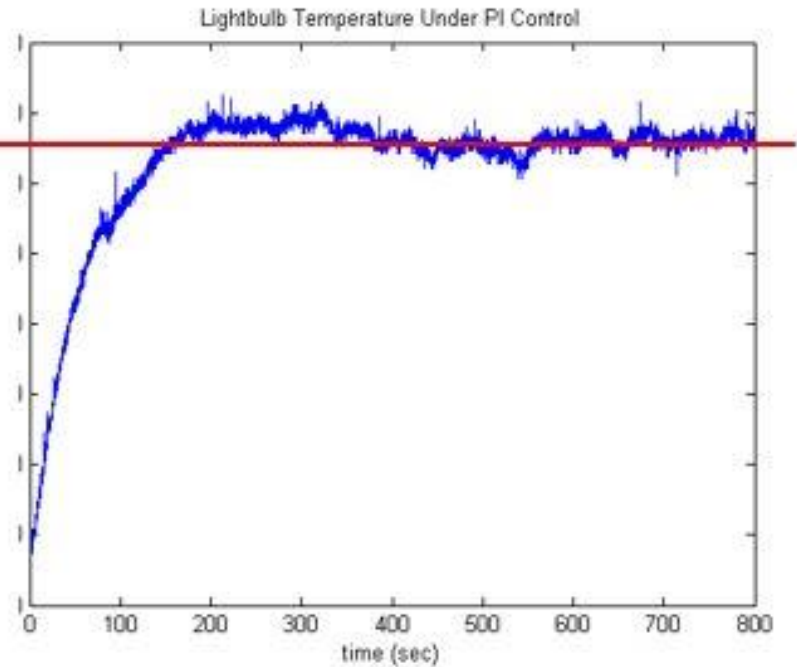
Comparison between P -control and I -control



A real comparison between P -control and I -control



P -control

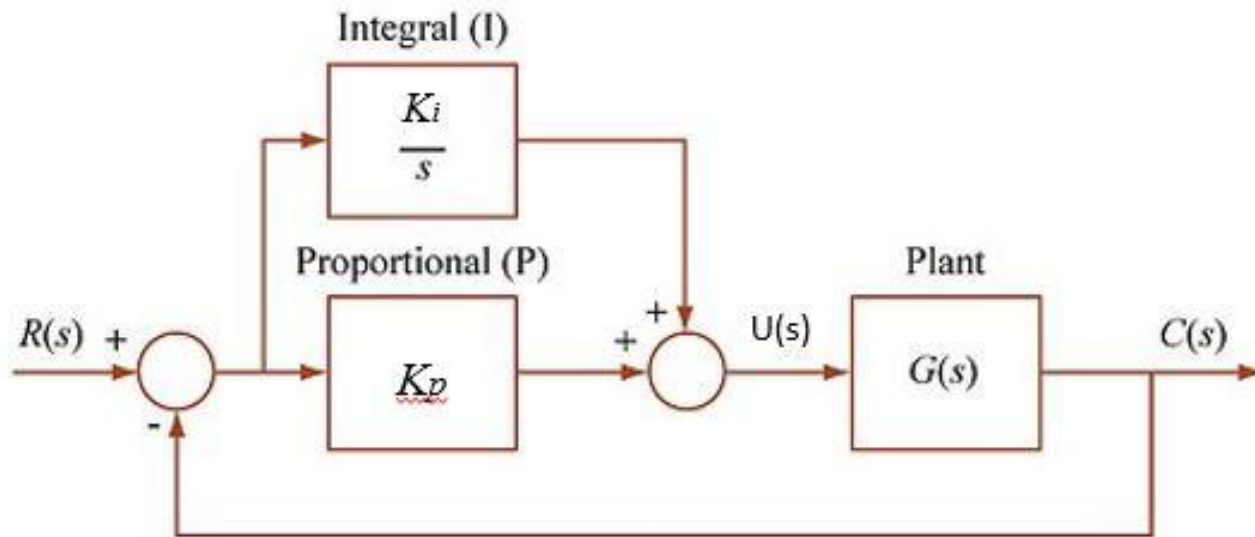


I -control

Lightbulb Temperature Control

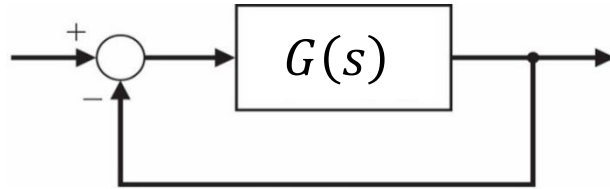
PI (Proportional-Integral) Control

This simple combination is much more powerful



PI-controller output $u(t) \propto e(t) + (\text{integral of } e(t))$

PI Control



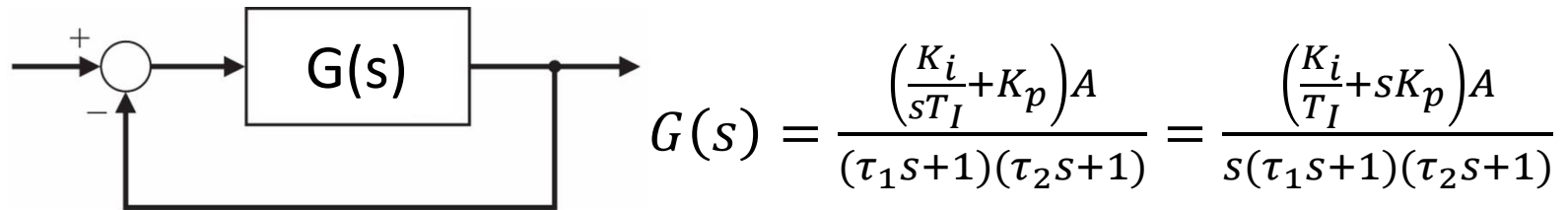
$$G(s) = \frac{\left(\frac{K_i}{sT_I} + K_p\right) A}{(\tau_1 s + 1)(\tau_2 s + 1)} = \frac{\left(\frac{K_i}{T_I} + sK_p\right) A}{s(\tau_1 s + 1)(\tau_2 s + 1)}$$

- $C(s) = K_p + \frac{K_i}{sT_I}$
- $u(t) = K_p e(t) + \frac{K_i}{T_I} \int e(t) dt$
(controller output $\propto e(t) + (\text{integral of } e(t))$)

Sometimes, write:

$$C(s) = K \left(1 + \frac{T_i}{s}\right)$$

Steady-State Performance



- System type = 1
- Steady-state error for Ramp Input of $r(t)$ is

$$\begin{aligned} e_{ss} &= \lim_{t \rightarrow \infty} (r(t) - y(t)) = \lim_{s \rightarrow 0} s(R(s) - Y(s)) \\ &= \lim_{s \rightarrow 0} sE(s) = \lim_{s \rightarrow 0} s \frac{s(\tau_1 s + 1)(\tau_2 s + 1)}{s(\tau_1 s + 1)(\tau_2 s + 1) + \left(\frac{K_i}{T_I} + sK_p\right)A} \frac{1}{s^2} = \frac{1}{K_i A / T_I} \end{aligned}$$

- Same as I -control: Larger $K_i \rightarrow$ Smaller e_{ss}
(but this is for ramp input $\rightarrow$ more powerful)

Transient Performance

- One more zero than I -control
- Better damped effect than I -control
- Worse transient performance than P -control
- Always stable for large K_i if $T_I > \frac{\tau_1 \tau_2}{\tau_1 + \tau_2}$

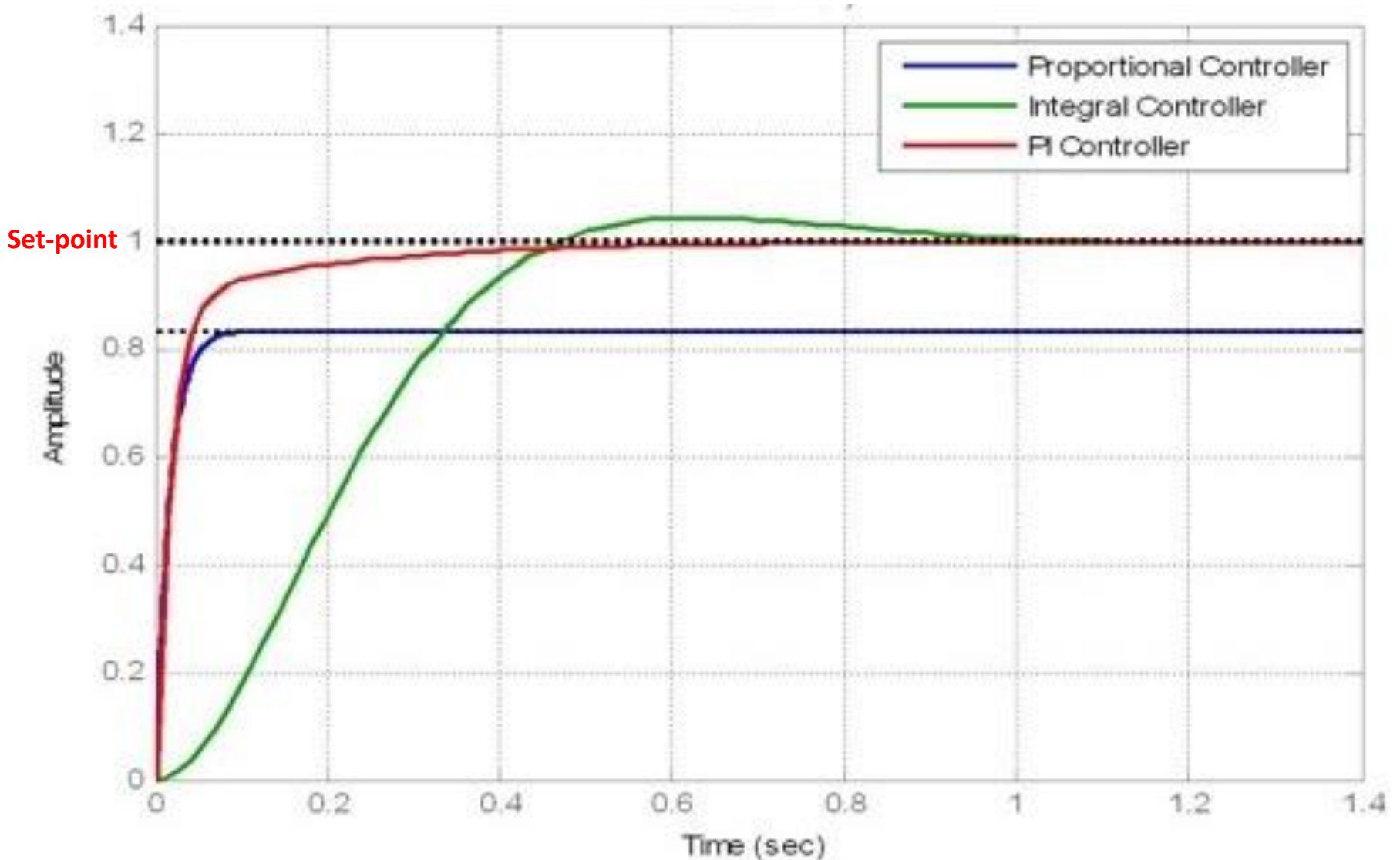
Verification:

Apply Routh criterion to

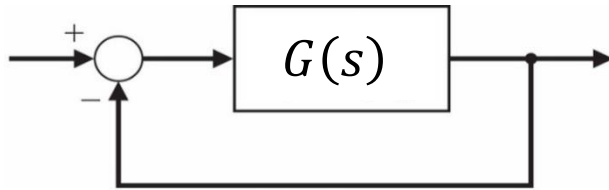
Closed-loop Transfer Function:

$$\frac{\left(\frac{K_i}{T_I} + sK_p\right)A}{\tau_1 \tau_2 s^3 + (\tau_1 + \tau_2)s^2 + (1 + K_p A)s + K_i A/T_I}$$

Comparison of Typical Unit Step Responses



Derivative Control



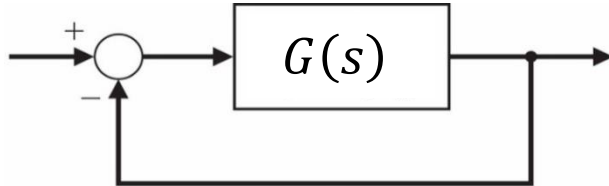
$$G(s) = \frac{sK_d T_D A}{(\tau_1 s + 1)(\tau_2 s + 1)}$$

- $C(s) = K_d T_D s$
- $u(t) = K_d T_D \frac{de(t)}{dt}$

Taking derivative on signals → instability + large errors

→ Usually, not being used alone

Steady-State Performance



$$G(s) = \frac{sK_d T_D A}{(\tau_1 s + 1)(\tau_2 s + 1)}$$

- System type = 0
- Steady-state error for Step Input of $r(t)$ is

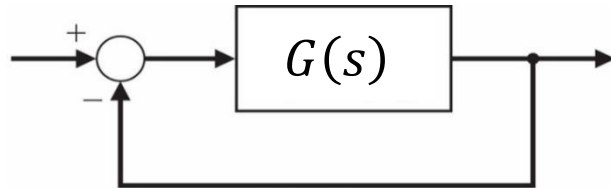
$$e_{ss} = \lim_{t \rightarrow \infty} (r(t) - y(t)) = \lim_{s \rightarrow 0} s(R(s) - Y(s))$$

$$= \lim_{s \rightarrow 0} sE(s) = \lim_{s \rightarrow 0} s \frac{(\tau_1 s + 1)(\tau_2 s + 1)}{(\tau_1 s + 1)(\tau_2 s + 1) + sK_d T_D A} \frac{1}{s} = 1$$

- Usually, e_{ss} = a constant

→ Steady-state errors cannot be reduced/eliminated

Transient Performance



$$G(s) = \frac{sK_d T_D A}{(\tau_1 s + 1)(\tau_2 s + 1)}$$

- Closed-loop Transfer Function:

$$H(s) = \frac{sK_d T_D A}{\tau_1 \tau_2 s^2 + (\tau_1 + \tau_2 + K_d T_D A)s + 1}$$

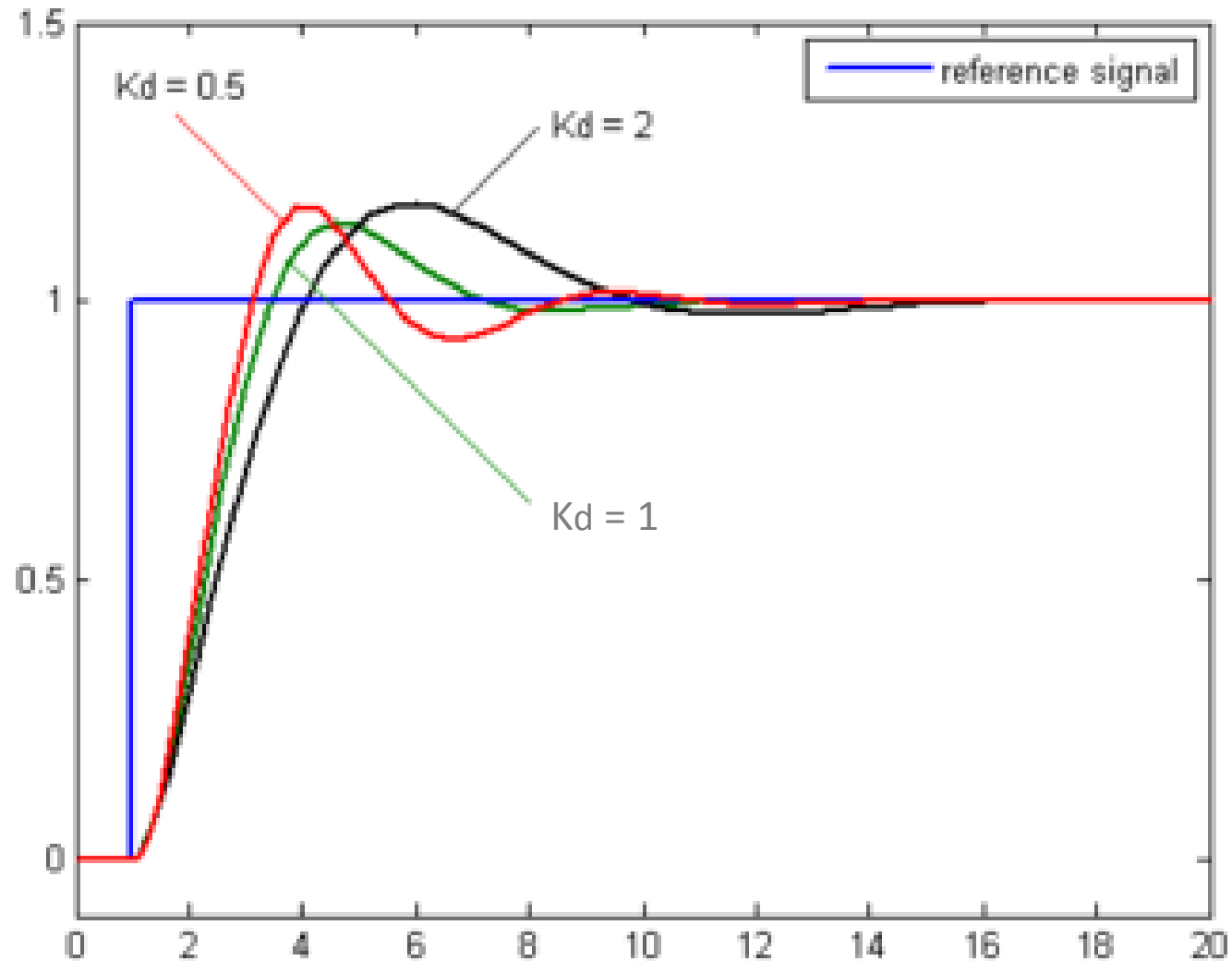
- T_D enters the damping ratio: $\zeta \propto \frac{1}{\sqrt{1 + K_d T_D A}}$

Potentially, making ζ larger



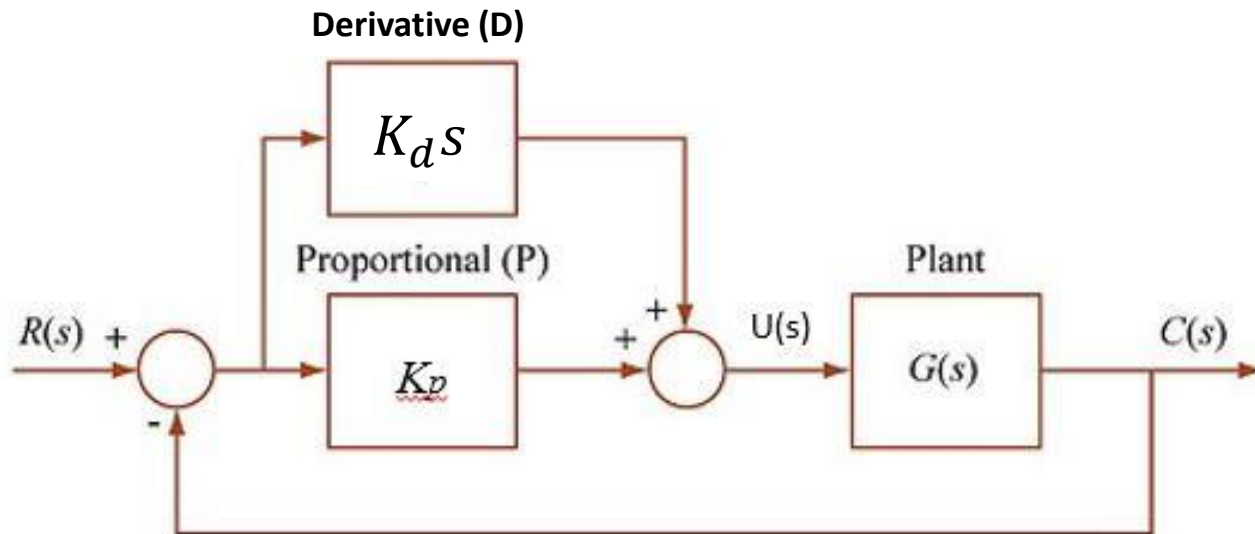
Stabilizing the transient of the D -controlled system

Typical Unit Step Response of *D*-Control



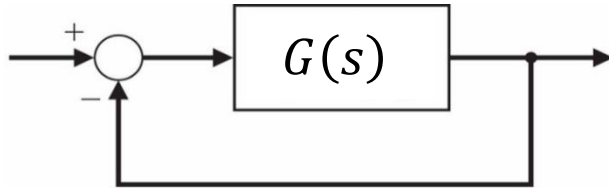
PD (Proportional-Derivative) Control

This simple combination is more powerful



PD-controller output $u(t) \propto e(t) + (\text{derivative of } e(t))$

PD Control



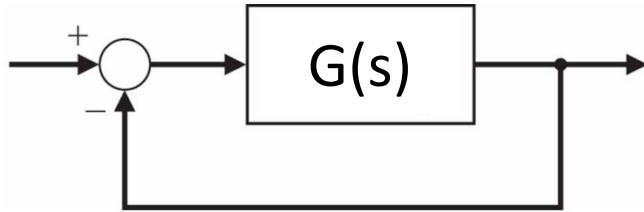
$$G(s) = \frac{(K_p + sK_d T_D)A}{(\tau_1 s + 1)(\tau_2 s + 1)}$$

- $C(s) = K_p + sK_d T_D$
- $u(t) = K_p e(t) + K_d T_D \frac{de(t)}{dt}$
(controller output $\propto e(t) + (\text{derivative of } e(t))$)

Sometimes write:

$$C(s) = K (1 + sT_d)$$

Steady-State Performance



$$G(s) = \frac{(K_p + sK_d T_D)A}{(\tau_1 s + 1)(\tau_2 s + 1)}$$

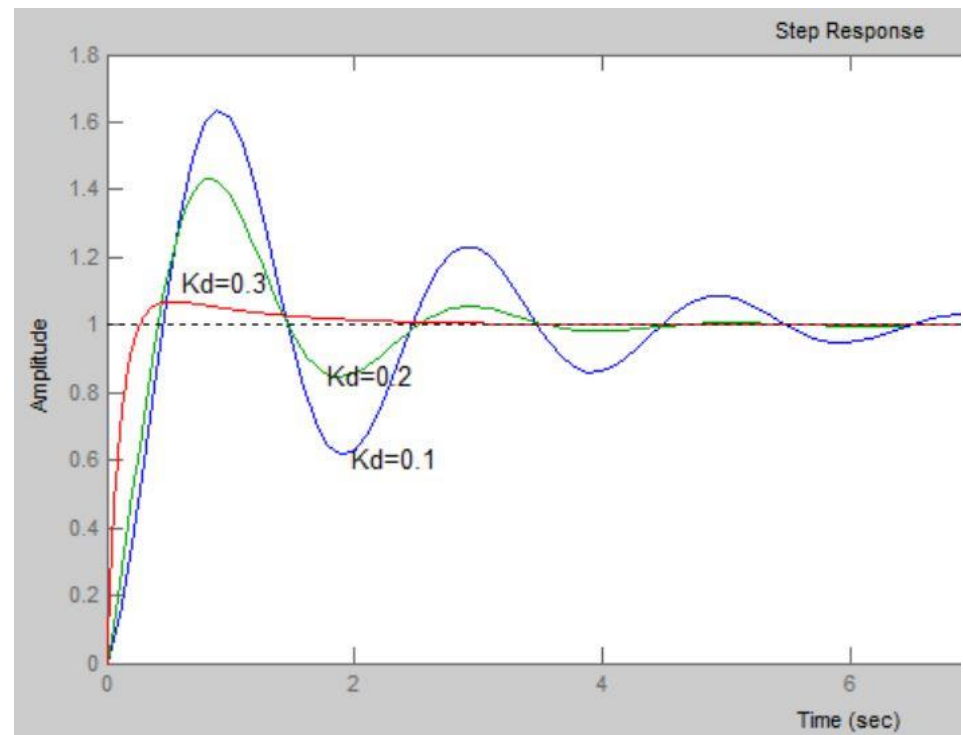
- System type = 0
- Steady state error for Step Input of $r(t)$ is

$$\begin{aligned} e_{ss} &= \lim_{t \rightarrow \infty} (r(t) - y(t)) = \lim_{s \rightarrow 0} s(R(s) - Y(s)) \\ &= \lim_{s \rightarrow 0} sE(s) = \lim_{s \rightarrow 0} s \frac{(\tau_1 s + 1)(\tau_2 s + 2)}{(\tau_1 s + 1)(\tau_2 s + 1) + (K_p + sK_d T_D)A} \frac{1}{s} = \frac{1}{1 + K_p A} \end{aligned}$$

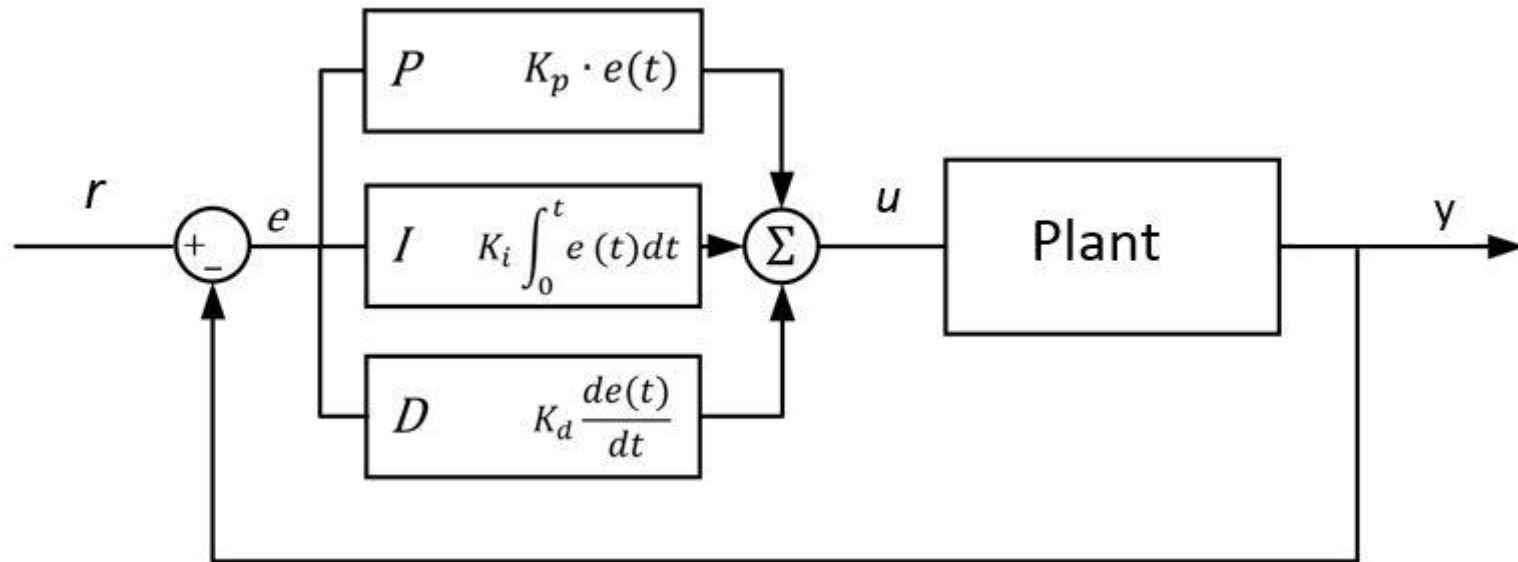
- Same as P -control: Large $K_p \rightarrow$ Small e_{ss}

Transient Performance

- PD-Control can add some damping effect to the output
 - PD-Control can decrease the settling time considerably
 - To control the steady-state error, the gain K_d (or time scale T_D) must be large
- ➔
- (This however may make the system sensitive to noise)



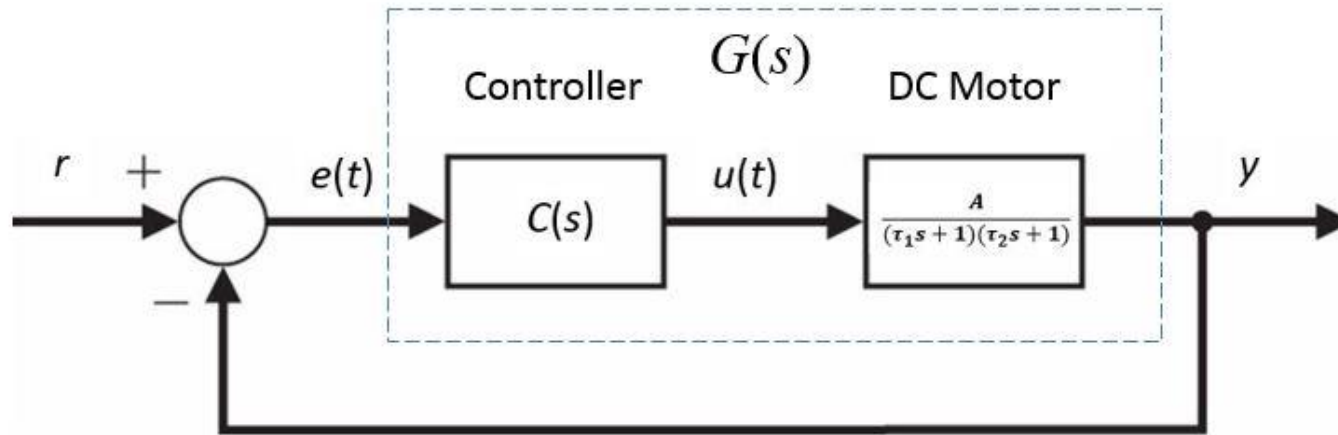
PID (Proportional-Integral-Derivative) Control



- $C(s) = K_p + \frac{K_i}{T_I s} + K_d T_D s = K \left(1 + \frac{1}{T_i s} + T_d s \right)$
- $u(t) = K_p e(t) + \frac{K_i}{T_I} \int_0^t e(\tau) d\tau + K_d T_D \frac{de(t)}{dt}$

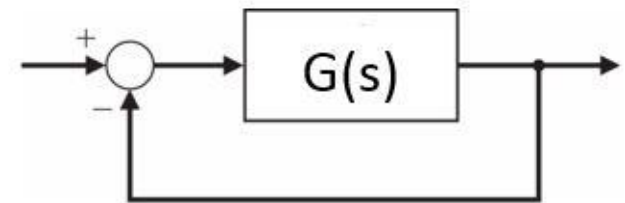
(output $\propto$ $e(t)$ + integral of $e(t)$ + derivative of $e(t)$)

PID (Proportional-Integral-Derivative) Control



- Unity feedback system with

$$G(s) = \frac{A \left(K_p + \frac{K_i}{T_I s} + K_d T_D s \right)}{s(\tau_1 s + 1)(\tau_2 s + 1)}$$



Steady-State Performance

$$G(s) = \frac{A\left(K_p + \frac{K_i}{T_I s} + K_d T_D s\right)}{s(\tau_1 s + 1)(\tau_2 s + 1)}$$

- System type = 1
- Steady state error for Parabolic Input $r(t) = t^2$ is

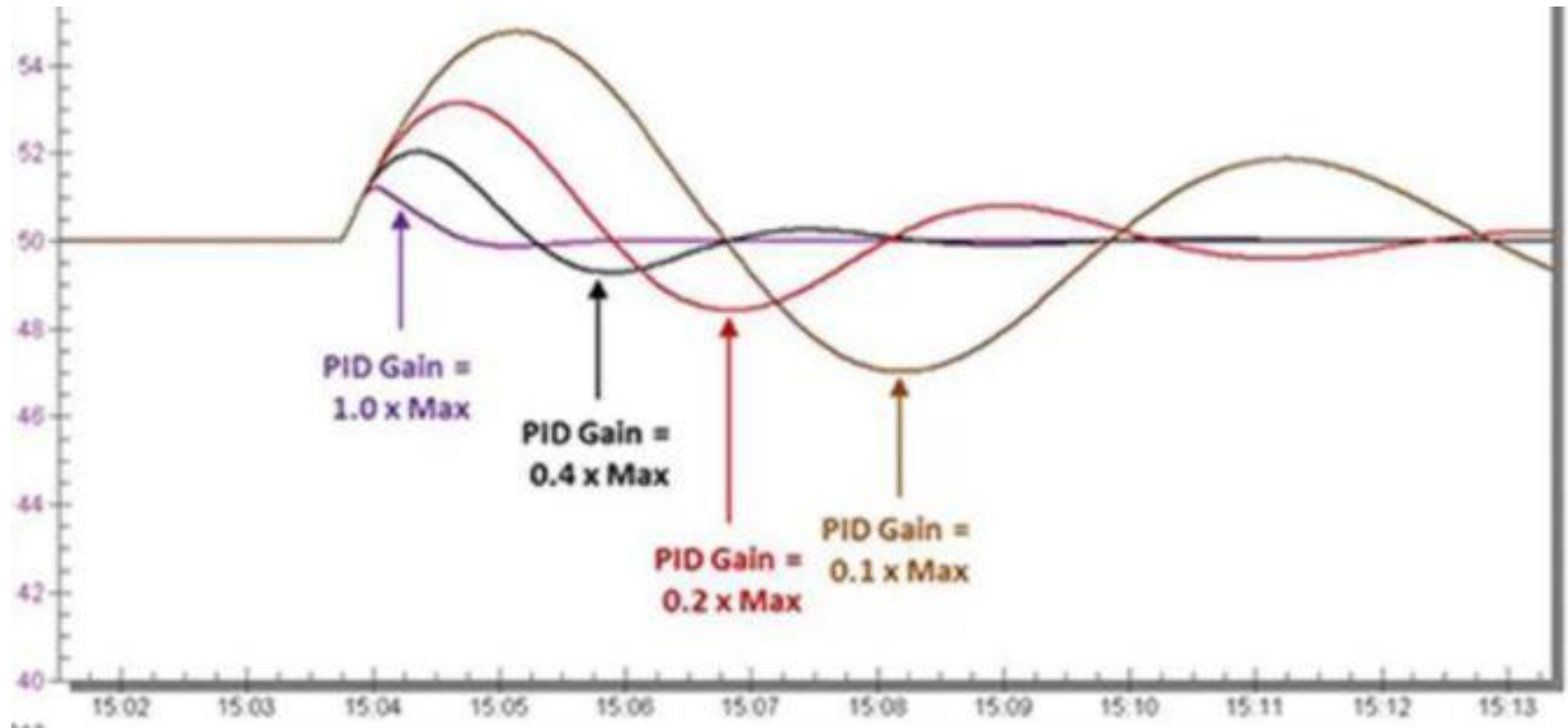
$$\begin{aligned} e_{ss} &= \lim_{t \rightarrow \infty} (r(t) - y(t)) = \lim_{s \rightarrow 0} s(R(s) - Y(s)) = \lim_{s \rightarrow 0} sE(s) \\ &= \lim_{s \rightarrow 0} s \frac{s^2(\tau_1 s + 1)(\tau_2 s + 1)}{s^2(\tau_1 s + 1)(\tau_2 s + 1) + A\left(K_d T_D s^2 + K_p s + \frac{K_i}{T_I}\right)} \frac{1}{s^3} = \frac{1}{K_i A / T_I} \end{aligned}$$

- Similarly improved performance as *PI*-control
(but this is for parabolic input → more powerful)
- It improves transient performance (next page)

Transient Performance

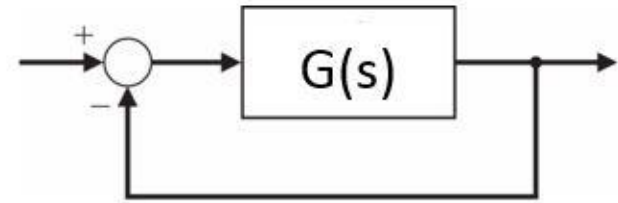
- One more zero than PI -control
- Poles further away from imaginary axis
 - better damped than P -control
- No instability problem for large K

Typical Unit Step Response of *PID*-Control



Unity feedback system with

$$G(s) = \frac{A \left(K_p + \frac{K_i}{T_I s} + K_d T_D s \right)}{s(\tau_1 s + 1)(\tau_2 s + 1)}$$



Example:

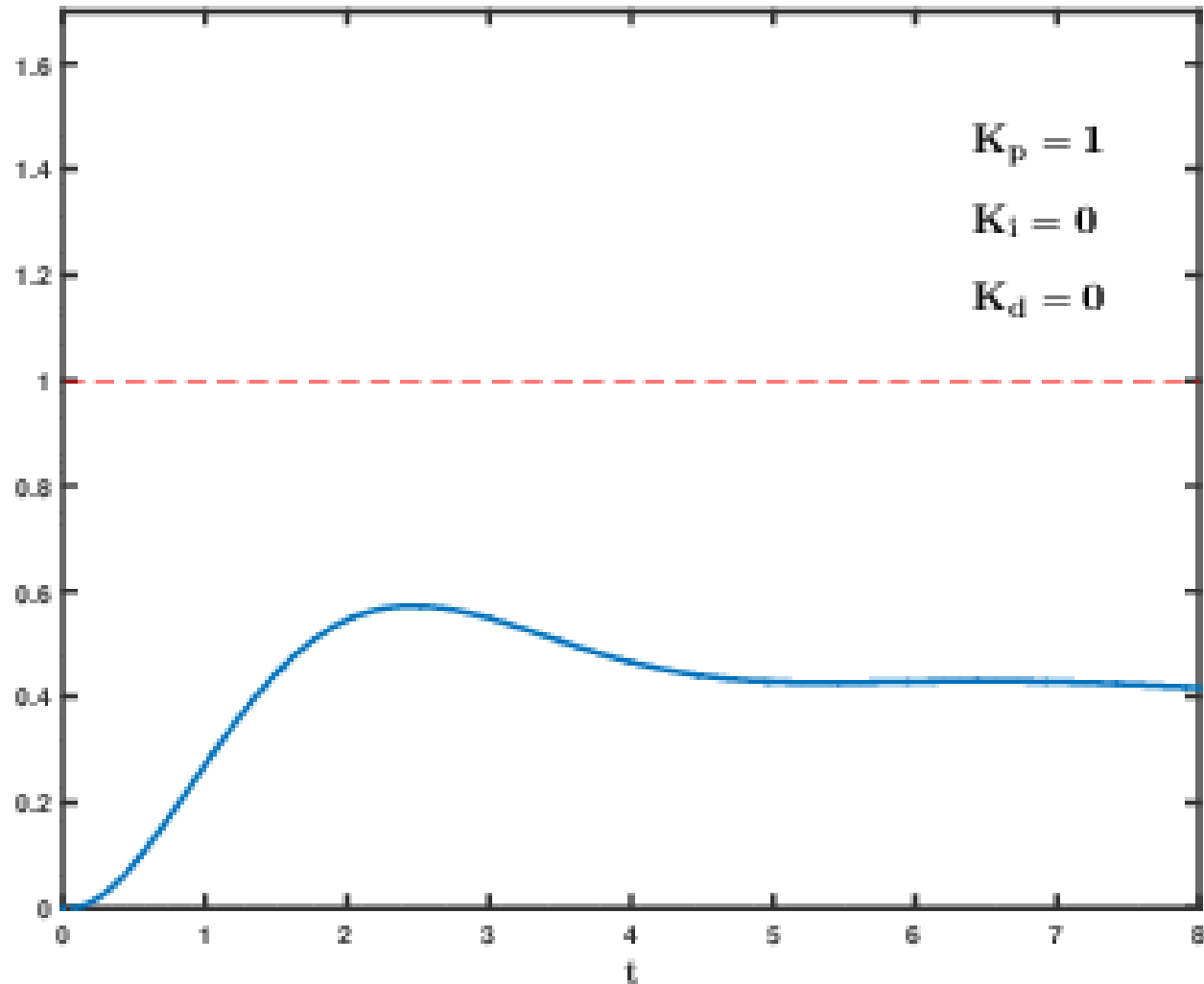
$$A = 1/6 ; \quad \tau_1, \tau_2 = 1/2, 1/3$$

Let $K_p = K$, $K_D = K T_D$ and $K_I = K/T_I$

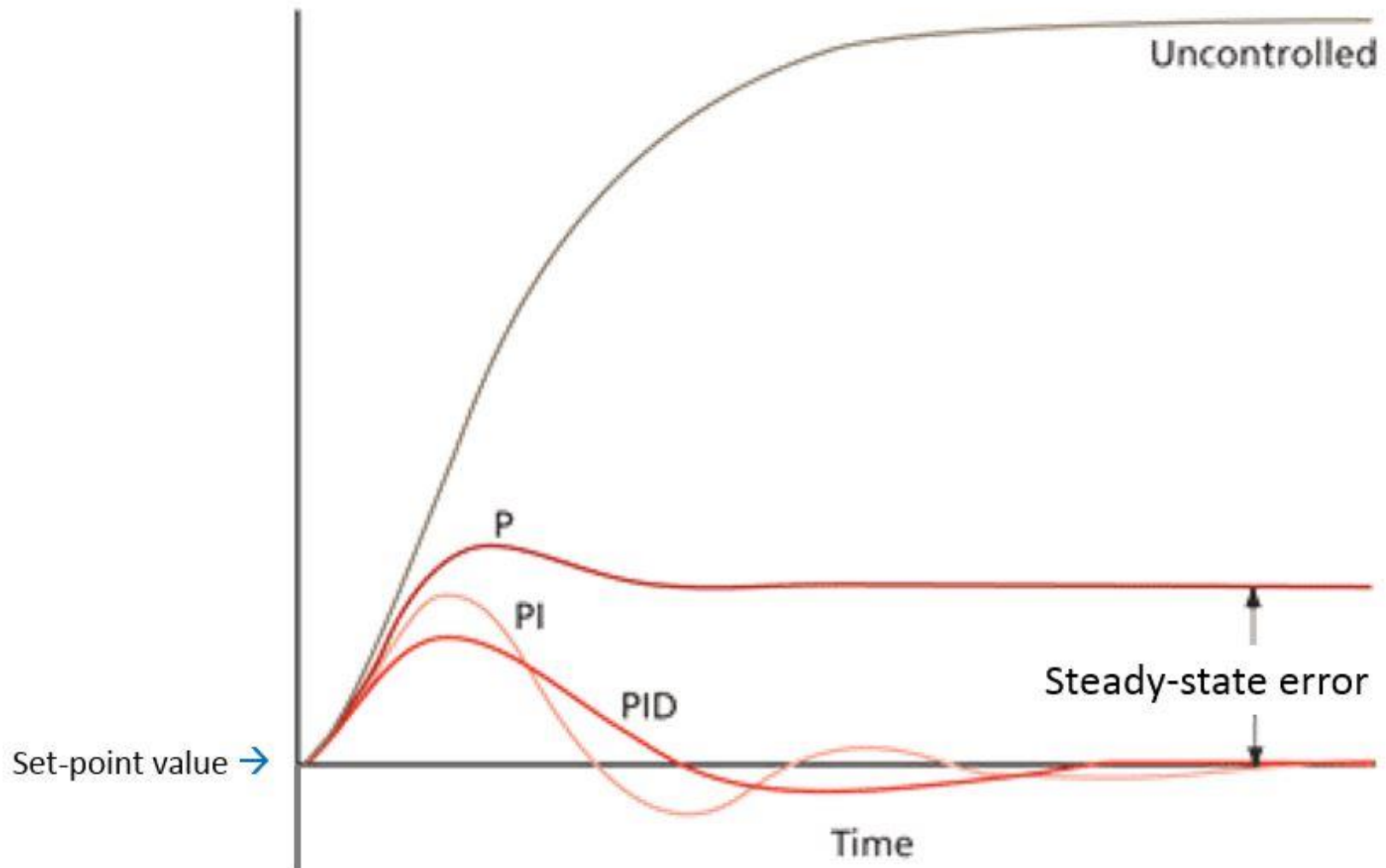
$$C(s) = K \left(1 + \frac{1}{T_i s} + T_d s \right) = \frac{K_D s^2 + K_P s + K_I}{s}$$

$$R(s) = \frac{1}{s}$$

Tuning the PID control gains K_p , K_i , K_d

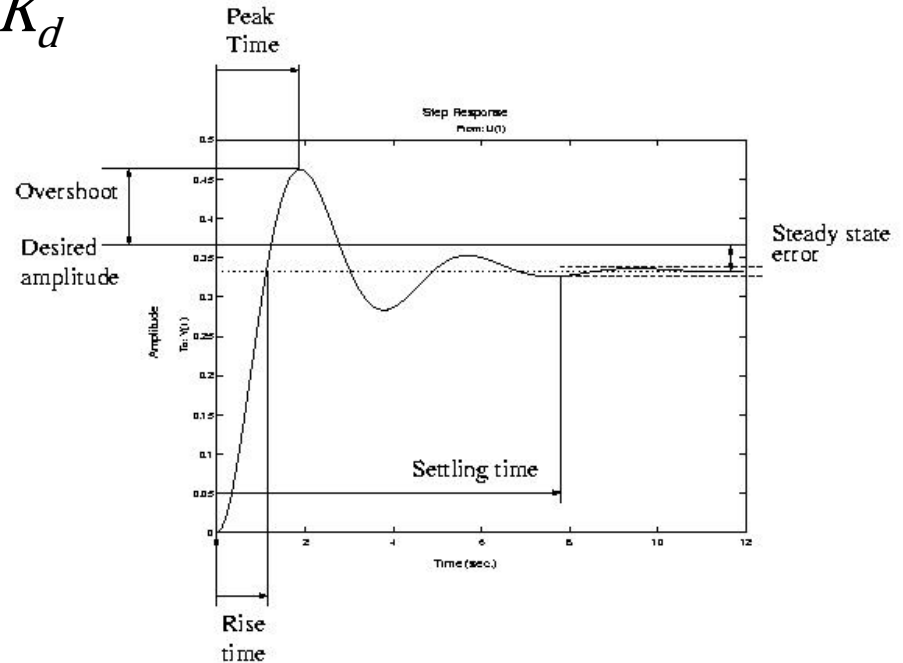


Comparison of Typical Unit Step Responses



PID Tuning

- Tuning parameters: K_p , K_i , K_d
- Control requirements:
 - Steady-State Error
 - Overshoot
 - Rise Time
 - Settling Time

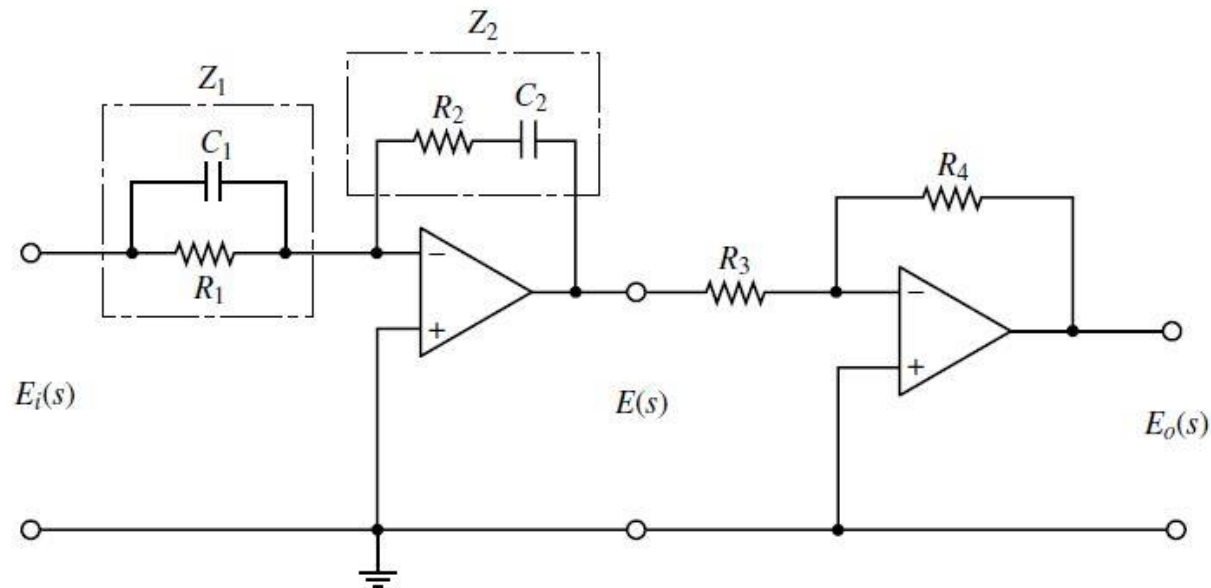


PID Gain	Steady-State Error	% Overshoot	Rise Time	Settling Time
K_p	Decrease	Increase	Decrease	Small change
K_i	Large decrease	Increase	Decrease	Increase
K_d	No effects	Decreases	Small decrease	Decreases

Comparison

Control	Advantages	Disadvantages
P	↓ Error	↑ Gain may destabilize system
I	• Improves e_{ss} • Provides robustness w.r.t. parameter variations	↓ Stability
D	• ↑ Damping • Improves stability	Amplify high-frequency error

PID Controller Implementation by Op-Amp



$$\frac{E_o(s)}{E_i(s)} = \frac{E_o(s)}{E(s)} \frac{E(s)}{E_i(s)} = \frac{R_4 R_2}{R_3 R_1} \frac{(R_1 C_1 s + 1)(R_2 C_2 s + 1)}{R_2 C_2 s} = \frac{R_4 (R_1 C_1 + R_2 C_2)}{R_3 R_1 C_2} \left[1 + \frac{1}{(R_1 C_1 + R_2 C_2)s} + \frac{R_1 C_1 R_2 C_2}{R_1 C_1 + R_2 C_2} s \right]$$

$$Z_1 = \frac{R_1}{R_1 C_1 s + 1}$$

$$Z_2 = \frac{R_2 C_2 s + 1}{C_2 s}$$

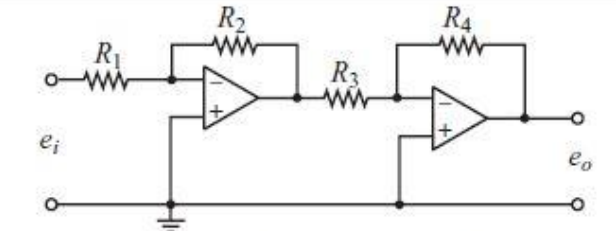
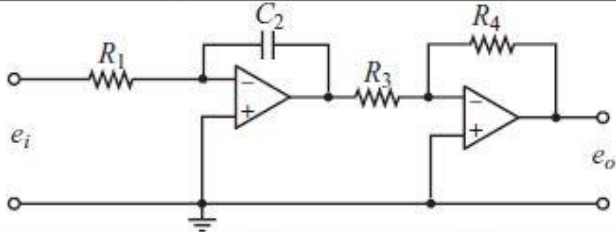
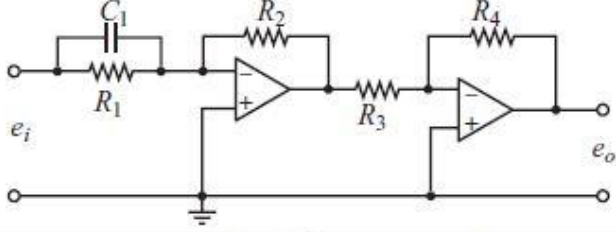
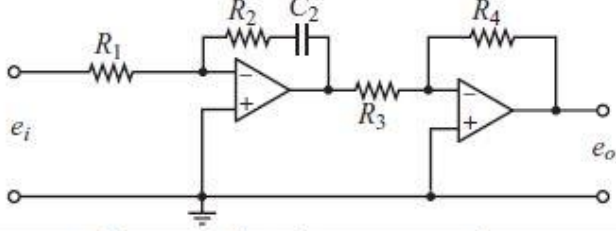
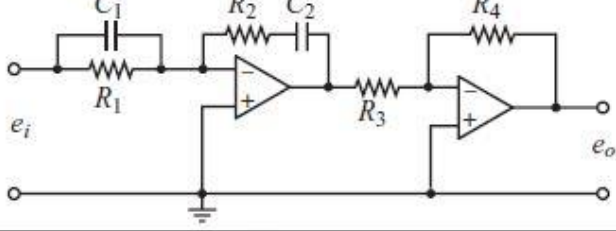
$$K_P = \frac{R_4 (R_1 C_1 + R_2 C_2)}{R_3 R_1 C_2}$$

$$T_I = R_1 C_1 + R_2 C_2$$

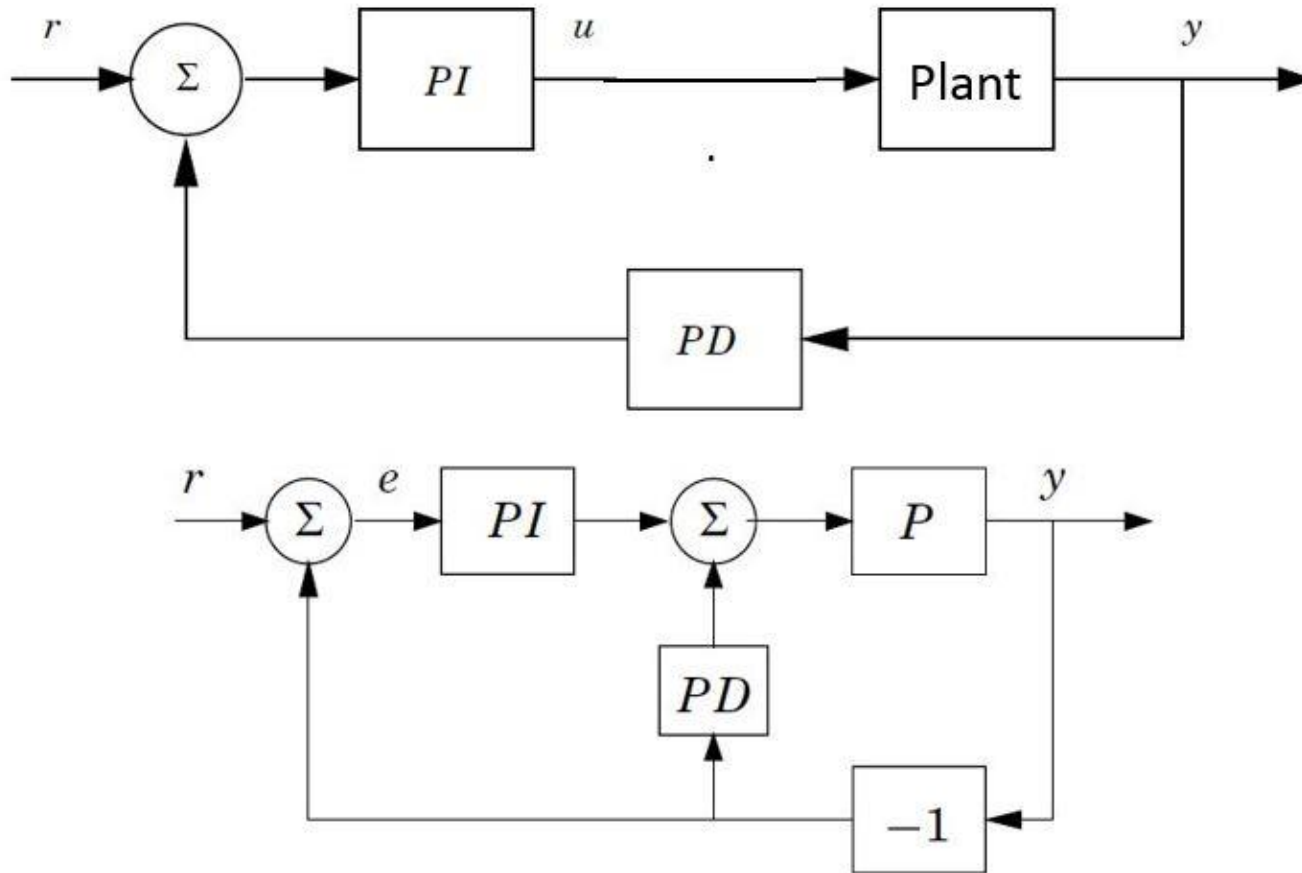
$$T_D = \frac{R_1 C_1 R_2 C_2}{R_1 C_1 + R_2 C_2}$$

$$K_I = \frac{R_4}{R_3 R_1 C_2}$$

$$K_D = \frac{R_4 R_2 C_1}{R_3}$$

Control Action	$G(s) = \frac{E_o(s)}{E_f(s)}$	Operational-Amplifier Circuits
P	$\frac{R_4}{R_3} \frac{R_2}{R_1}$	
I	$\frac{R_4}{R_3} \frac{1}{R_1 C_2 s}$	
PD	$\frac{R_4}{R_3} \frac{R_2}{R_1} (R_1 C_1 s + 1)$	
PI	$\frac{R_4}{R_3} \frac{R_2}{R_1} \frac{R_2 C_2 s + 1}{R_2 C_2 s}$	
PID	$\frac{R_4}{R_3} \frac{R_2}{R_1} \frac{(R_1 C_1 s + 1) (R_2 C_2 s + 1)}{R_2 C_2 s}$	

PI + PD Control



Ziegler-Nichols Rules for Tuning PID Controllers

- Tuning the gains K_P , K_D , K_I are not easy
- Ziegler and Nichols proposed **two heuristic methods** for tuning these parameters for good closed-loop responses

(This is a stable operation but may yield large overshoots)

- **Very useful when mathematical model of plant is unknown**

(Of course, still can be applied to the design of systems with known plant models)

Ziegler-Nichols First Method

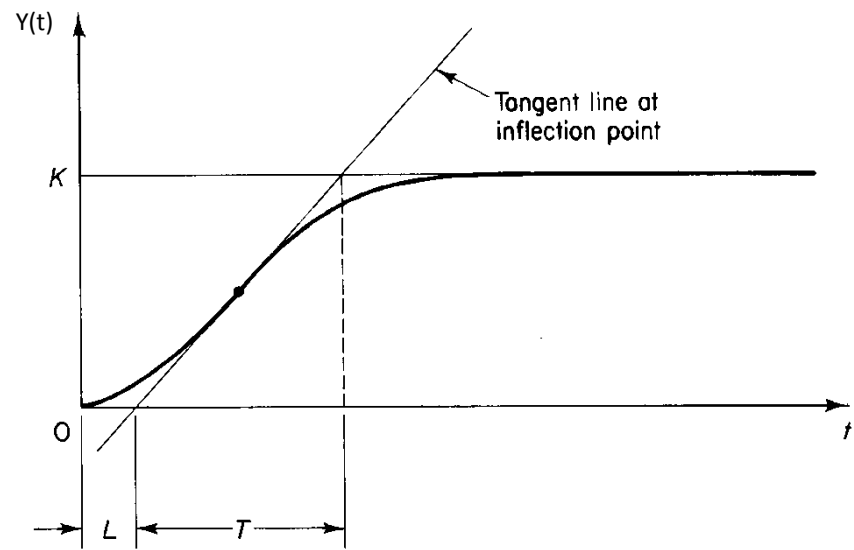
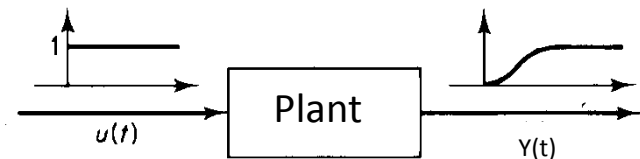
Record experimental output response of the plant to unit step input

Conditions:

1. Plant involves no integrators ($\frac{1}{s}$) nor complex-conjugate poles
2. If the response does not exhibit an S-shaped curve, this method cannot be applied

Then, the transfer function of the plant can be approximated by

$$\frac{Y(s)}{U(s)} = \frac{Ke^{-Ls}}{Ts + 1}$$



Ziegler-Nichols first rule (see figure) gives:

$$Y(s) = 1.2 \frac{T}{L} \left(1 + \frac{1}{2Ls} + 0.5Ls \right) = 0.6T \frac{\left(s + \frac{1}{L} \right)^2}{s}$$

It has double zeros: $z_1 = z_2 = -\frac{1}{L}$

It suggests the following PID control parameters			
	K	T_I	T_D
P	T/L	∞	0
PI	$0.9T/L$	$L/0.3$	0
PID	$1.2T/L$	$2L$	$0.5L$

Example:

Consider the plant response curve shown in the figure:

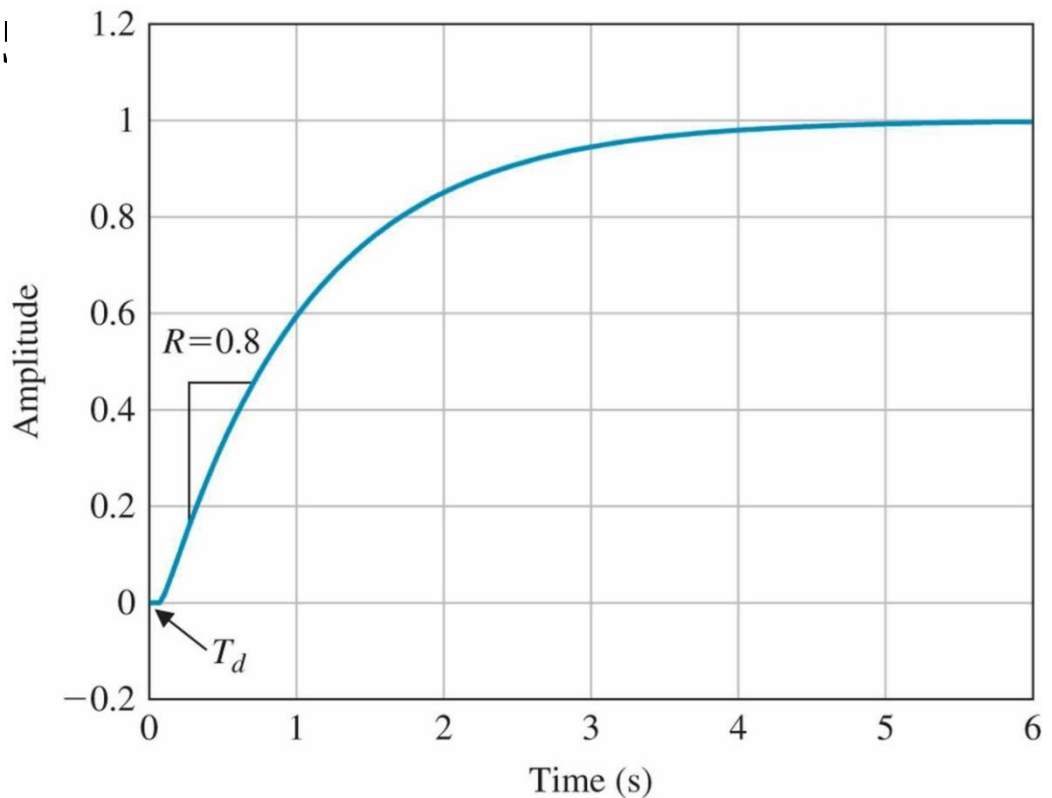
Estimating: $L \approx 0.1$ and $T \approx \frac{1}{0.8} = 1.25$

Design PI control by Ziegler-Nichols first rule:

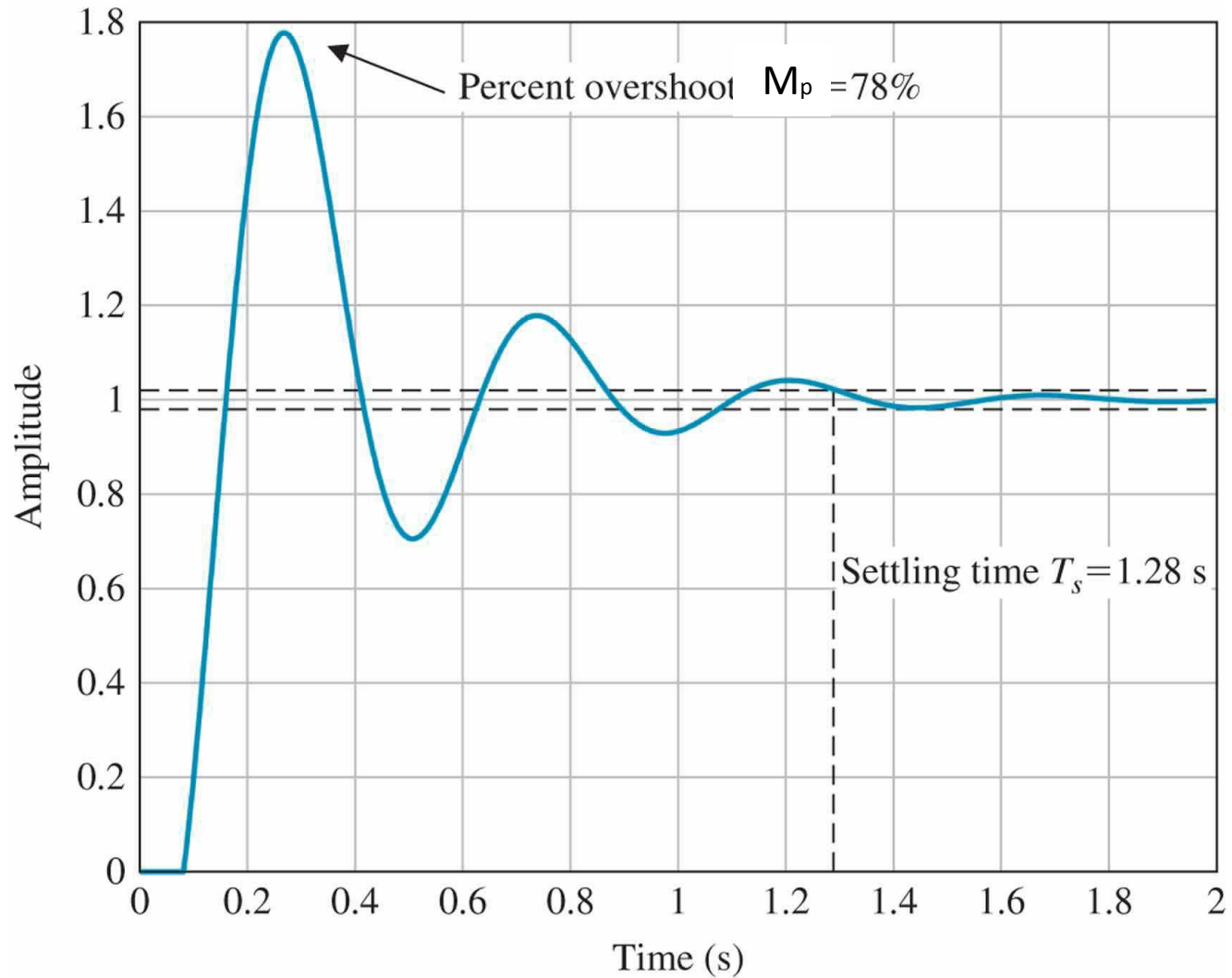
$$K = K_p = 0.9T/L = 11.25$$

$$T_I = L/0.3 = 1/3$$

$$K_i = K/T_I = 33.75$$

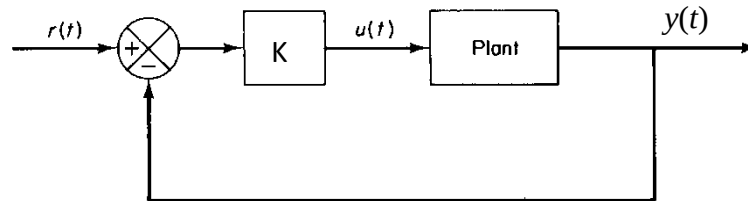


Time Response of PI control with $K_P = 11.25$ and $K_i = 33.75$

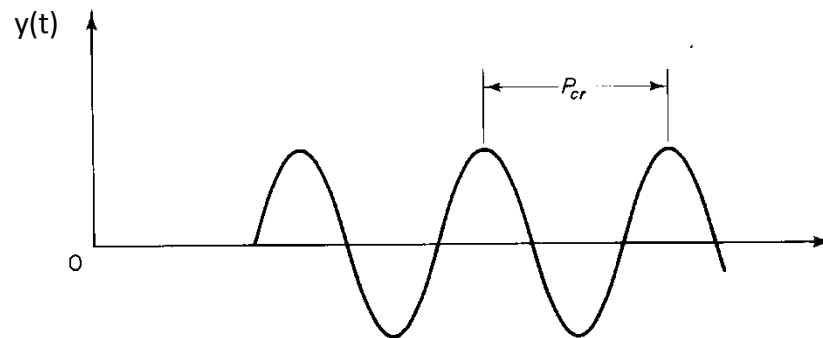


Ziegler-Nichols Second Method

- Set $T_I = \infty$ and $T_D = 0$



- Using P -control only, increase K from 0 to a critical value K_{cr} at which the output exhibits sustained oscillations (marginally stable):



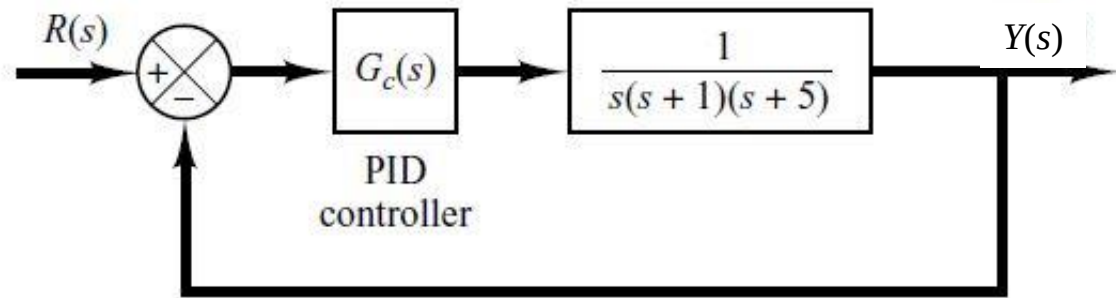
- If output does not exhibit oscillation for any value of K then this method cannot be applied

Ziegler-Nichols second rule (see figure) gives:

$$G_c(s) = 0.6K_{cr} \left(1 + \frac{1}{0.5P_{cr}s} + 0.125P_{cr}s \right) = 0.075K_{cr}P_{cr} \frac{\left(s + \frac{4}{P_{cr}} \right)^2}{s}$$

It suggests PID control parameters:			
	K	T_I	T_D
P	$0.5K_{cr}$	∞	0
PI	$0.45K_{cr}$	P_{cr}	0
PID	$0.6K_{cr}$	$0.5P_{cr}$	$0.125P_{cr}$

Example:



Plant has integrator $\frac{1}{s}$, so Ziegler–Nichols second method is used

Set $T_i = \infty$ and $T_d = 0 \Rightarrow G_c(s) = K \left(1 + \frac{1}{T_i s} + T_d s \right)$

$\Rightarrow G_c(s) = K = K_p \quad (P\text{-controller})$

$\Rightarrow \frac{Y(s)}{R(s)} = \frac{K_p}{s(s+1)(s+5) + K_p}$

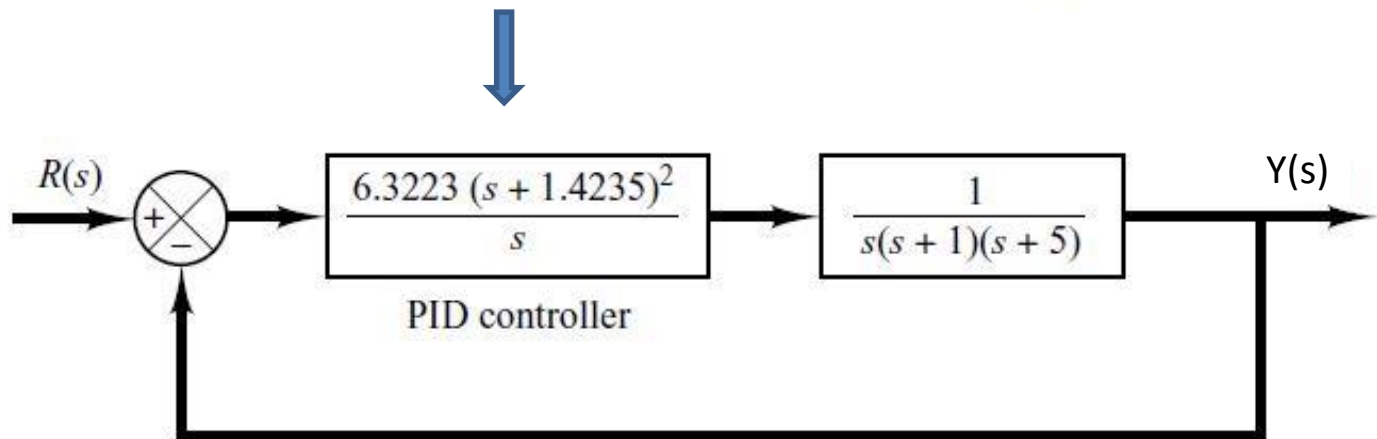
Routh Table

s^3	1	5
s^2	6	K_p
s^1	$\frac{30 - K_p}{6}$	
s^0	K_p	

$\Rightarrow K_{cr} = 30$

Parameters Table:

$$\begin{aligned} K_p &= 0.6K_{cr} = 18 & \longrightarrow G_c(s) &= K_p \left(1 + \frac{1}{T_i s} + T_d s \right) \\ T_i &= 0.5P_{cr} = 1.405 & &= 18 \left(1 + \frac{1}{1.405s} + 0.35124s \right) \\ T_d &= 0.125P_{cr} = 0.35124 & &= \frac{6.3223(s + 1.4235)^2}{s} \end{aligned}$$

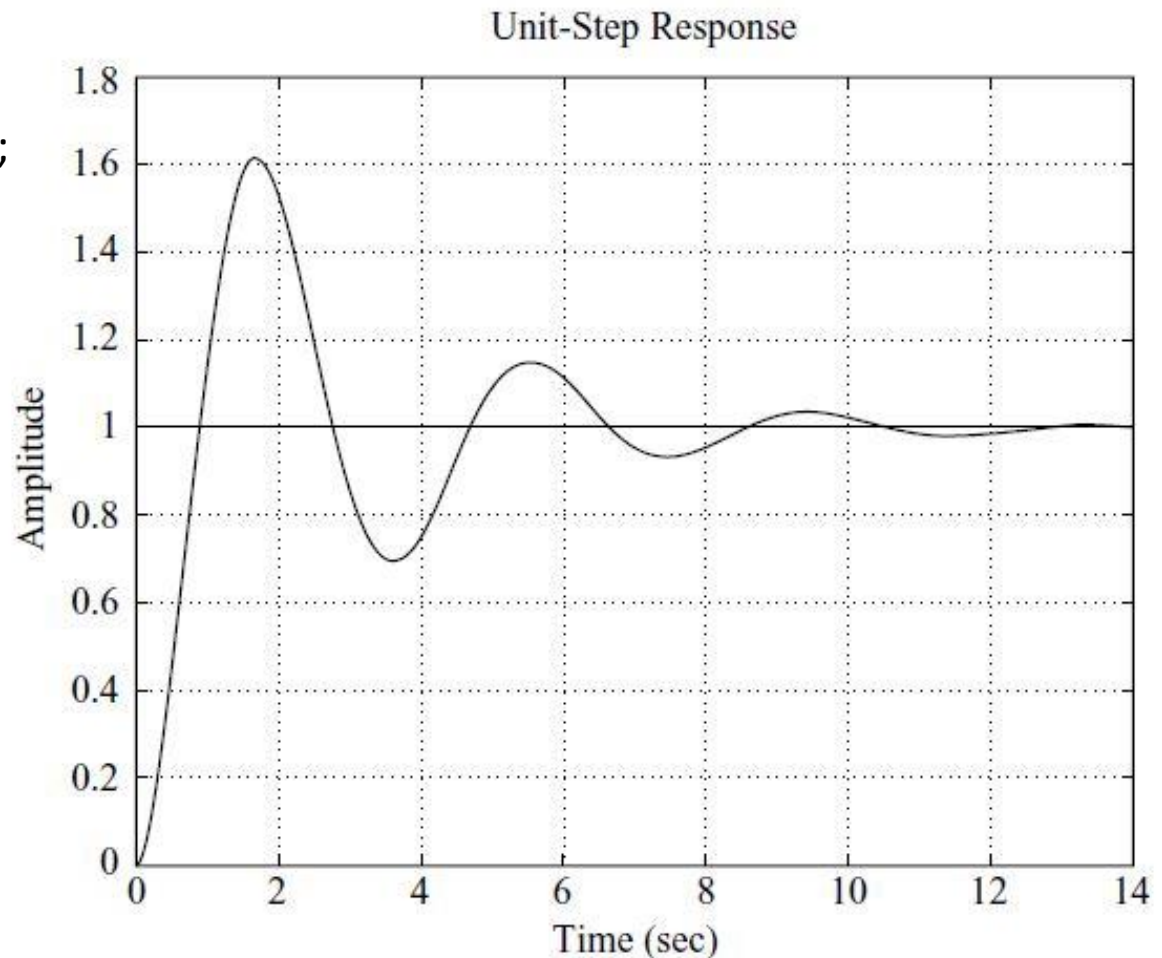


$$G_c(s) = \frac{6.3223(s + 1.4235)^2}{s}$$

%MatLab code

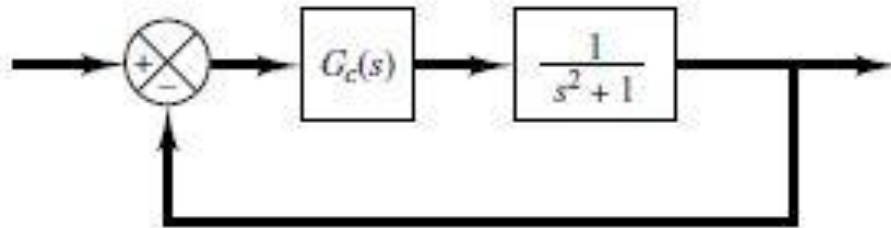
```
num = [6.3223 18 12.811];
den = [1 6 11.3223 18 12.811];
step(num,den)
grid
```

Note: The overshoot can be further reduced by increasing the gain K_p while keeping the double zero ($s = -0.65$), e.g. till $K_p = 39.42$ (numerically determined)



PID Design Based on Frequency Responses

Example: For a given plant, design a PID controller



$$G_c(s) = K \left(1 + \frac{1}{T_I s} + T_D s \right) = \frac{K_D s^2 + K_P s + K_I}{s}$$

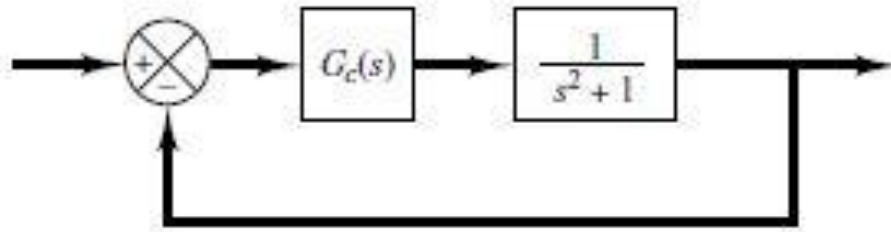
In this example, set $G_c(s) = \frac{K(as+1)(bs+1)}{s}$

$$\lim_{s \rightarrow 0} s \frac{K(as+1)(bs+1)}{s} \frac{1}{s^2+1} = K \quad (\text{static velocity error})$$

Now, suppose that the specifications are:

$K = 4$, gain margin > 10 dB, phase margin $> 50^\circ$

Need to determine a and b (to satisfy the two margins)



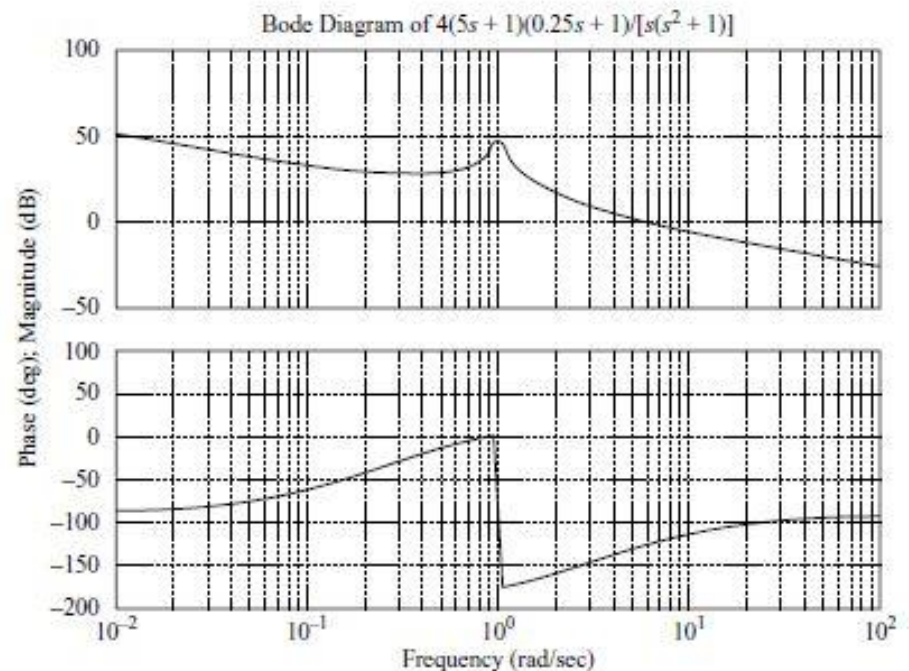
In steady state, $G_c(s) = K = 4$, so for step input, to find the gain margin and phase margin, consider the Transfer Function

$$KG(s) = \frac{4}{s(s^2 + 1)}$$

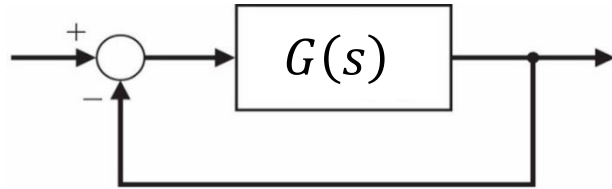
Using MatLab, with $a = 5$, $b = 0.25$

```
%MatLab code
num = [5 21 4];
den = [1 0 1 0];
w = logspace(-2,2,100);
bode(num,den,w)
title('Bode Diagram of
4(5s+1)(0.25s+1)/[s(s^2+1)]')
```

→ Specifications are satisfied



Frequency Responses of PI Control



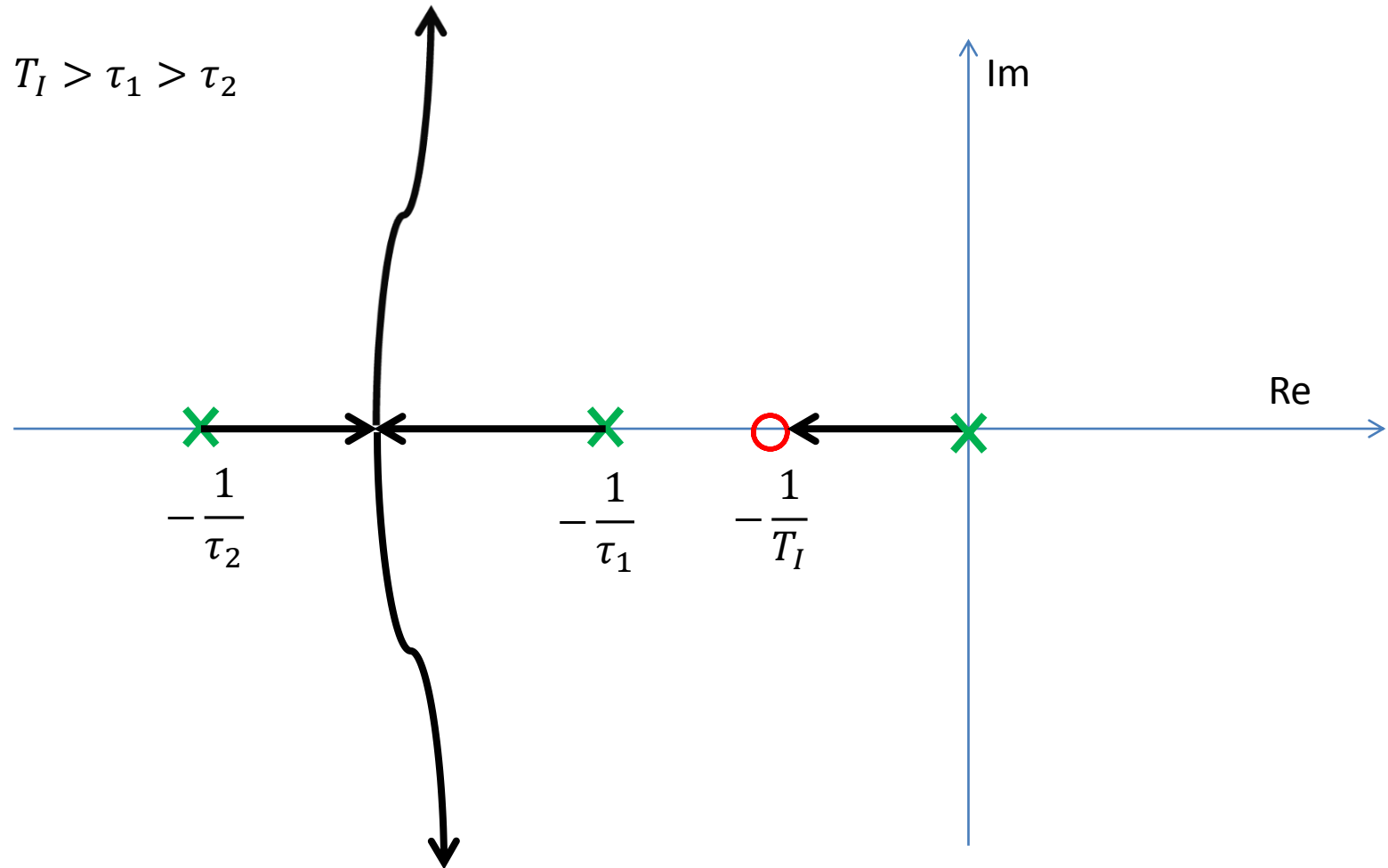
$$G(s) = \frac{\left(\frac{K_i}{sT_I} + K_p\right) A}{(\tau_1 s + 1)(\tau_2 s + 1)} = \frac{\left(\frac{K_i}{T_I} + sK_p\right) A}{s(\tau_1 s + 1)(\tau_2 s + 1)}$$

$$u(t) = K_p e(t) + \frac{K_i}{T_I} \int e(t) dt$$

$$C(s) = K_p + \frac{K_i}{sT_I} = K \left(1 + \frac{T_i}{s}\right)$$

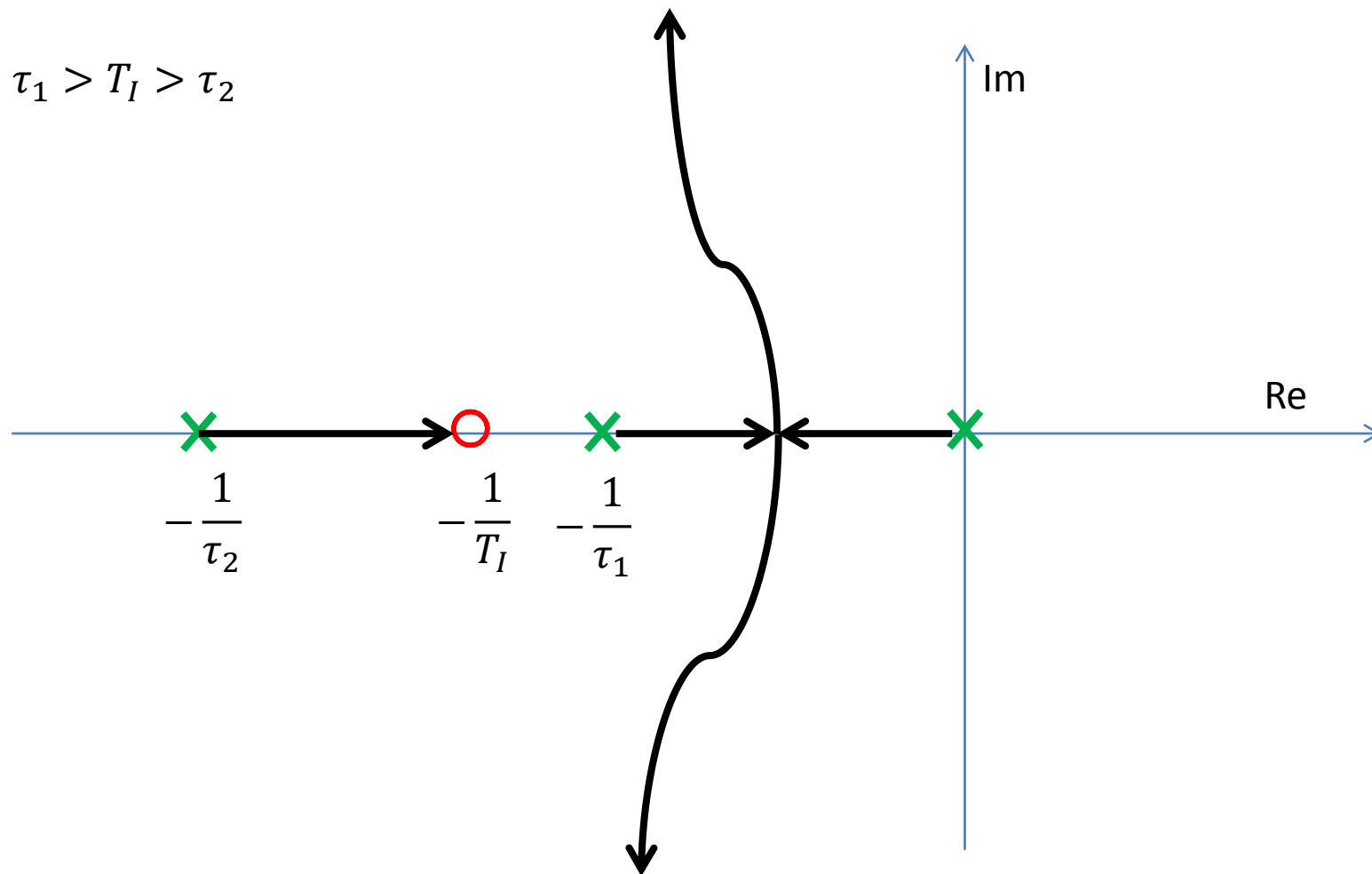
To determine what part of the locus is acceptable, use MatLab

Root Locus for DC Motor with *PI*-Control

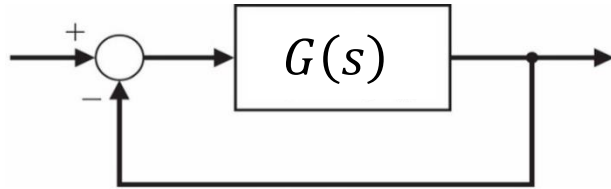


MatLab plot shows all possible closed-loop pole locations for the control gain K : $0 \rightarrow \infty$

$$\tau_1 > T_I > \tau_2$$



Frequency Response of PD Control



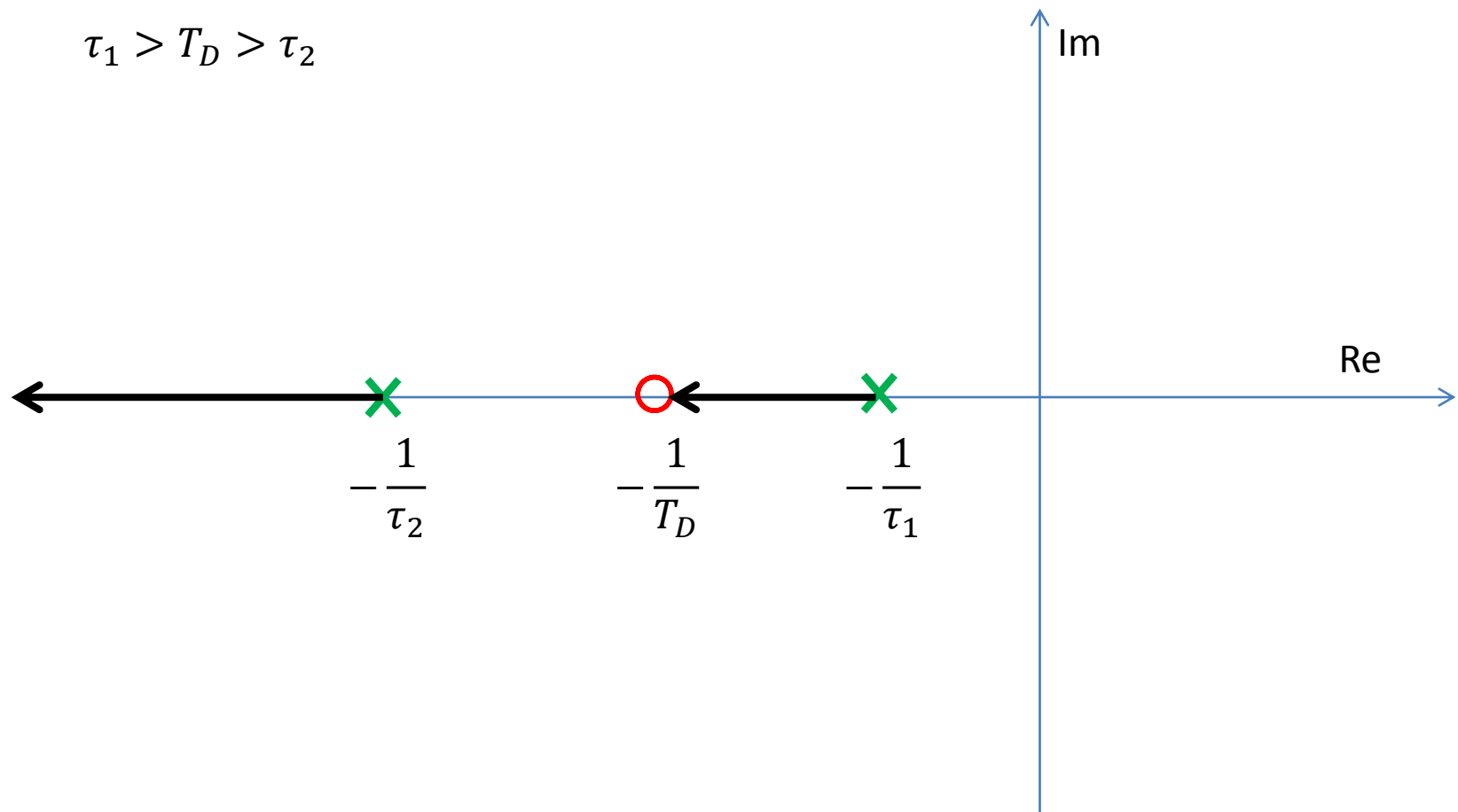
$$G(s) = \frac{(K_p + sK_d T_D)A}{(\tau_1 s + 1)(\tau_2 s + 1)}$$

$$u(t) = K_p e(t) + K_d T_D \frac{de(t)}{dt}$$

$$C(s) = K_p + sK_d T_D = K(1 + sT_d)$$

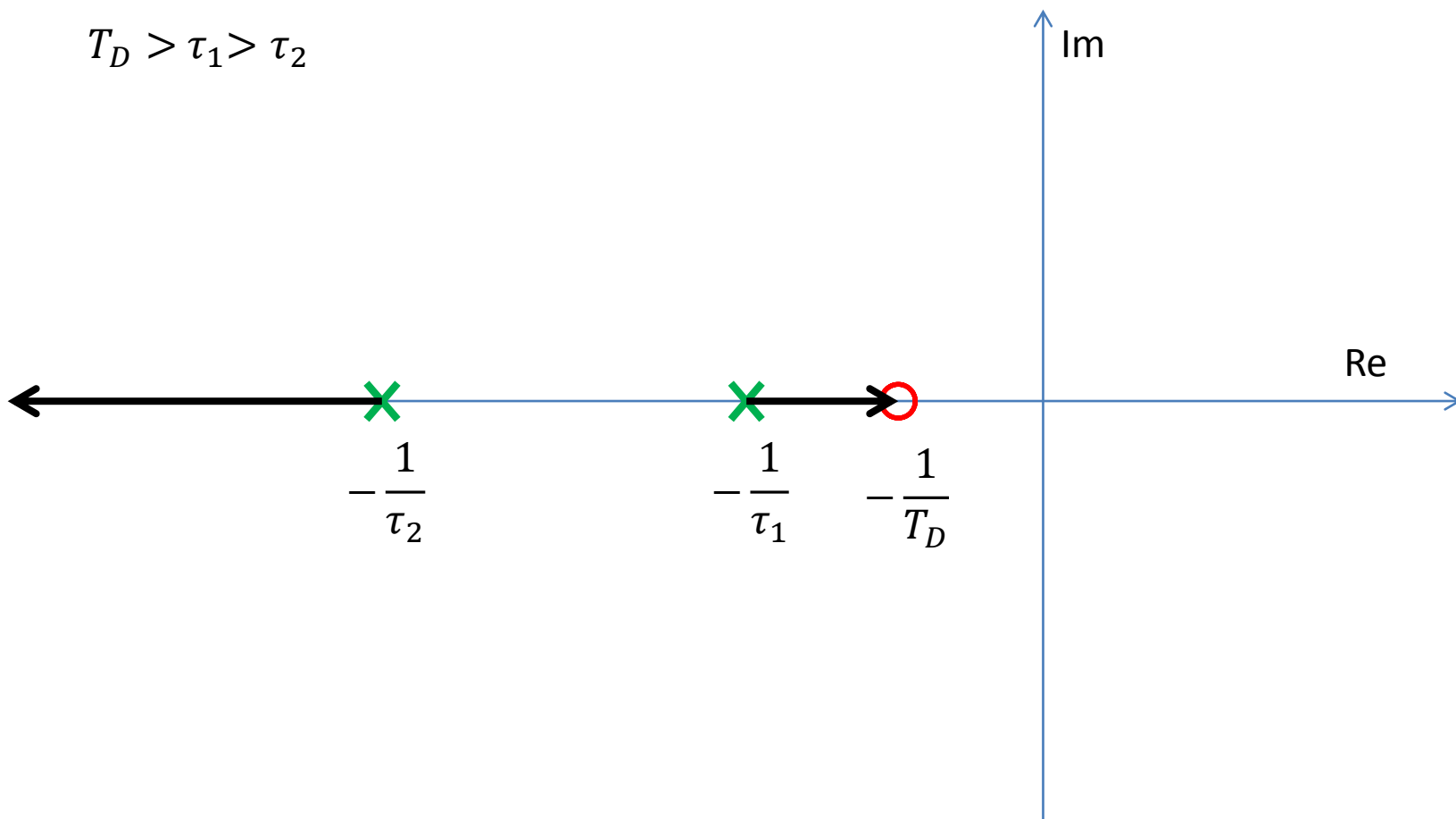
To determine what part of the locus is acceptable, use MatLab

Root Locus for DC Motor with *PD*-Control

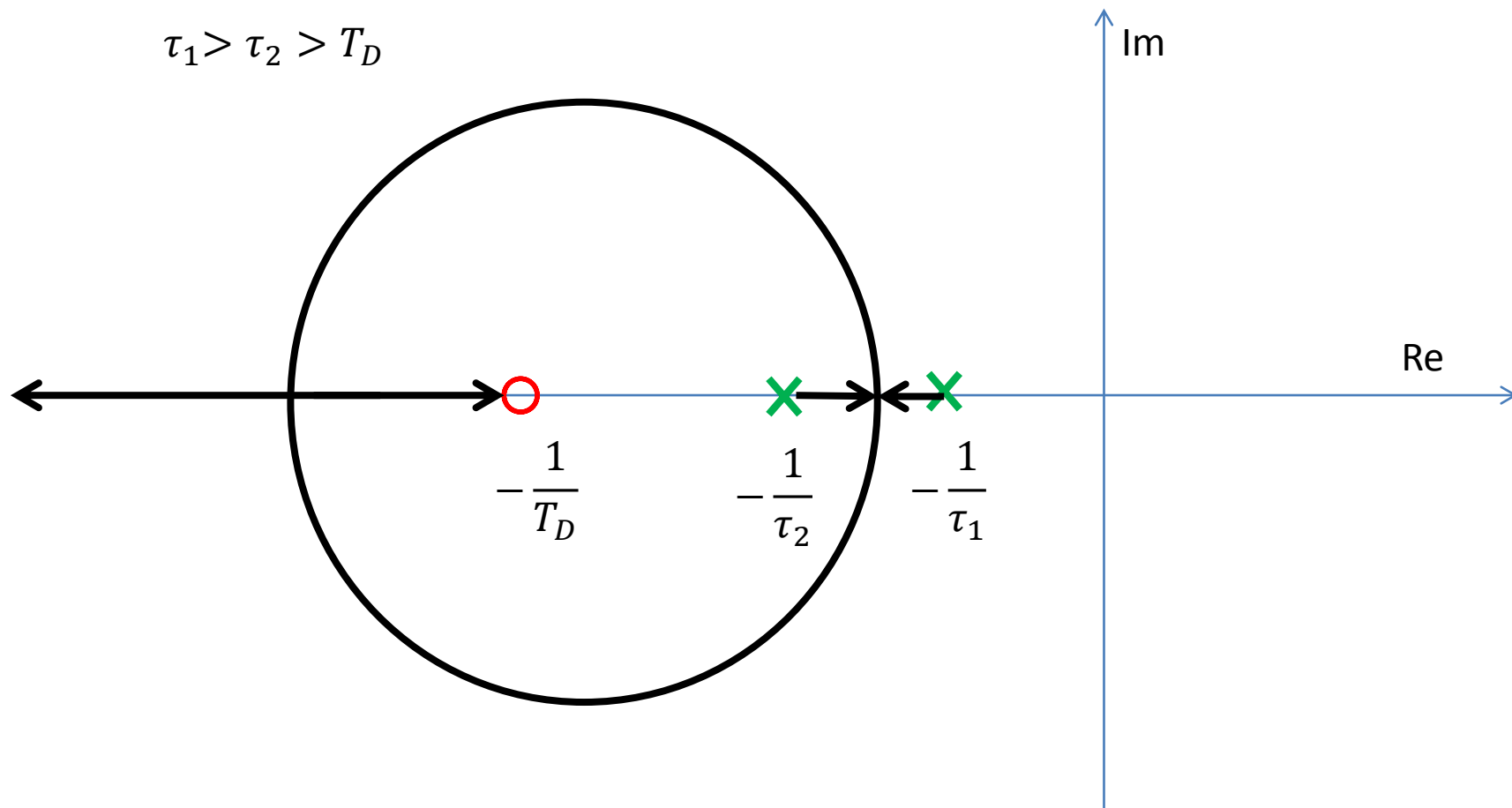


MatLab plot shows all possible closed-loop pole locations for the control gain K : $0 \rightarrow \infty$

$$T_D > \tau_1 > \tau_2$$



$$\tau_1 > \tau_2 > T_D$$



END

